



# Signal processing: a networking perspective

## Part 2 – option A

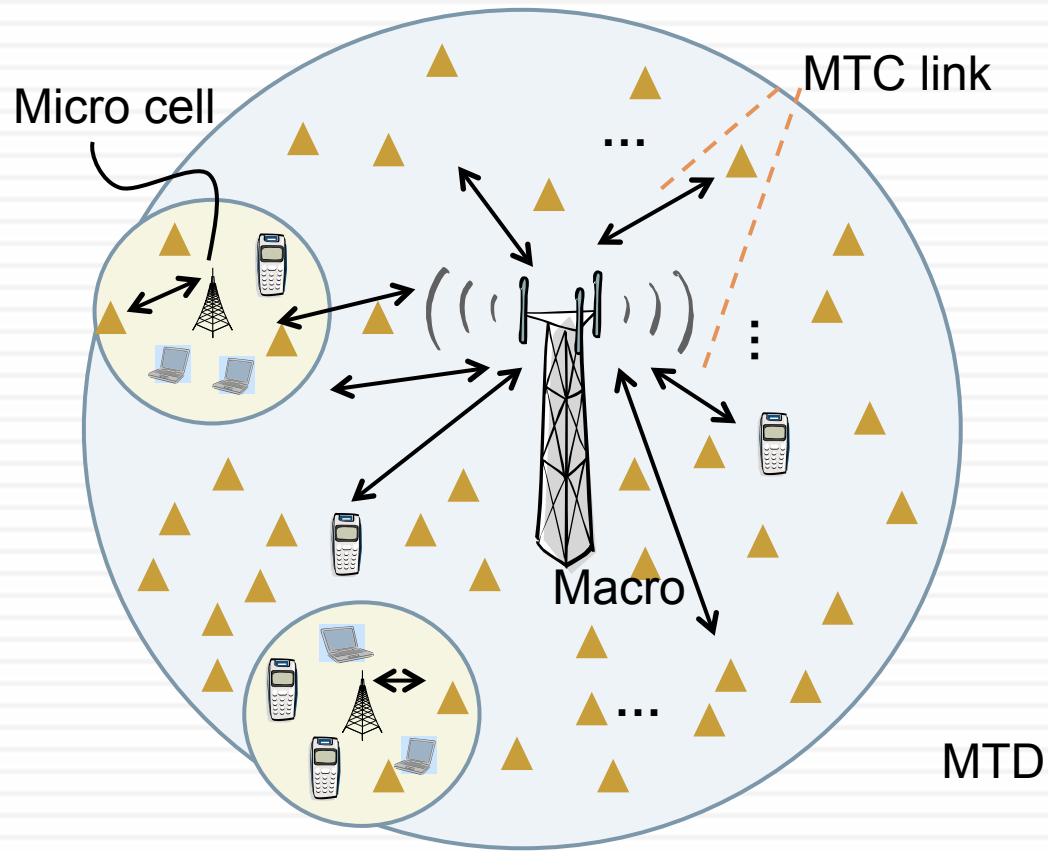
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**SIG**nals processing &  
**NET**working research group

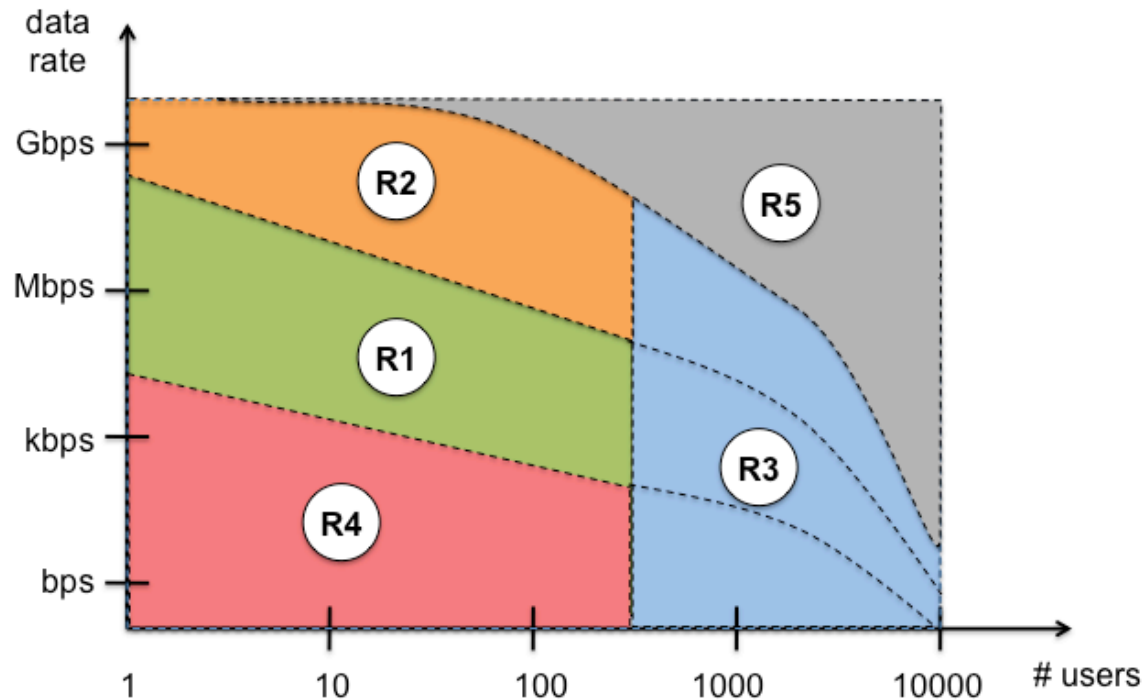
- **Part 2: Signal processing at PHY/MAC layers**
  - The challenge of massive M2M access
    - RFTag counting and identification
  - Interference models & system capacity
    - Multi-packet reception and Successive Interference Cancellation



## The challenge of massive M2M access



# The shape of wireless to come



**R1:** today's systems

**R2:** high-speed versions of today's systems

**R3:** massive access for sensors and machines

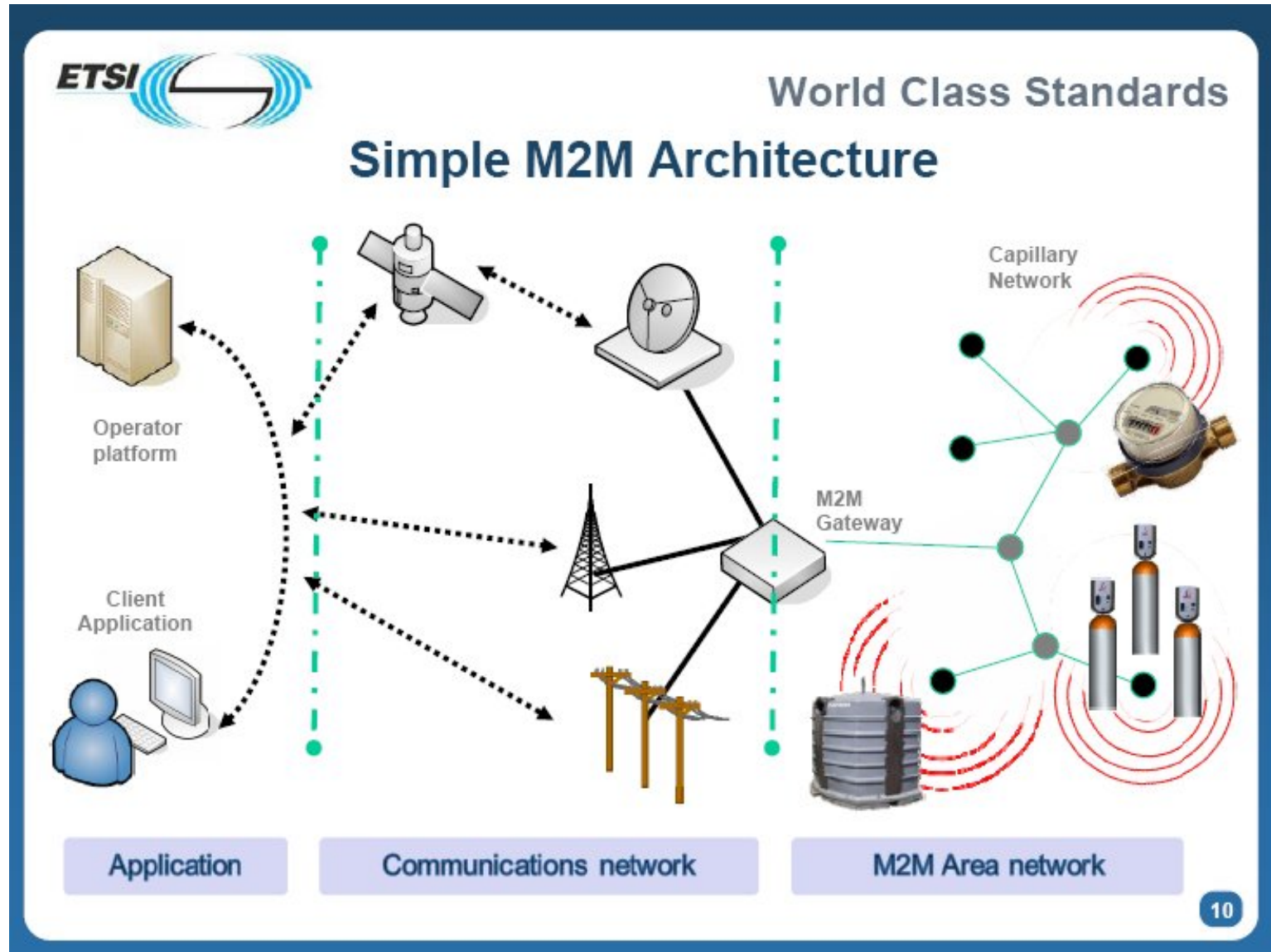
**R4:** ultra-reliable connectivity at minimal rate

**R5:** physically impossible

$$5G = R1 + R2 + R3 + R4$$



# M2M reference architecture





# Machine Network Traffic

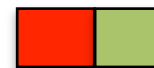
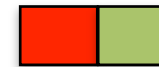
- M2M devices generate traffics of the following types
  - ▣ Periodic: smart metering application
  - ▣ Event-driven: emergency event report
  - ▣ Continuous: surveillance camera
- Large volume of different types of traffic at core network
  - ▣ Guarantee of diverse QoS traffic requirements
  - ▣ Reliability of both human-to-human and M2M traffic

# high-speed wireless vs. M2M

- high-speed systems built from information-theoretic principles with **small control info** and **large data**



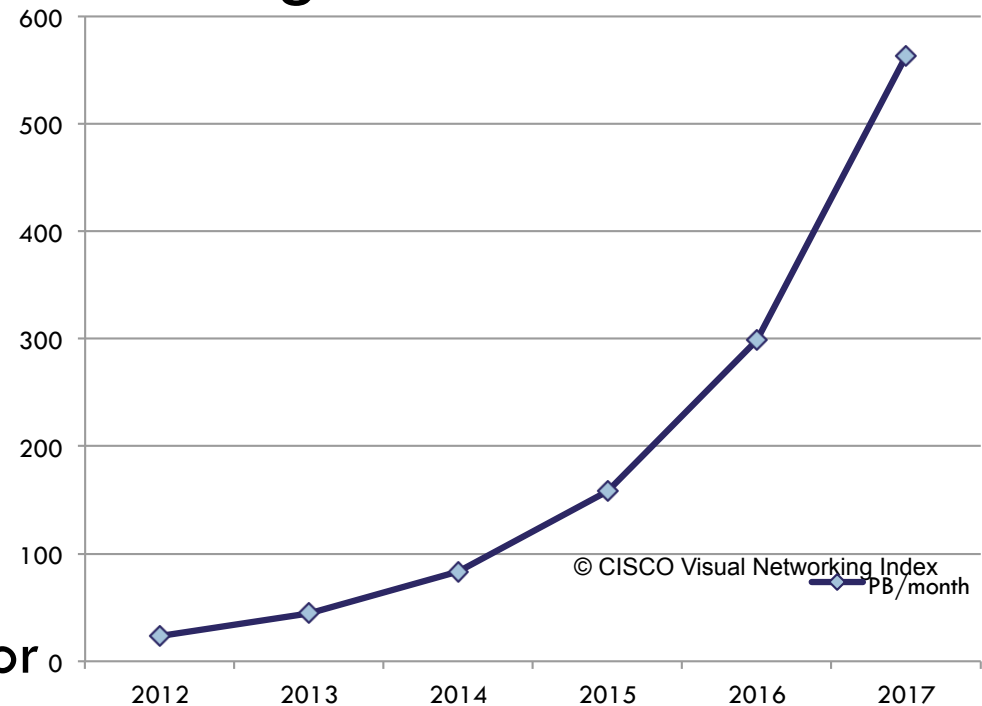
- M2M require short data packets from massive number of devices each transmitting sporadically



# enormous M2M growth expected

- 24-fold traffic growth from 2012 to 2017
- 4.6-fold growth of M2M #subscriptions
  - from 369 million in 2012 to 1,7 billion in 2017
- M2M traffic will account for approximately 5 % of overall mobile traffic in 2017

## global M2M traffic





  
 DIPARTIMENTO  
DI INGEGNERIA  
DELL'INFORMAZIONE

# The batch resolution problem



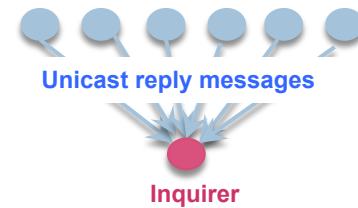
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research group

# Problem statement

- What's a "batch"?
  - Set of mutually interfering nodes simultaneously solicited to send a packet
    - RF tags illuminated by a reader
    - Wireless nodes that reply to neighbour-discovery request
    - Mobile terminals that compete to reserve a channel slot
- What's the "Batch resolution problem"?
  - Simultaneous transmissions by multiple nodes result into collision → all packets are lost!
  - Nodes need to arbitrate the channel access in order to transmit their packet avoiding collisions
    - A node that successfully transmits is said to be **resolved**
- What's a "Batch Resolution Algorithm" (BRA)?
  - The BRA arbitrates the channel access in order to minimizing the batch resolution interval, ie, the mean time required to resolve all the nodes in the batch

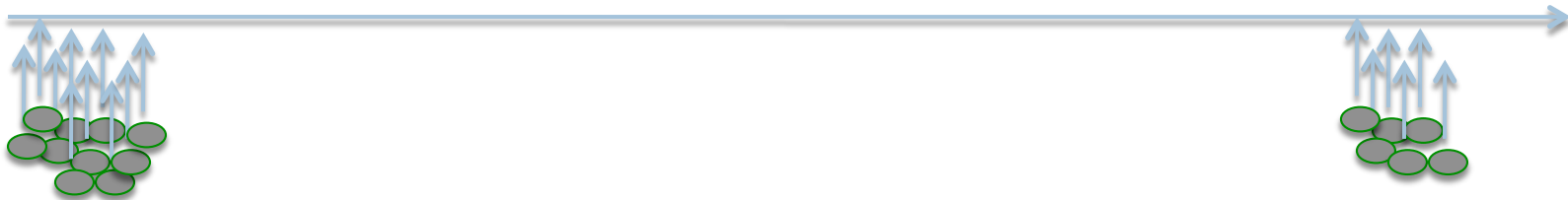


Solicited nodes form the "batch"



# Batch resolution problem vs MAC

- The batch resolution problem resembles the MAC problem but...
  - ▣ MAC protocols generally look at the channel contention as a *steady-state phenomenon*
  - ▣ BRAs address scenarios where contention has a *bursty nature*





# Performance measures

- Batch resolution interval (BRI)
  - ▣  $T(n) = E[\text{time required to resolve a batch of size } n]$
- Batch Throughput

$$\lambda(n) = \frac{n}{T(n)}$$

- Asymptotic throughput  $\Lambda = \lim_{n \rightarrow \infty} \lambda(n)$ 
  - ▣ Maximal sustainable arrival rate when BRA is used as *obvious MAC*:
    - solve a batch and queue packets arriving in the meanwhile, the form the next batch (gate policy)



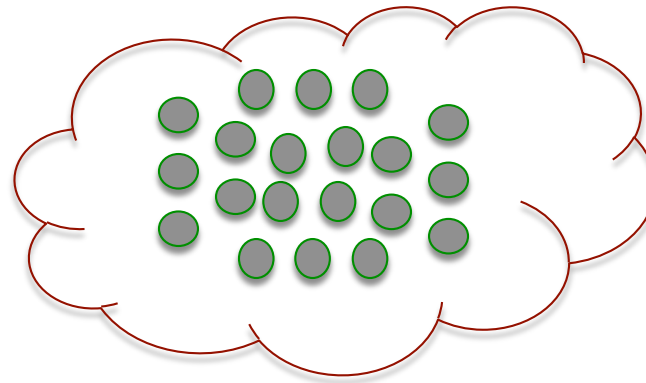


# Splitting-tree BRAs

- Time is slotted
  - Slots may have unequal duration in CSMA networks
- In each slot, some nodes are **activated**, i.e., transmit
- **Immediate feedback**: returned after each slot
  - **Idle** slot: no nodes transmit
  - **Successful** slot: a single node transmit
  - **Collided** slot: two or more nodes transmit
- BRA works recursively, driven by feedback



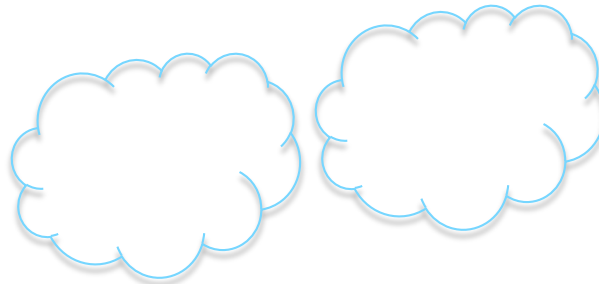
# Divided batch in “m” subgroups



Initial batch  
(expected size  $m_n$ )



Subgroup 1



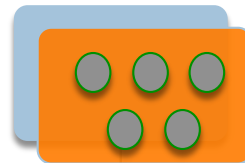
Subgroup 2



Subgroup m

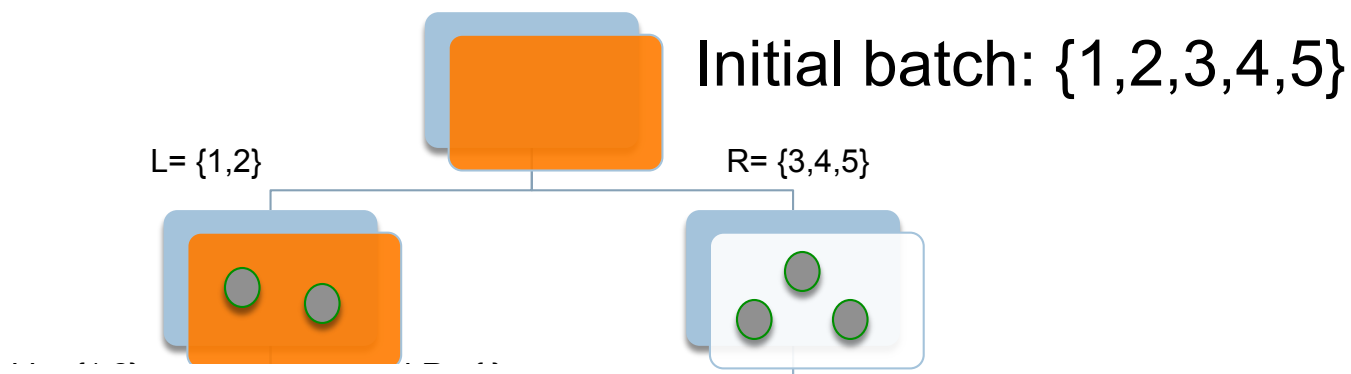
(expected size  $g = m_n/m$  optimal)

# Slot 1: collided

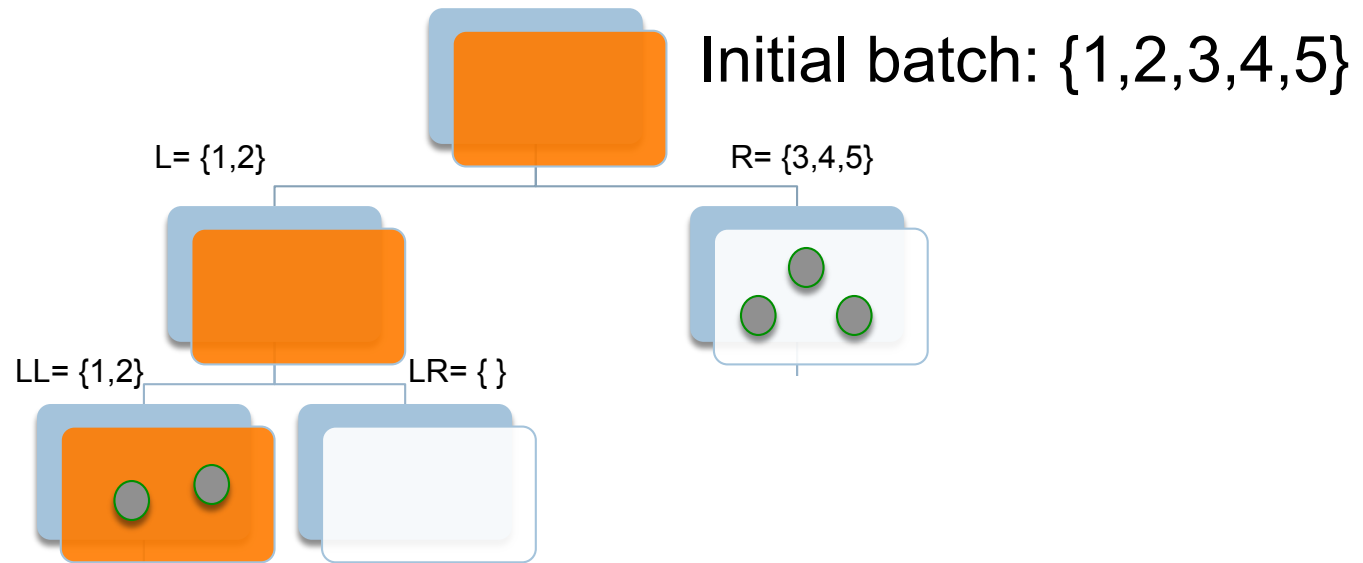


Initial batch: {1,2,3,4,5}

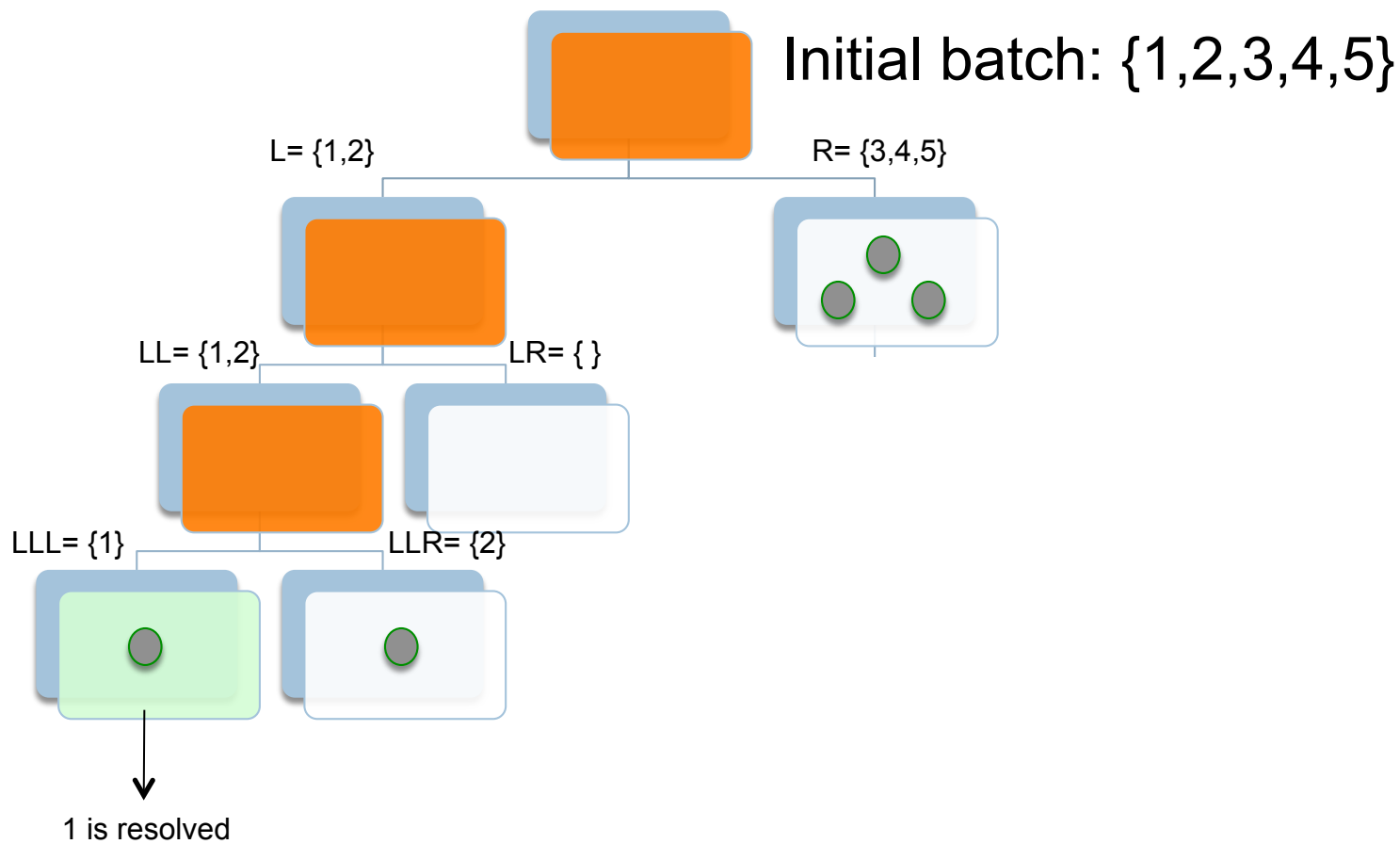
# Slot2: collided



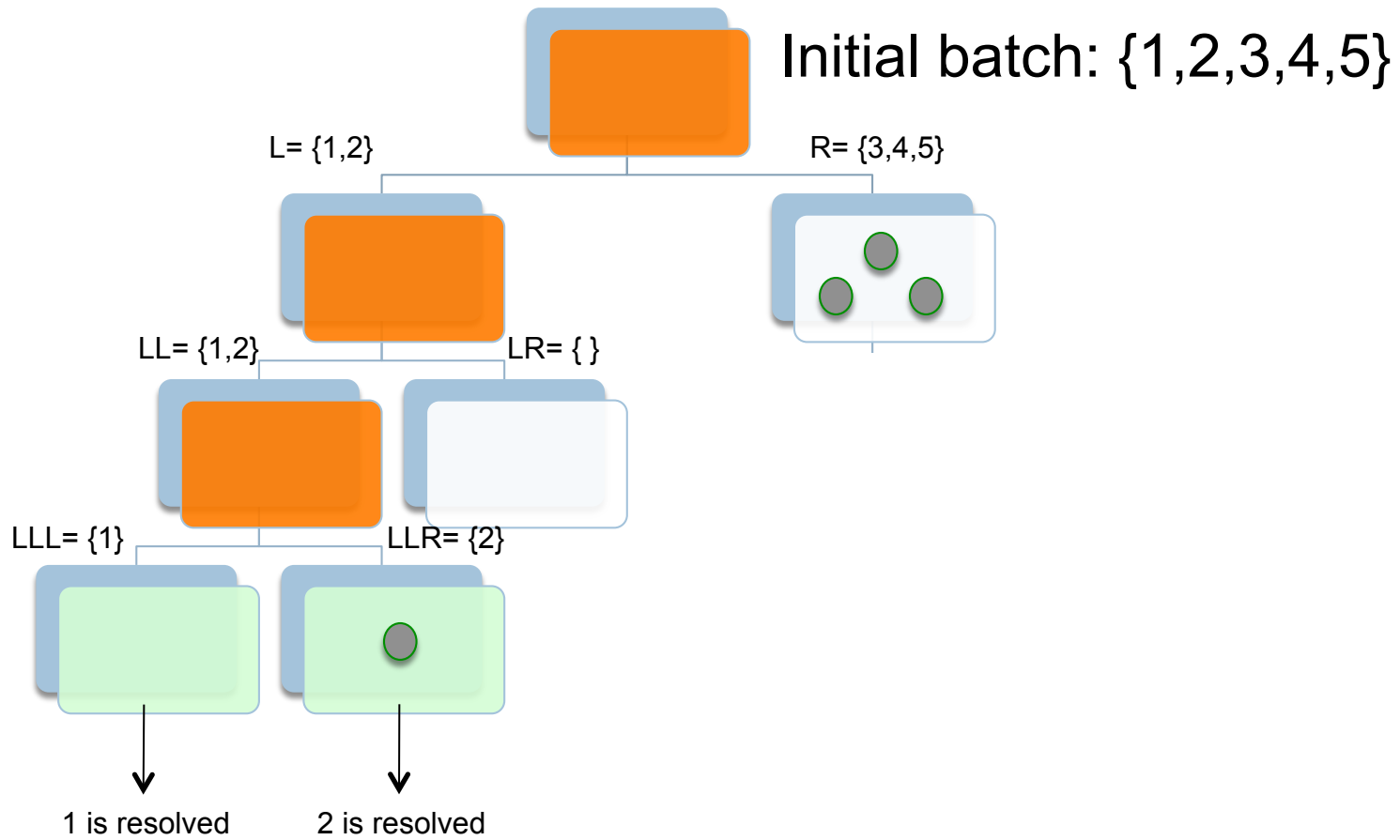
# Slot3: collided



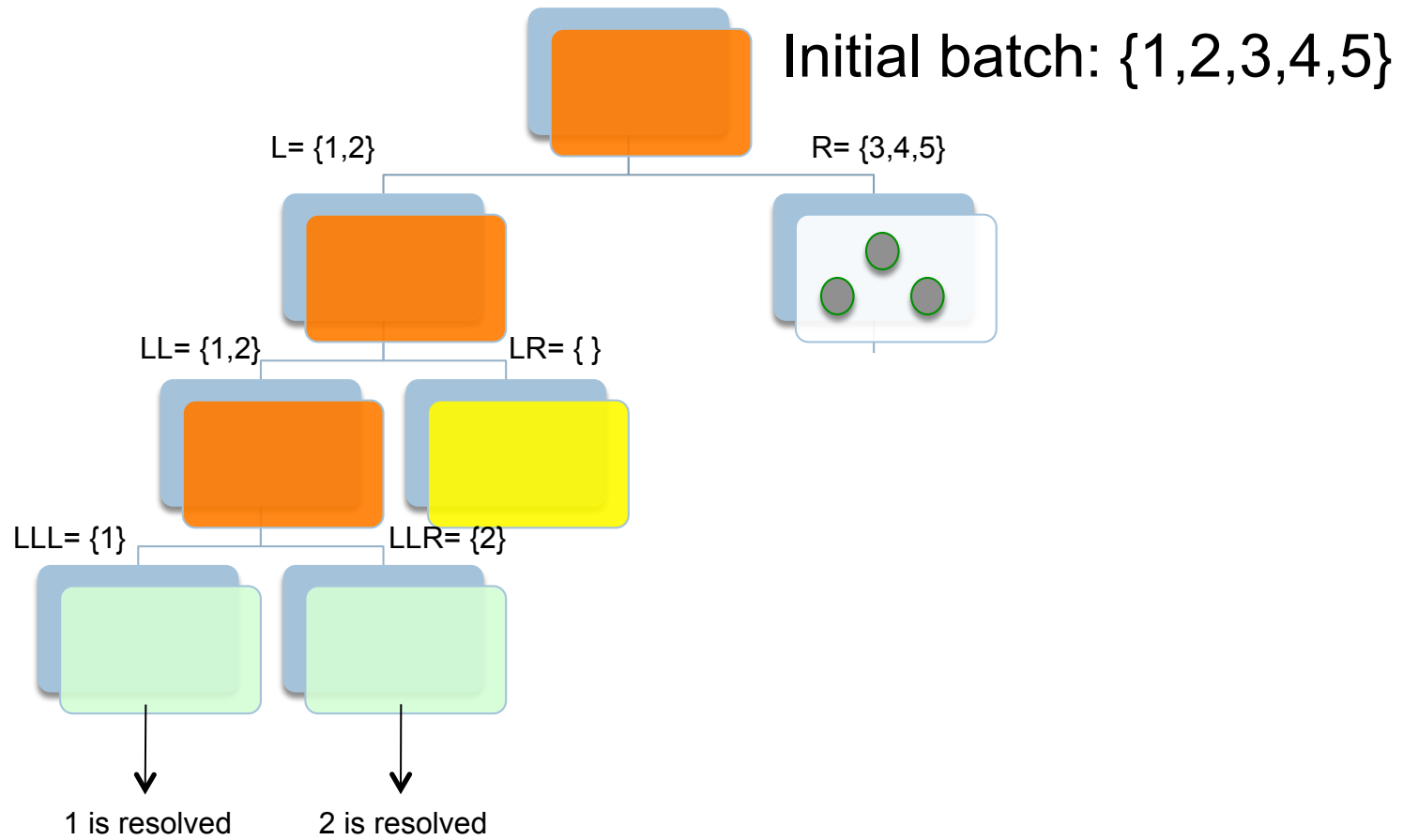
# Slot4: successful



# Slot5: successful

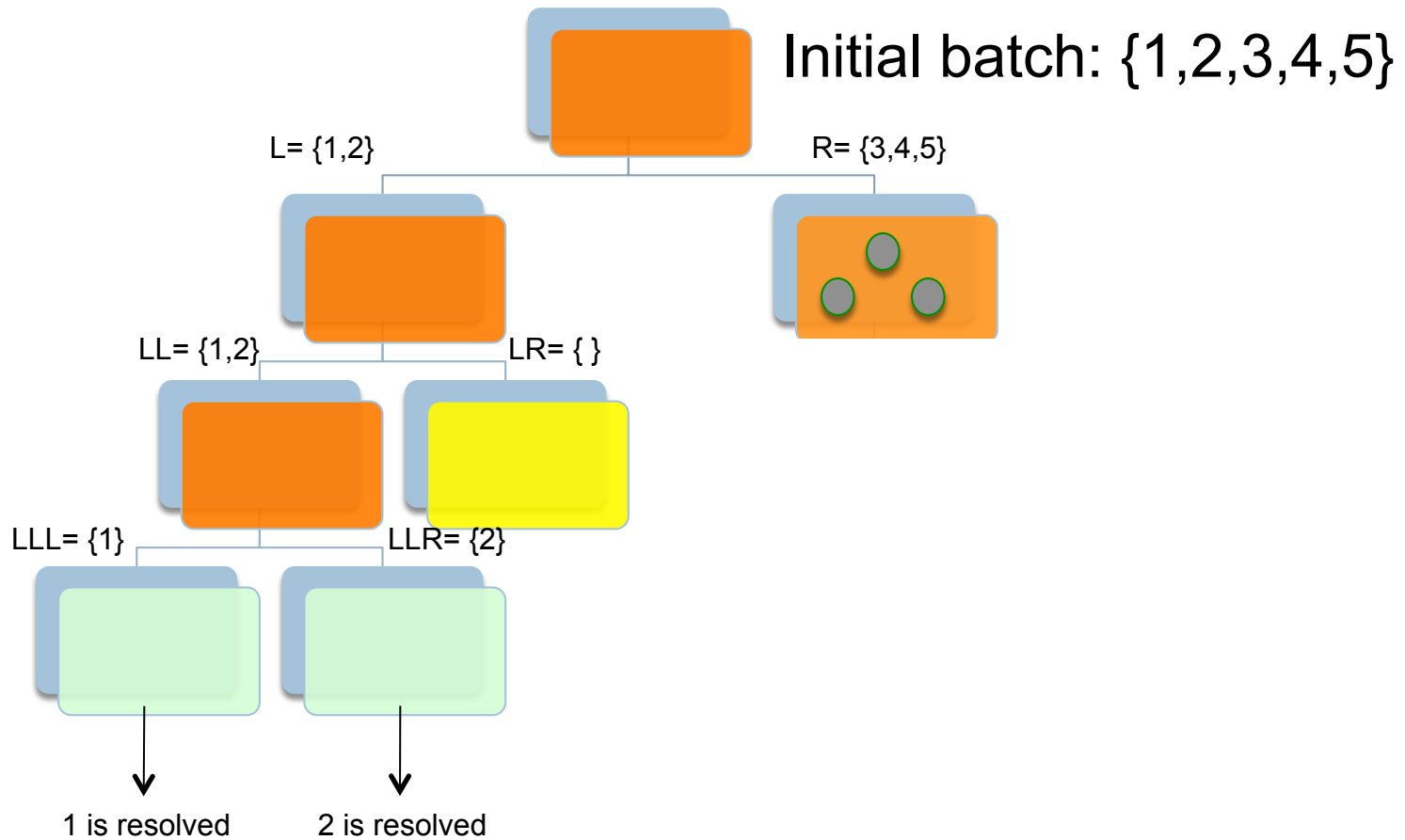


# Slot6: idle

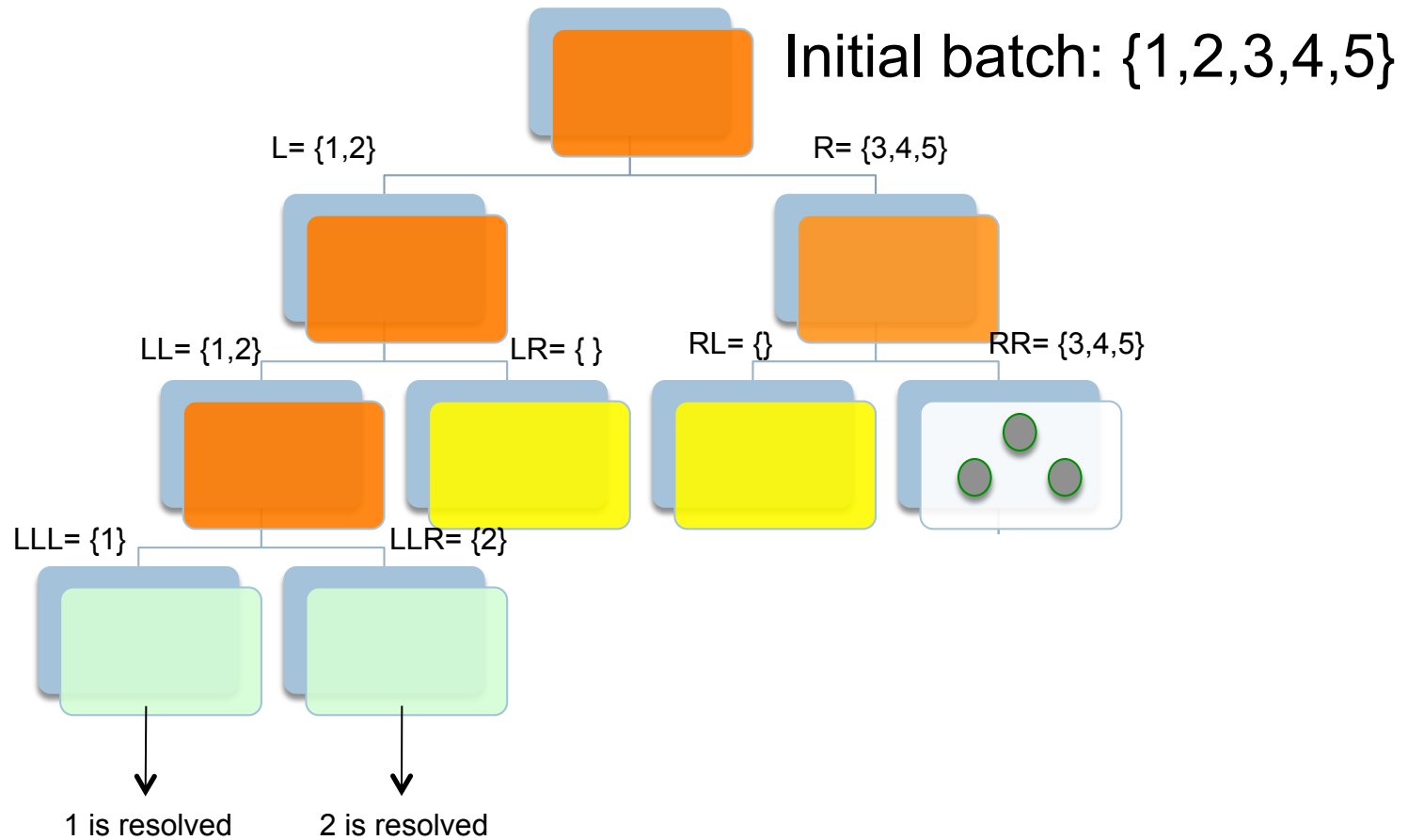




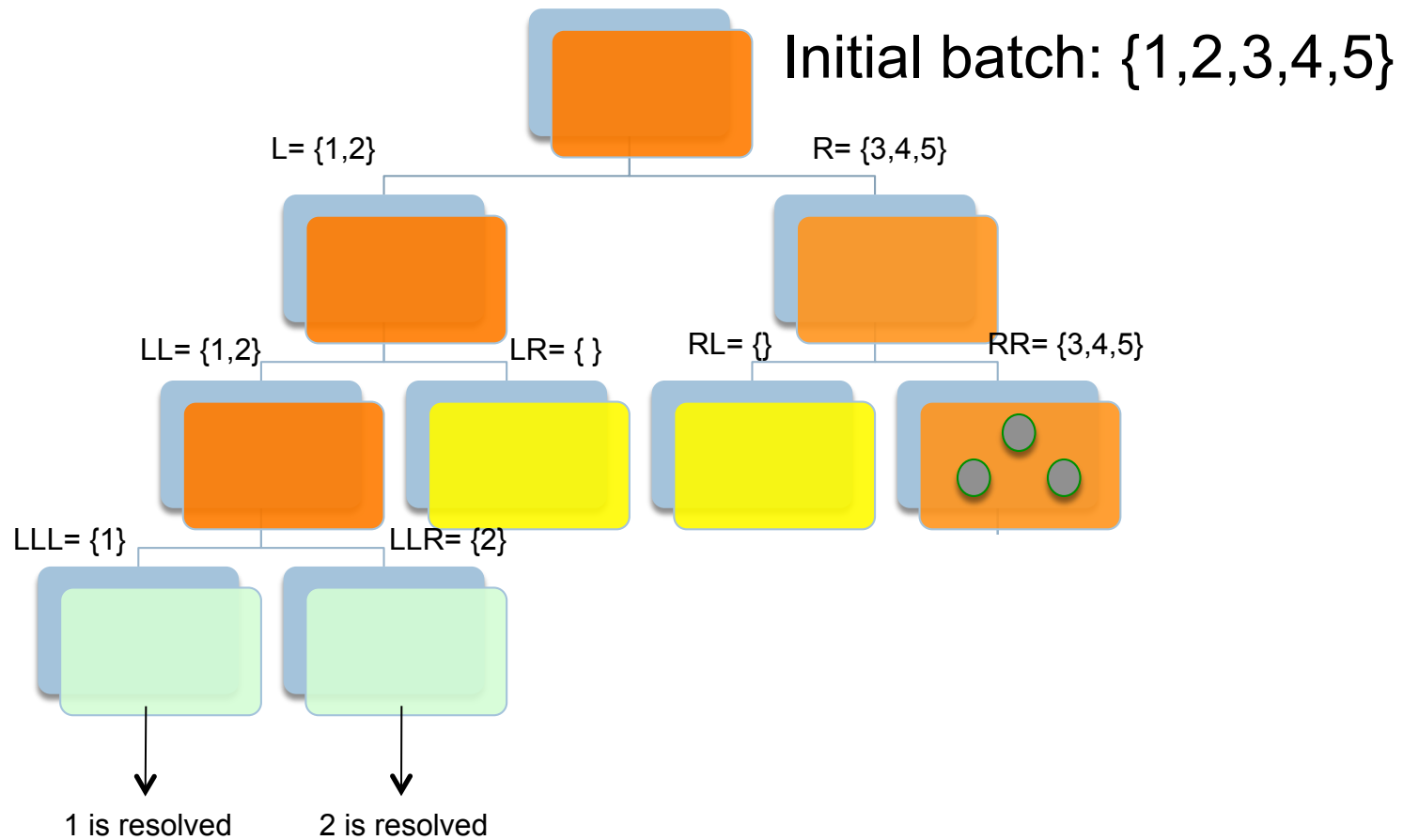
# Slot7: collided



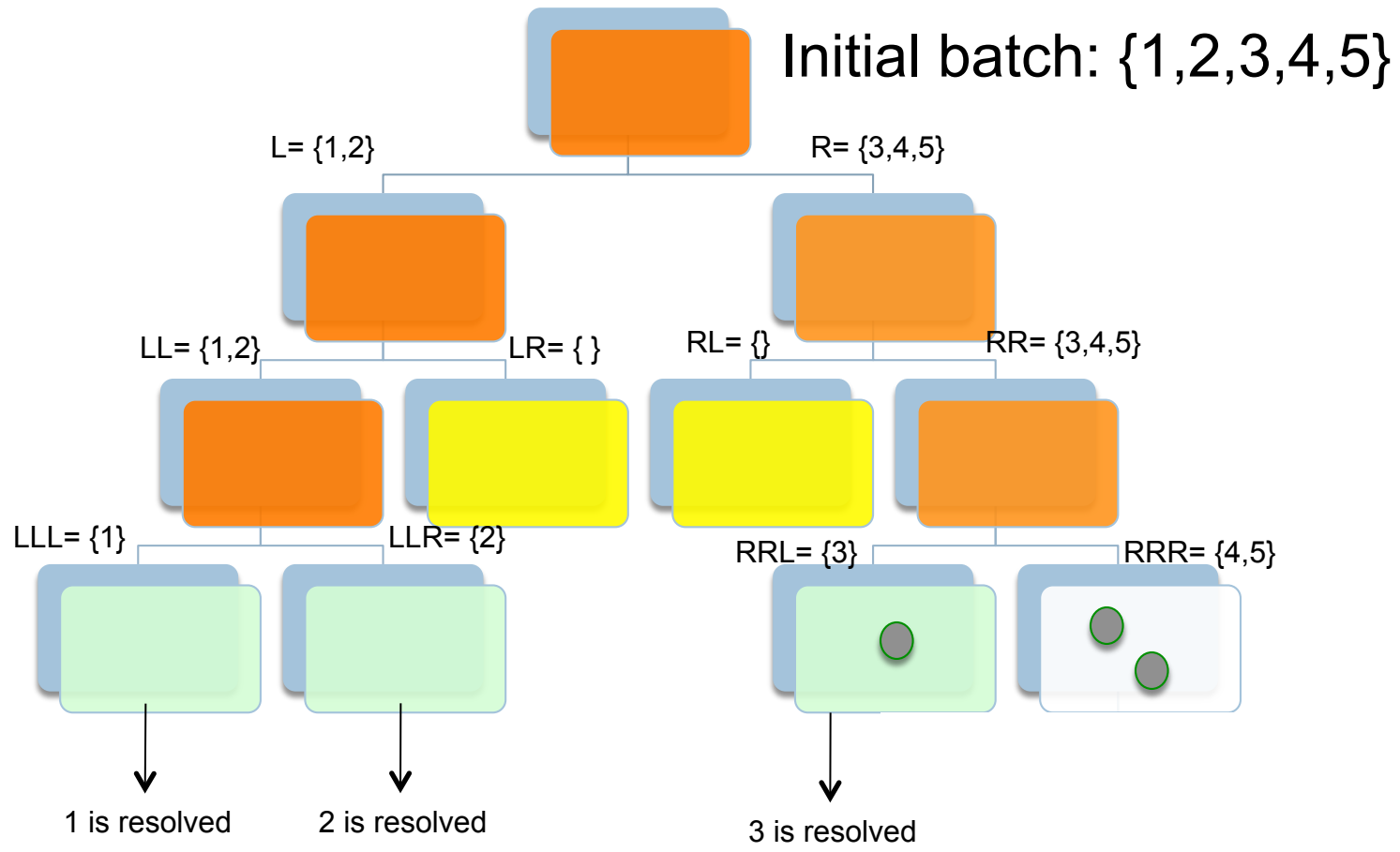
# Slot8: idle



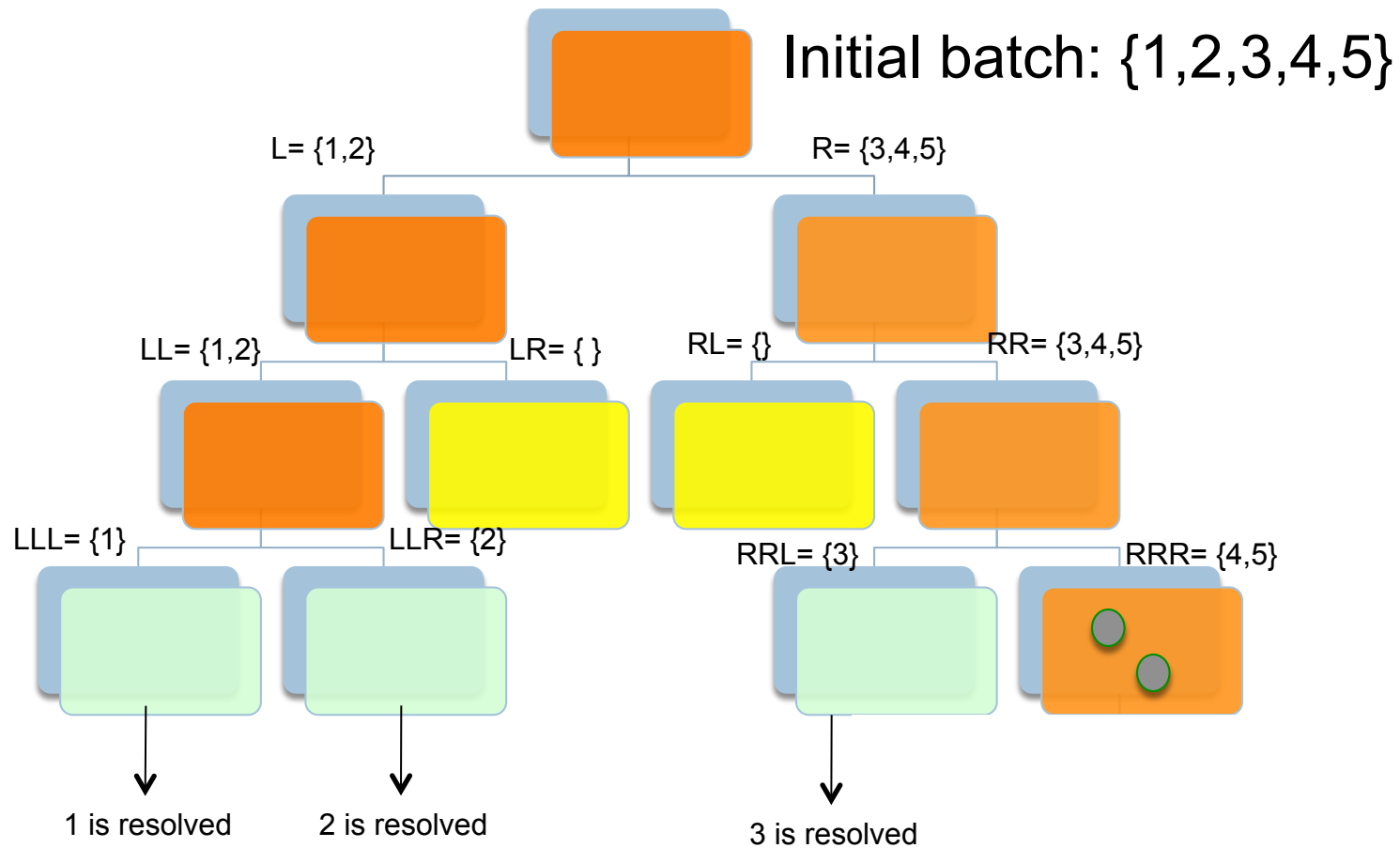
# Slot9: collided



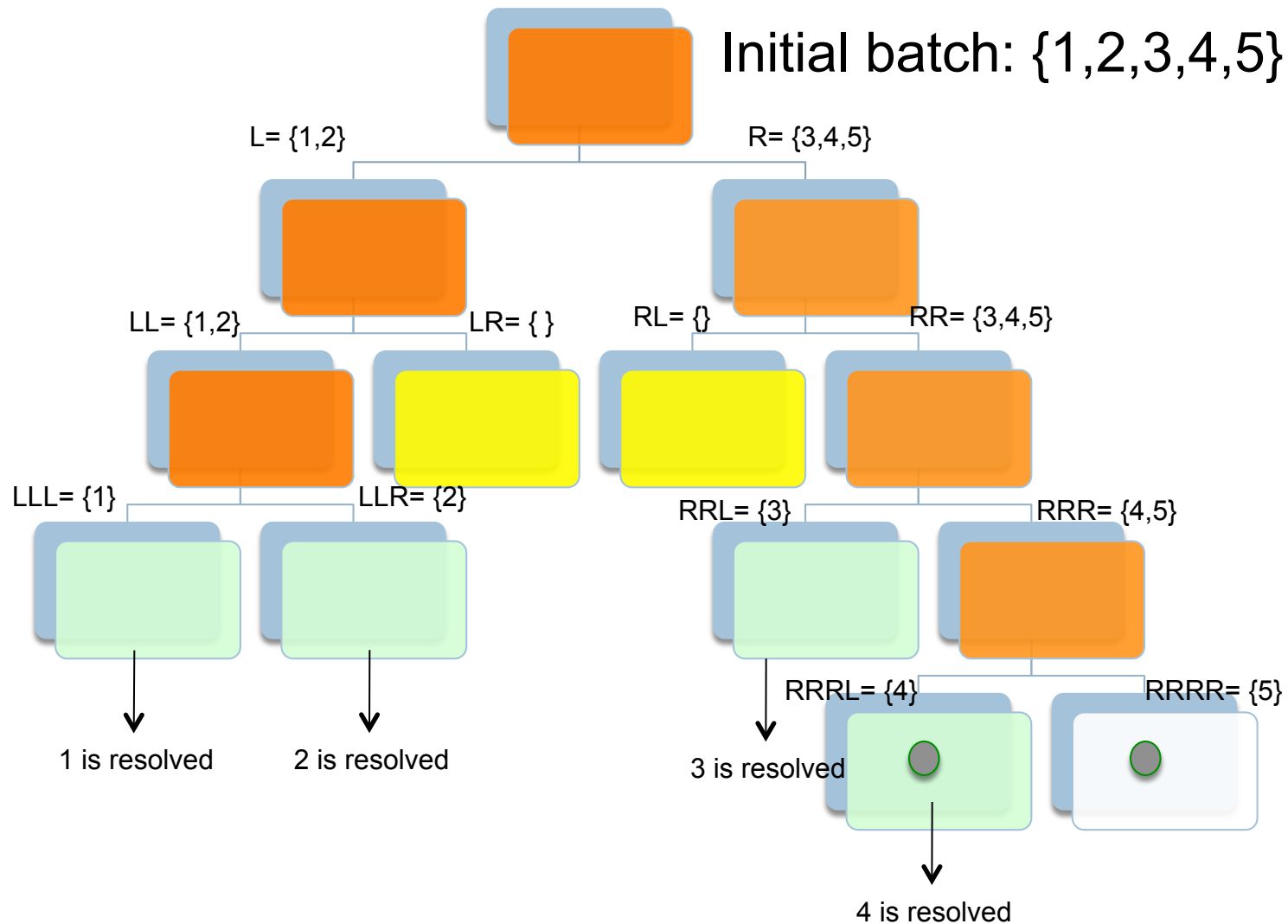
# Slot 10: successful



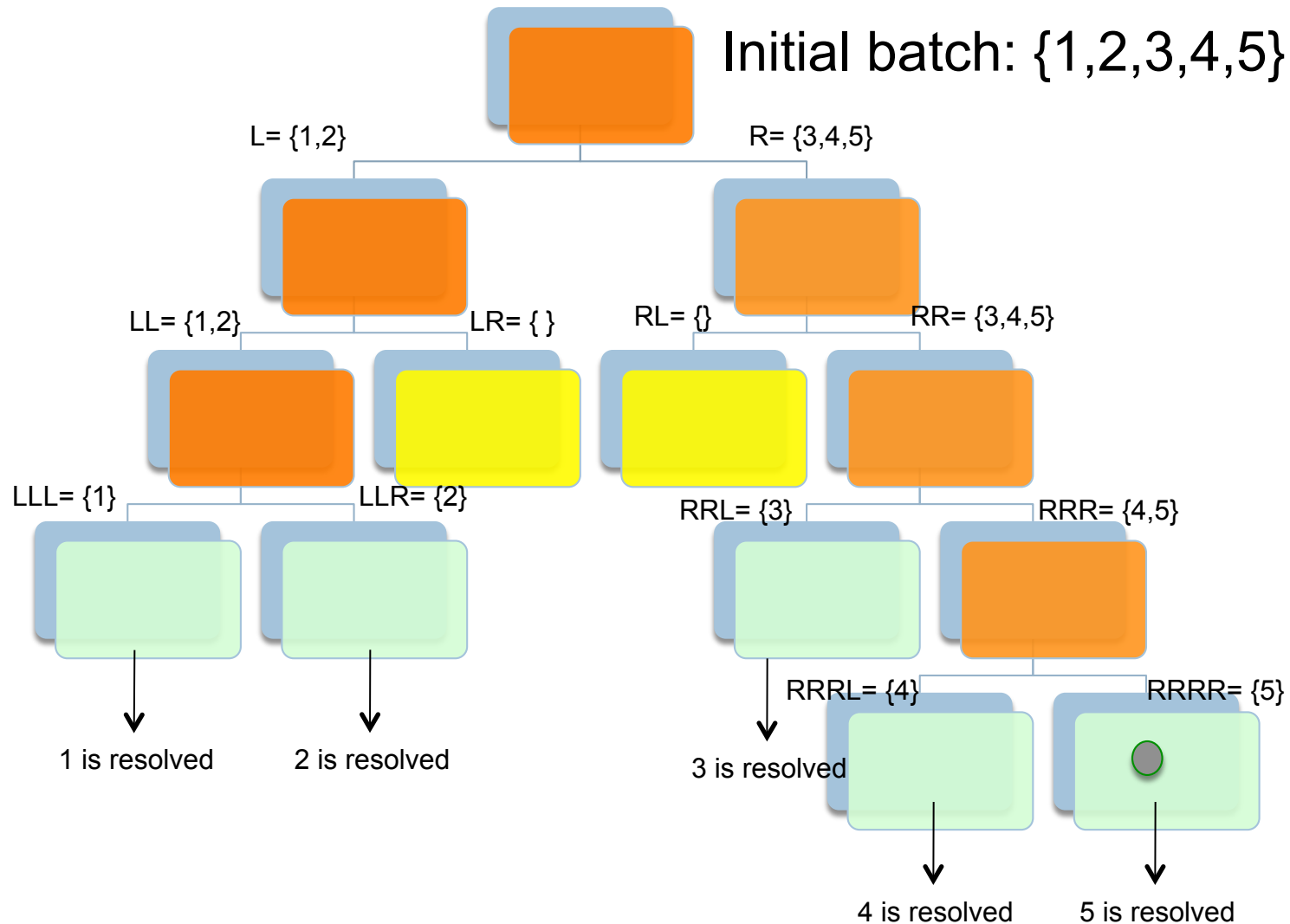
# Slot 1 1: collided



# Slot 1 2: successful

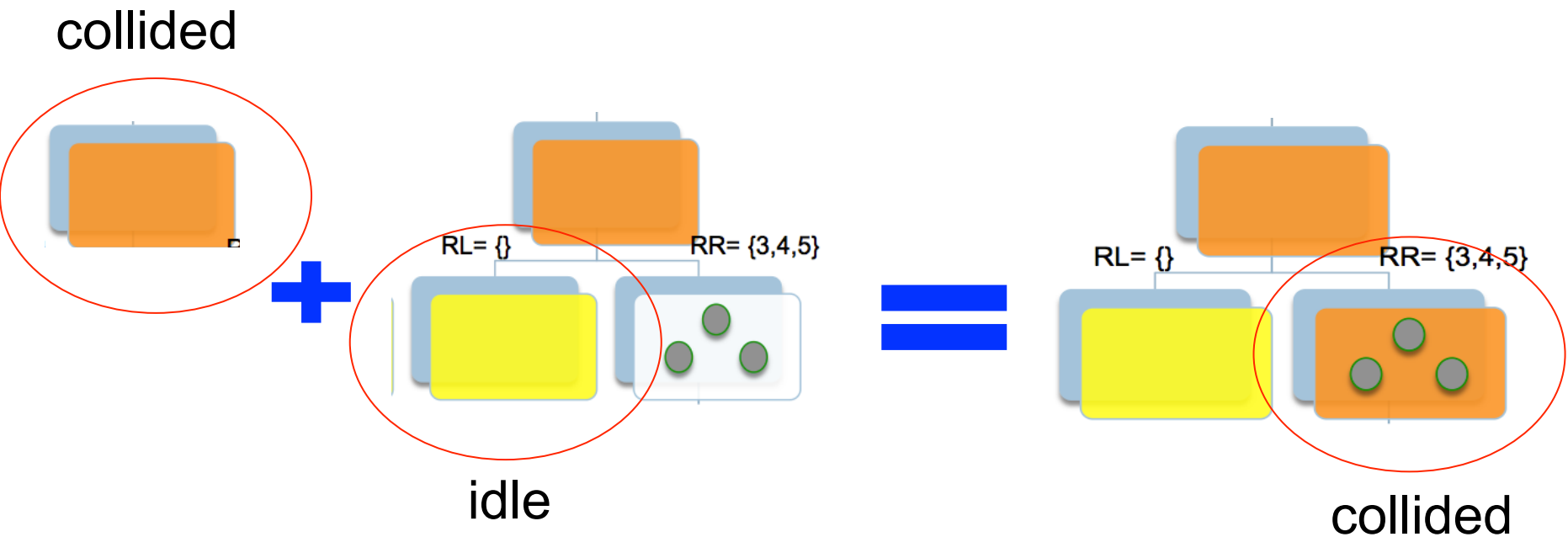


# Slot 1 3: successful



# Modified Binary Tree

- Avoid predictable collision
  - ▣ A “collided  $\rightarrow$  idle” sequence is always followed by a collided slot!



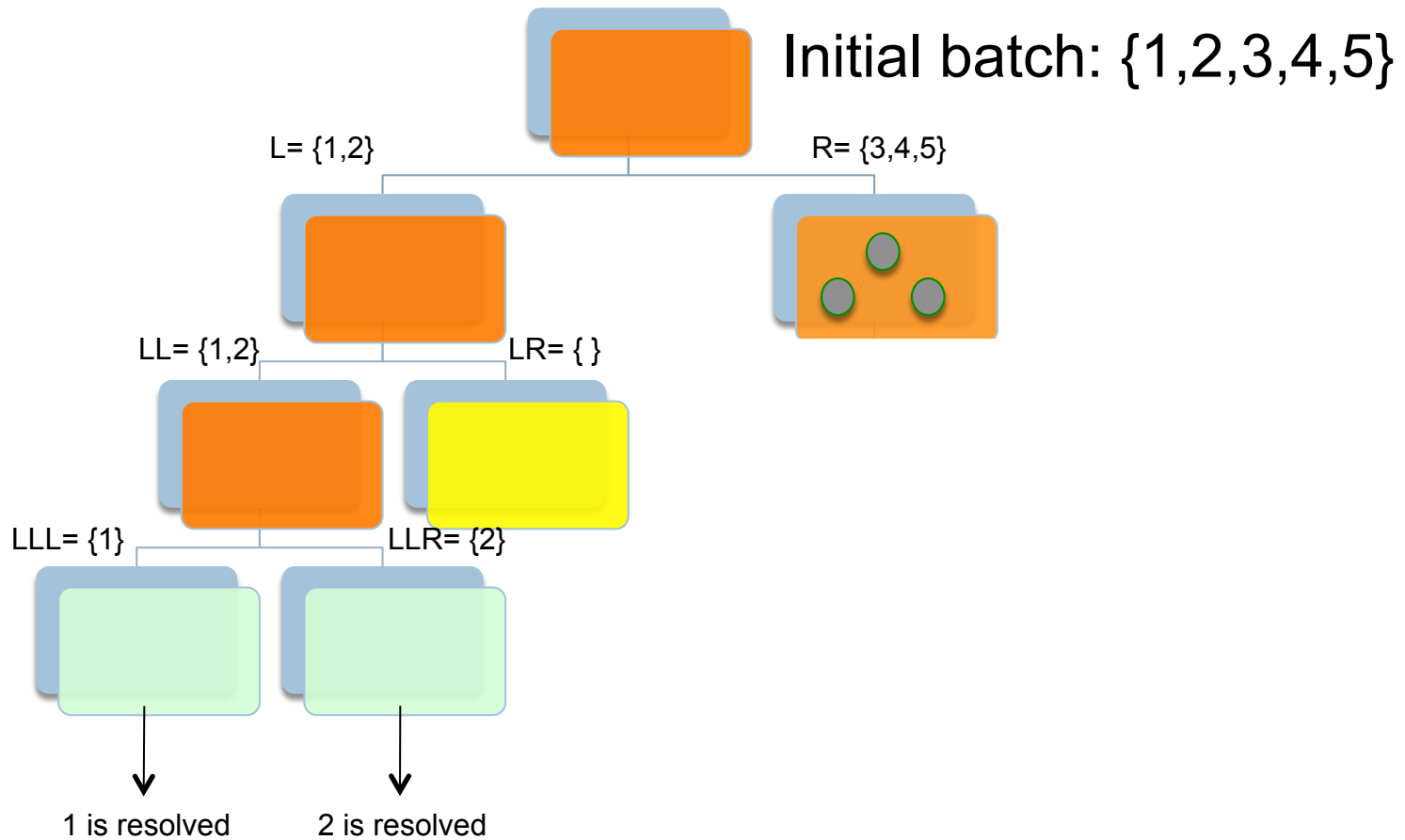




# Modified Binary Tree

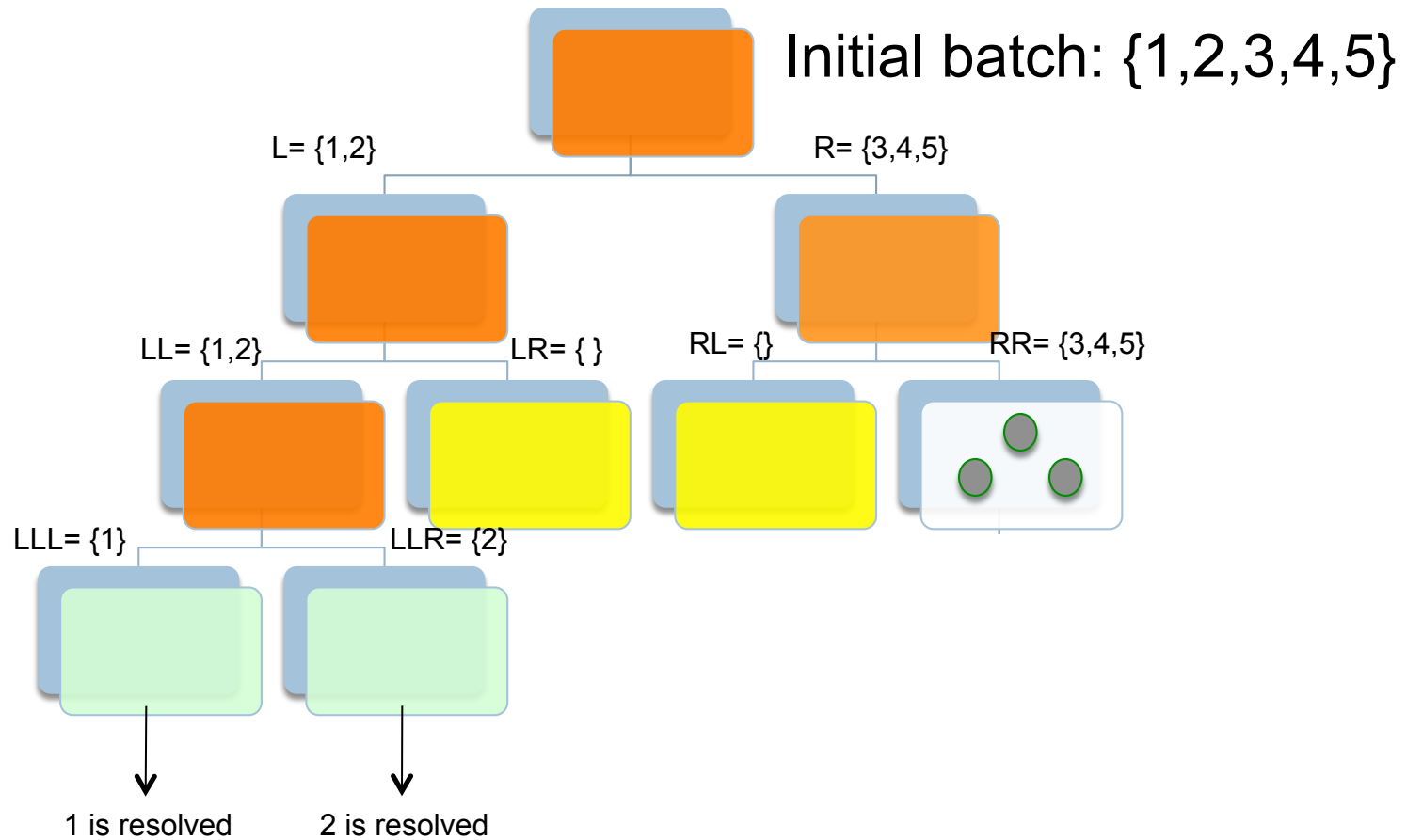
- Solution: *virtualize predictable collisions*
- Any time a collided slot is followed by an idle slot, *do not activate the right subset*, but rather, split it in two subsets *as if a collision had occurred* (but without wasting a slot)

# Slot7: collided

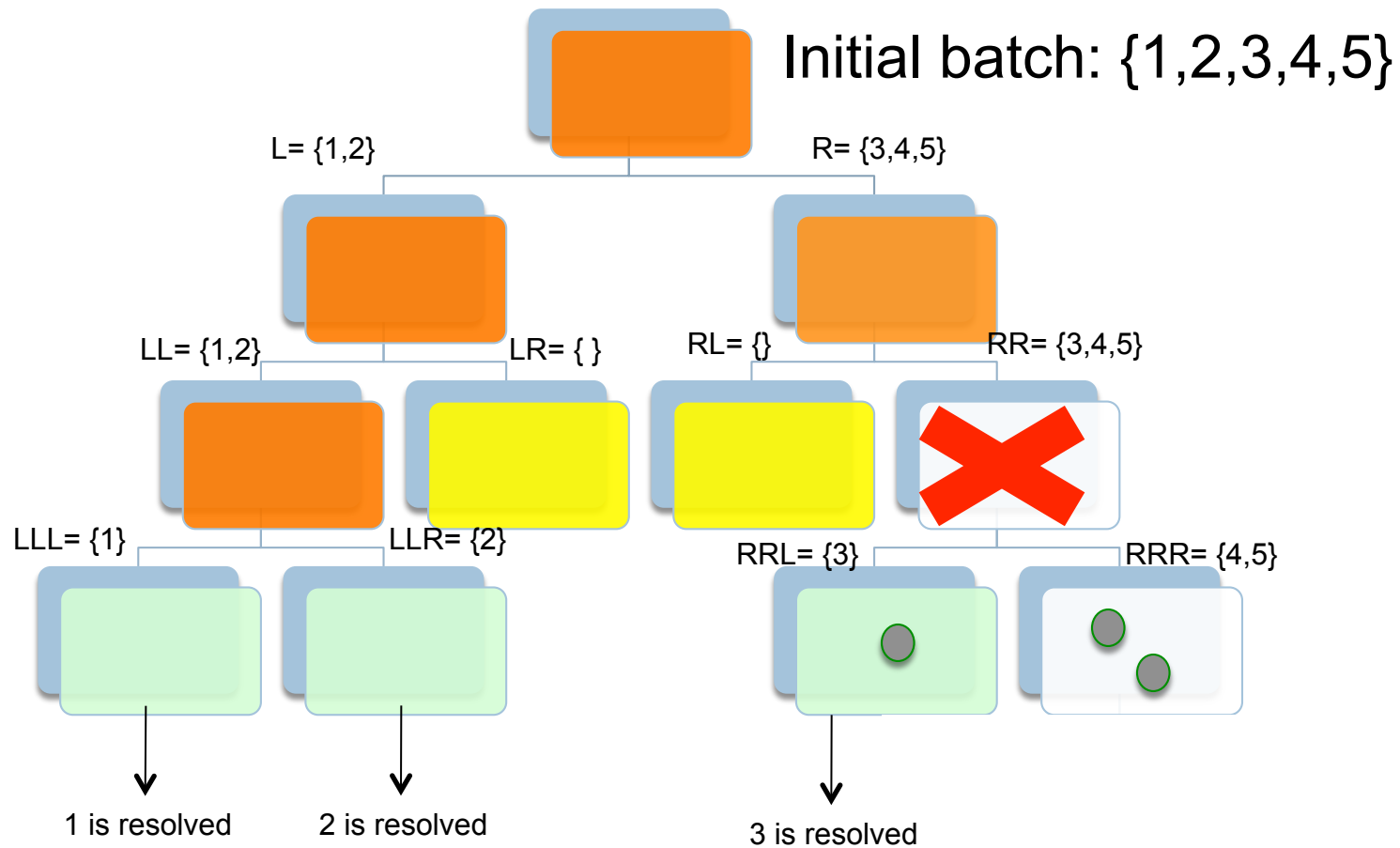




# Slot8: idle $\rightarrow$ split immediately!!!



# Slot9: successful



# Clipping mechanism

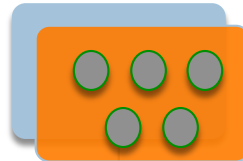
- Valid for batches with **Poisson distributed size**
- When a collided slot is followed by another collided slot, the right branches of the tree can be clipped
  - ▣ The second collision “erase” the prior information on the cardinality of the nodes in the right subsets
  - ▣ It is more convenient to return these nodes to the original set and divide again in optimal subsets



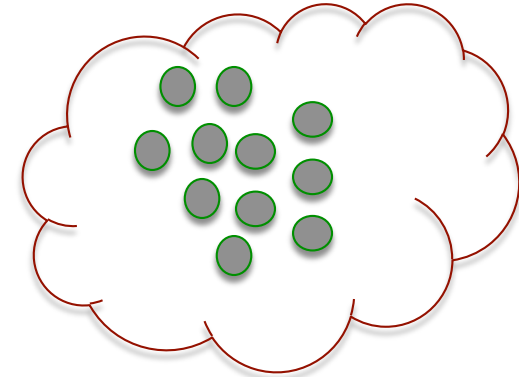
- Let  $X$  be the size of the activated interval
  - $X$  is a Poisson rv with parameter  $m$
- Imagine to split BEFORE observing the outcome
  - $L$  and  $R$  are independent Poisson rvs with parameters  $mp$  and  $m(1-p)$
- Now, you observe a collision  $\rightarrow X > 1 \rightarrow R$  is not Poisson!
 
$$\Pr[R=k \mid L+R > 1]$$
- If another collision  $\rightarrow L > 1 \rightarrow R$  is again Poisson!
 
$$\Pr[R=k \mid L+R > 1, L > 1] = \Pr[R=k \mid L > 1] = \Pr[R=k]$$

# Slot 1: collided

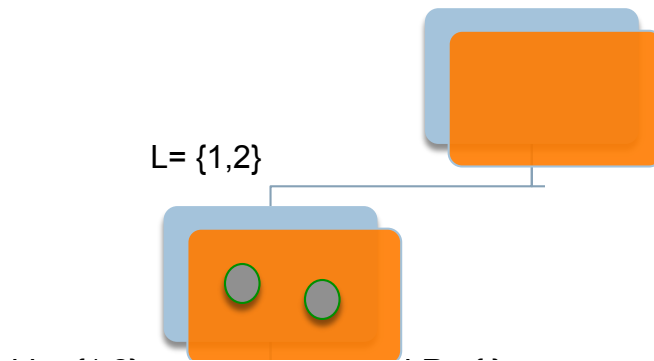
subgroup: {1,2,3,4,5}



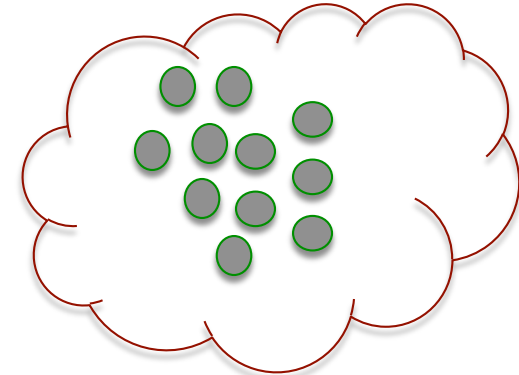
Residual batch



# Slot2: collided

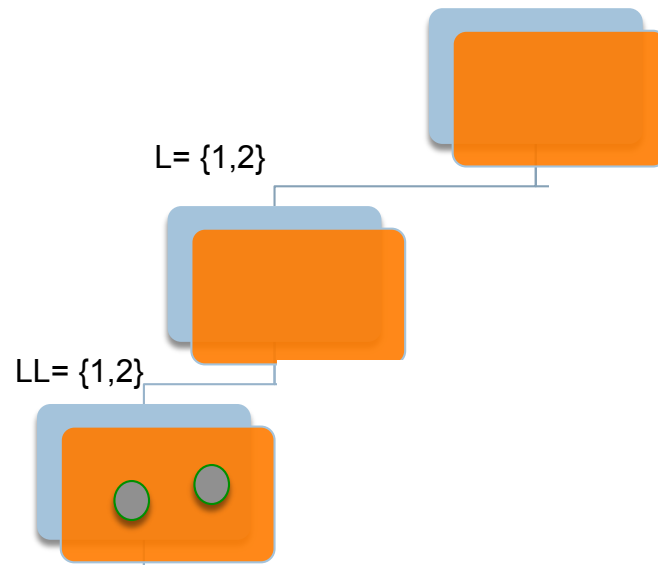


Residual batch

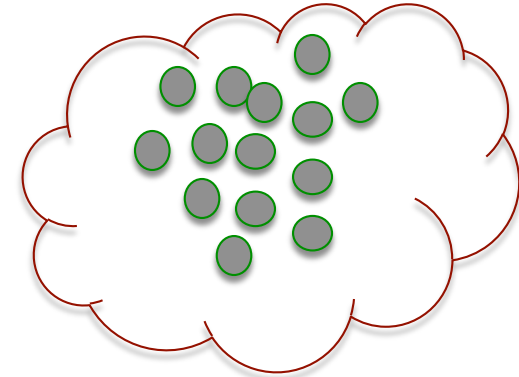




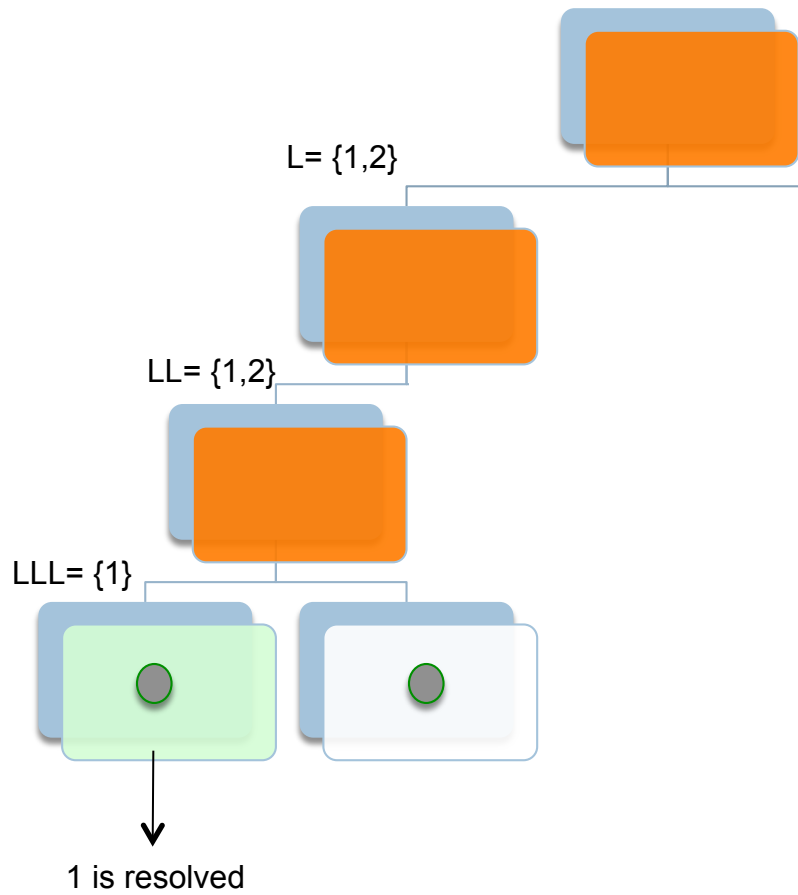
# Slot3: collided



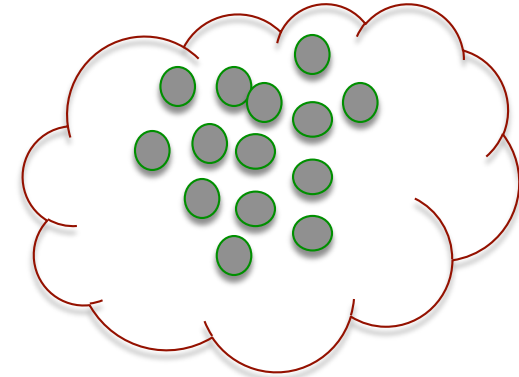
Residual batch



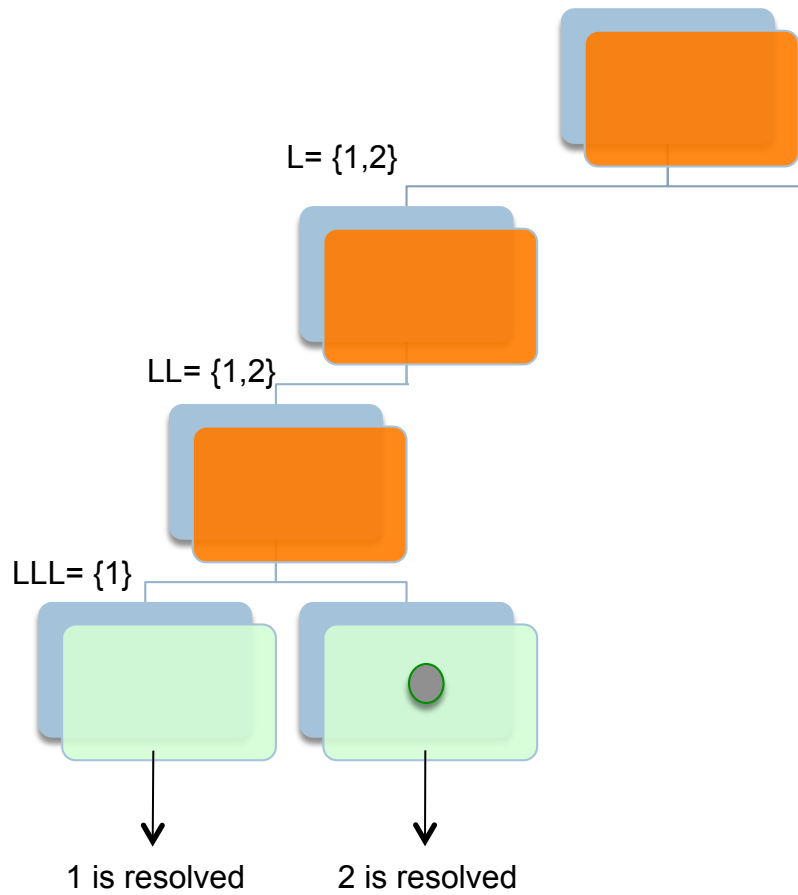
# Slot4: successful



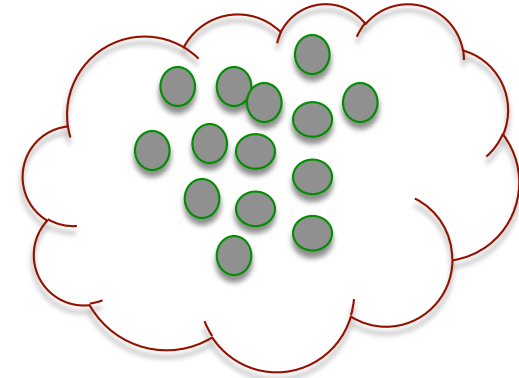
Residual batch



# Slot5: successful



Residual batch



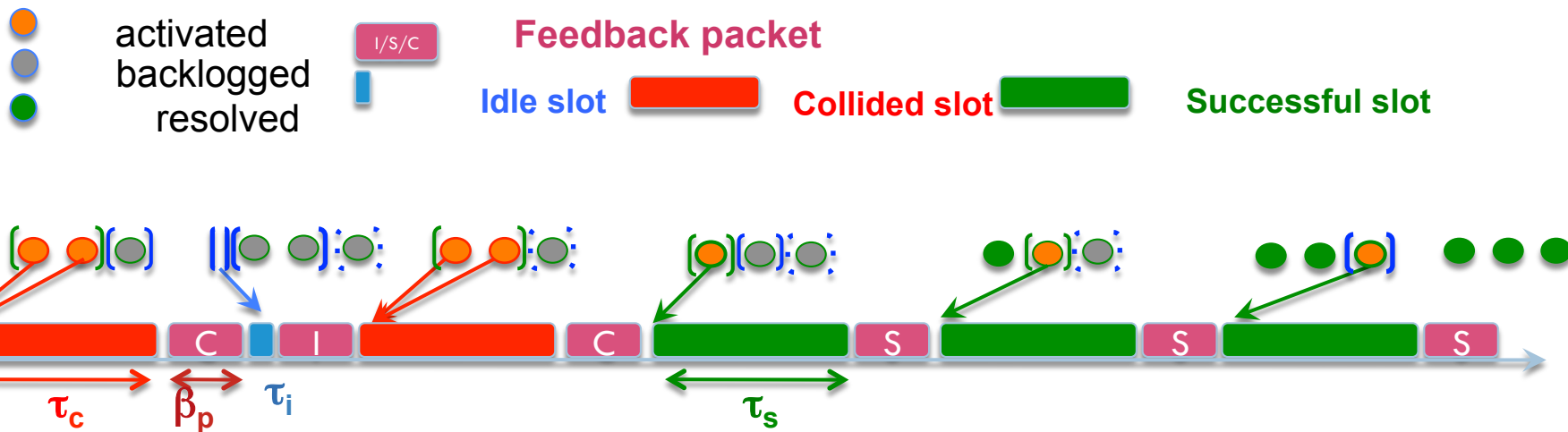


# Interval Estimate Collision Resolution algorithm (IECR)

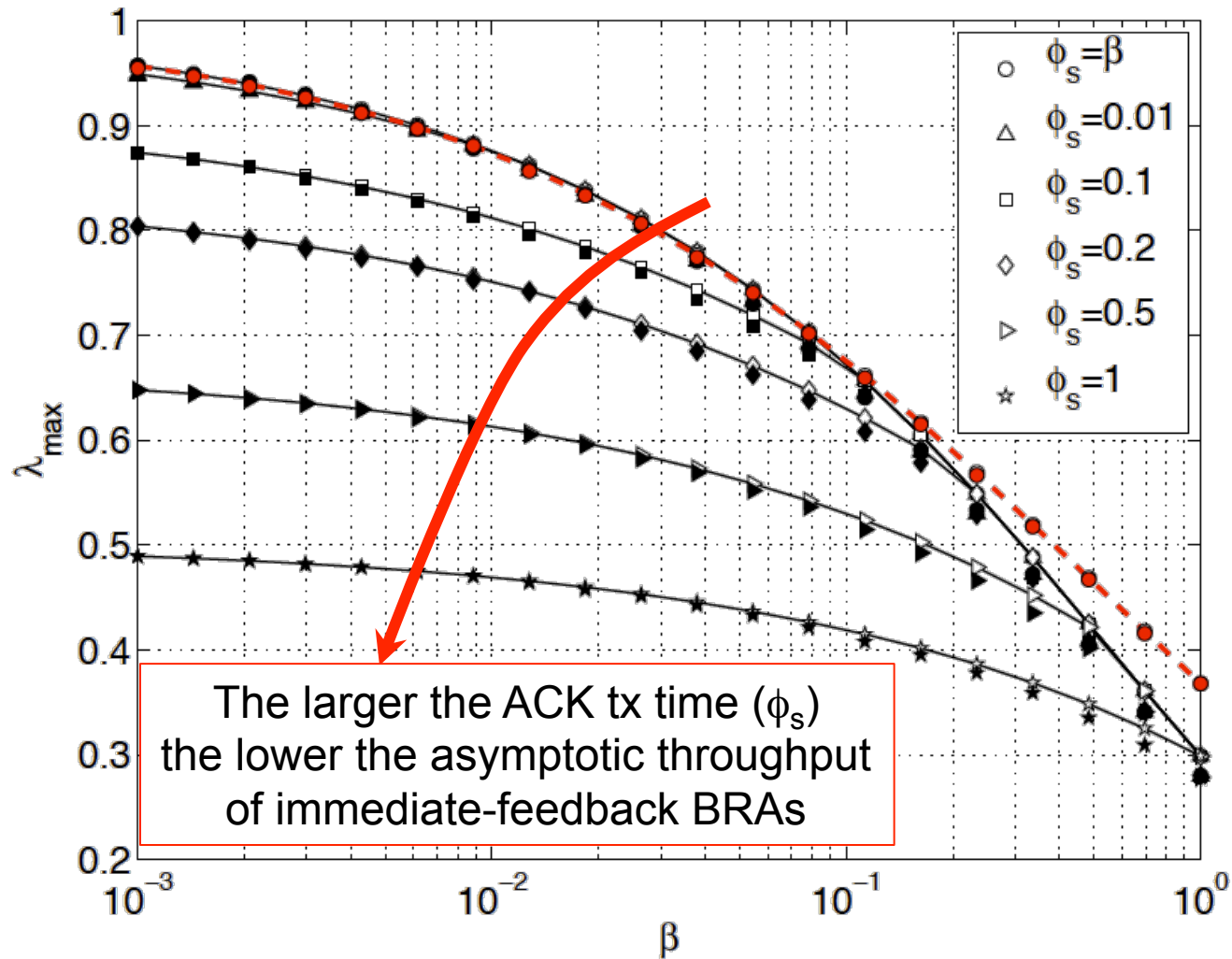
- CMBT is optimal if mean batch size  $m_n$  is known
- If the batch size is unknown, it must be estimated before applying CMBT
- IECR couple CMBT with a **batch size estimate phase**
  - ▣ Apply CMBT to whole batch until the first successful transmission
  - ▣ Estimate the residual batch size  $n$  on the basis of the number of consecutive collisions undergone so far
  - ▣ Repeat by assuming  $n$  as estimated above

# Timing

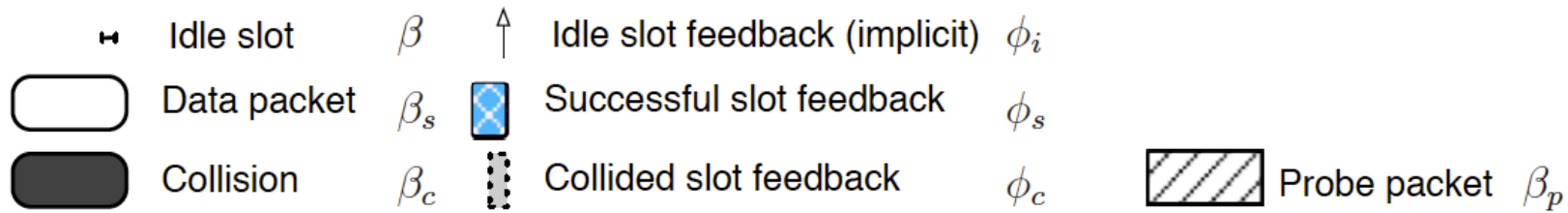
Event	Slot duration	Feedback duration		
			Classical	Practical (CSMA)
Successful	$T_{data}=1$	$\phi_s$	negligible	<i>significant</i>
Idle	$\beta_i \ll 1$	$\phi_i$	negligible	negligible
Collision	$\beta_c \sim 1$	$\phi_c$	negligible	<i>small</i>



# The cost of neglecting feedback cost...



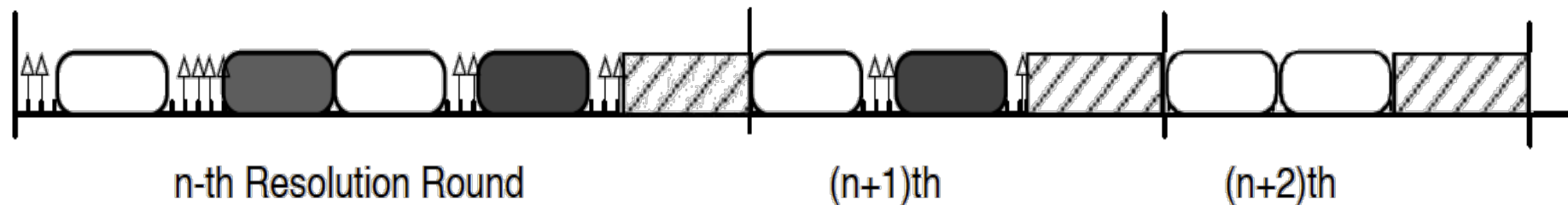
# A different approach: deferred feedback strategies



## Immediate feedbacks



## Deferred feedback





# Classical Dynamic Framed Slotted Aloha

- Framed Slotted ALOHA (FSA)
  - ▣ Slots are organized in frames of  $W$  slots
  - ▣ In each frame, nodes transmit in random slots
  - ▣ Feedback is returned only at the end of the frame by using a *probe packet*
- Dynamic FSA
  - ▣ Frame size is adjusted dynamically to maximize the expected per-frame throughput
- Drawback: probing cost is neglected!
  - ▣ Maximizing per-frame throughput does not necessarily minimize the overall batch resolution interval



# Our proposal

- ABRADe: Adaptive Batch Resolution Algorithm with Deferred Feedback
  - ▣ Basically a dynamic framed slotted ALOHA with a novel frame-adaptation strategy that keeps into account all costs!
  - ▣ Batch size  $n$  is assumed to be known beforehand!
- ABRADe+: couple ABRADe with a batch size estimate algorithm
  - ▣ No prior knowledge about the batch size

# ABRADE in a nutshell

- Assumption: the (residual) batch size “n” is known
- The frame size  $w_n$  of the next round is selected in order to minimize the **overall Batch Resolution Interval (BRI)** for that batch

number of successful collided idle slots in the frame

$$T(n) = E \left[ s + c + i\tau + b_p + T(n - s) \mid w, n \right]$$

BRI for batch  
of size n

Frame  
duration

Residual batch  
resolution interval

Frame  
Size

Batch  
Size

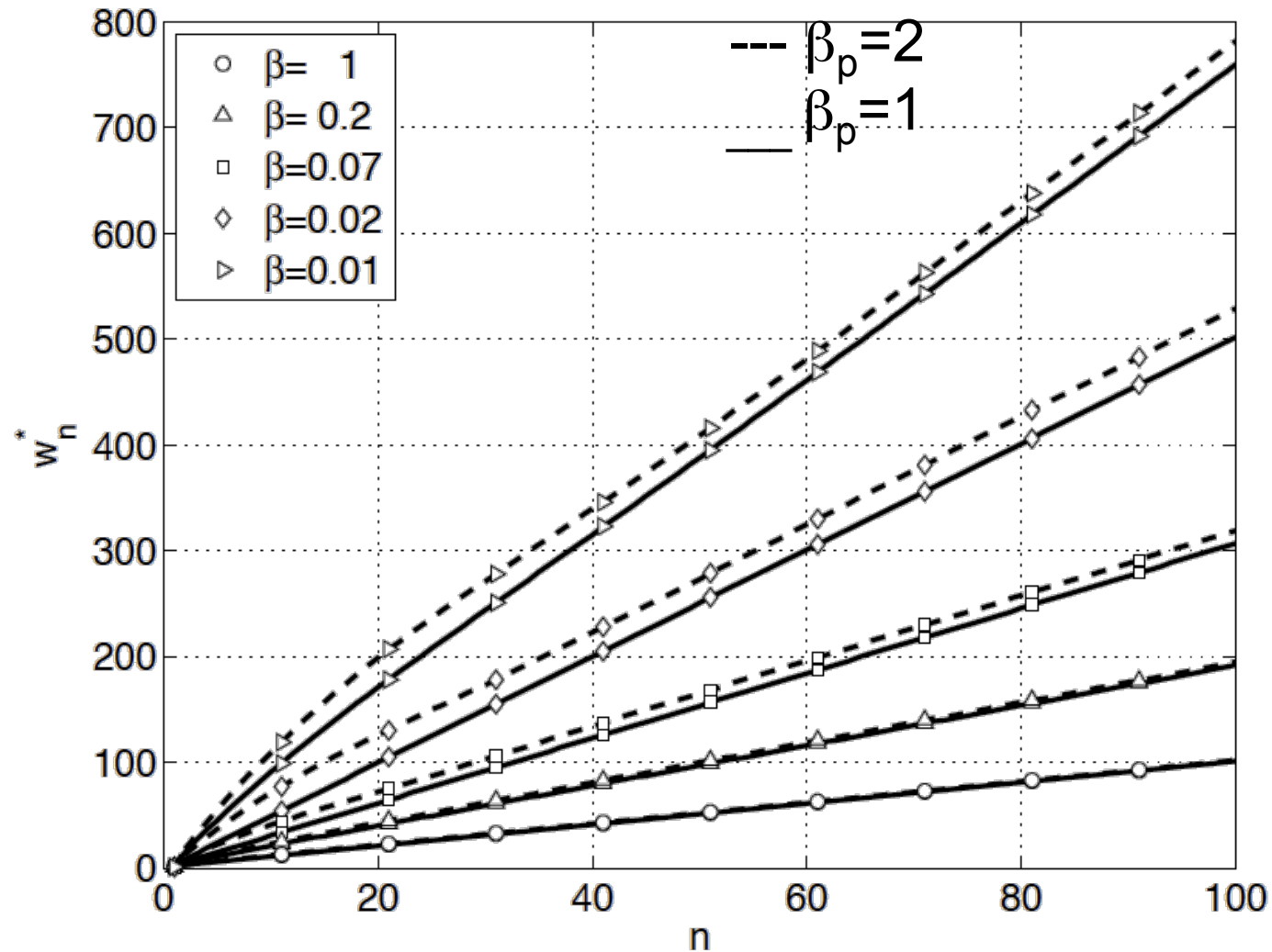
$w_n^*$ : optimal frame length for a batch of size n

$$p_{w,n}(s) = \Pr[s \mid w, n]$$

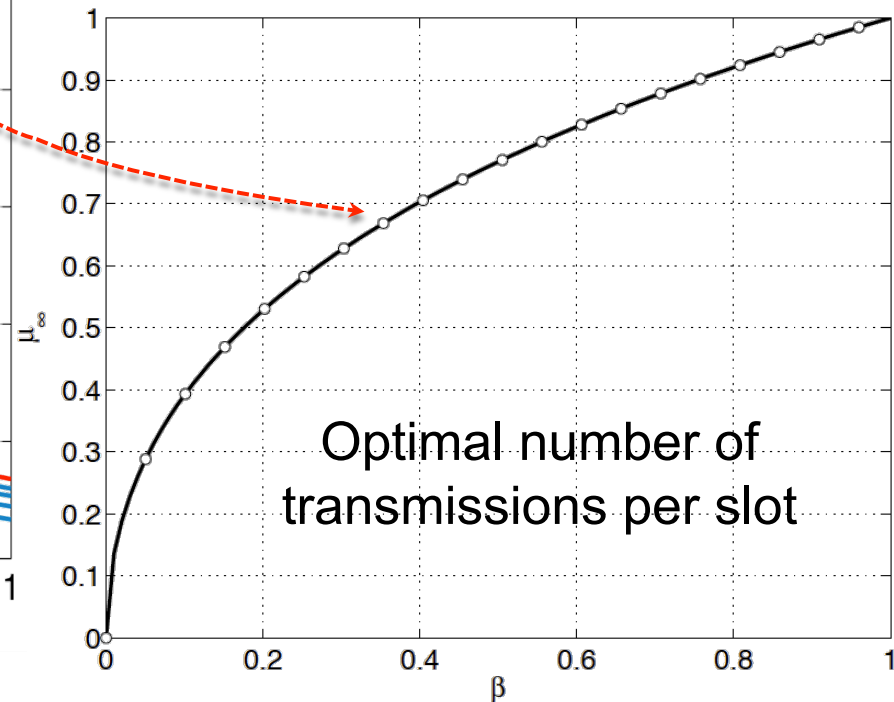
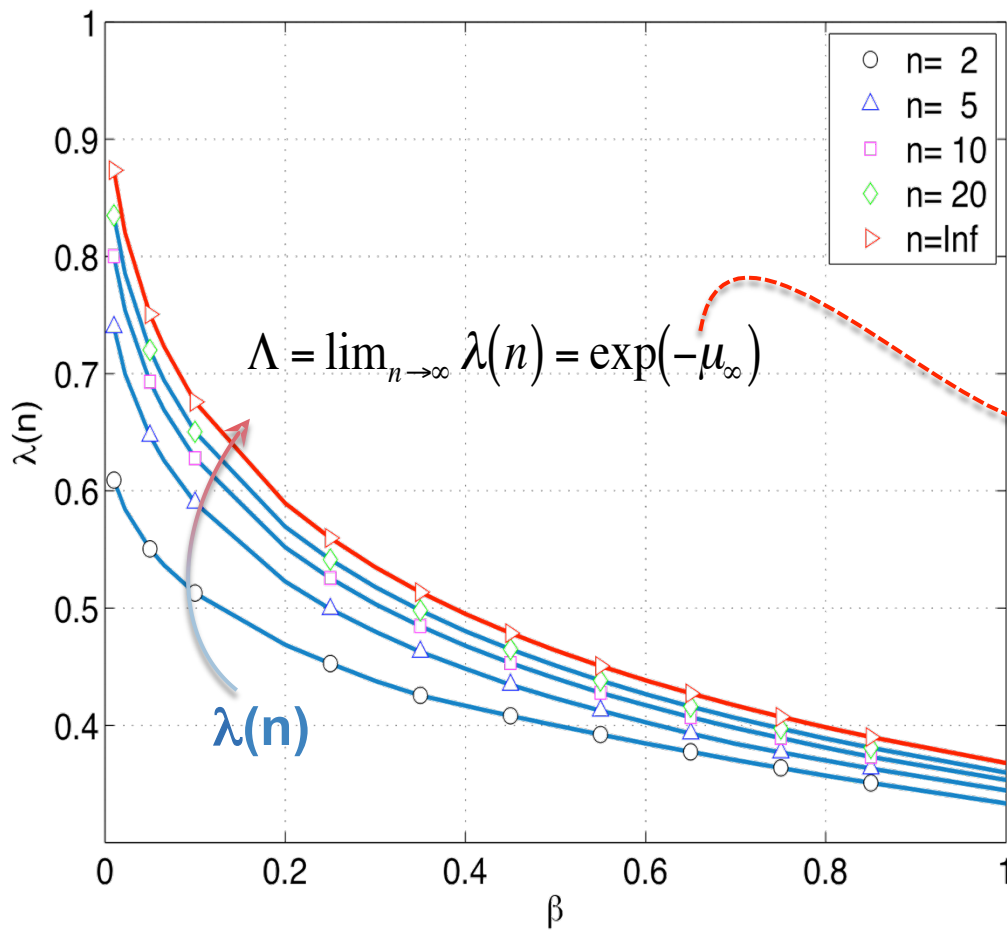
$$w_n^* = \operatorname{argmin}_w \left\{ \frac{\beta_p + w\beta_c + n(1 - \beta_c) \left(1 - \frac{1}{w}\right)^{n-1} + w \left(1 - \frac{1}{w}\right)^n (\beta - \beta_c) + \sum_{s=1}^{\infty} T^*(n - s) p_{w,n}(s)}{1 - p_{w,n}(0)} \right\}$$

**Dynamic  
programming  
optimization**

# Optimal frame length



# ABRADE's throughput



# ABRADE+: batch size estimate

- ABRADE needs prior knowledge of the batch size  $n$
- In most cases,  $n$  is unknown and needs to be estimated
- Estimate can be refined as the batch resolution proceeds
- ABRADE+ is as ABRADE with two add-ons
  1. Batch Size Estimate Function (BSEF)
  2. Start up phase



# Quick survey of most-known BSEFs

□  $V = \langle s, c, i \rangle \rightarrow \hat{n}$  estimate of  $n$

□ [Schoute83]:  $\hat{n} = s + 2c$

□ [Cha&Kim05]:  $\hat{n} = s + 2.39c$

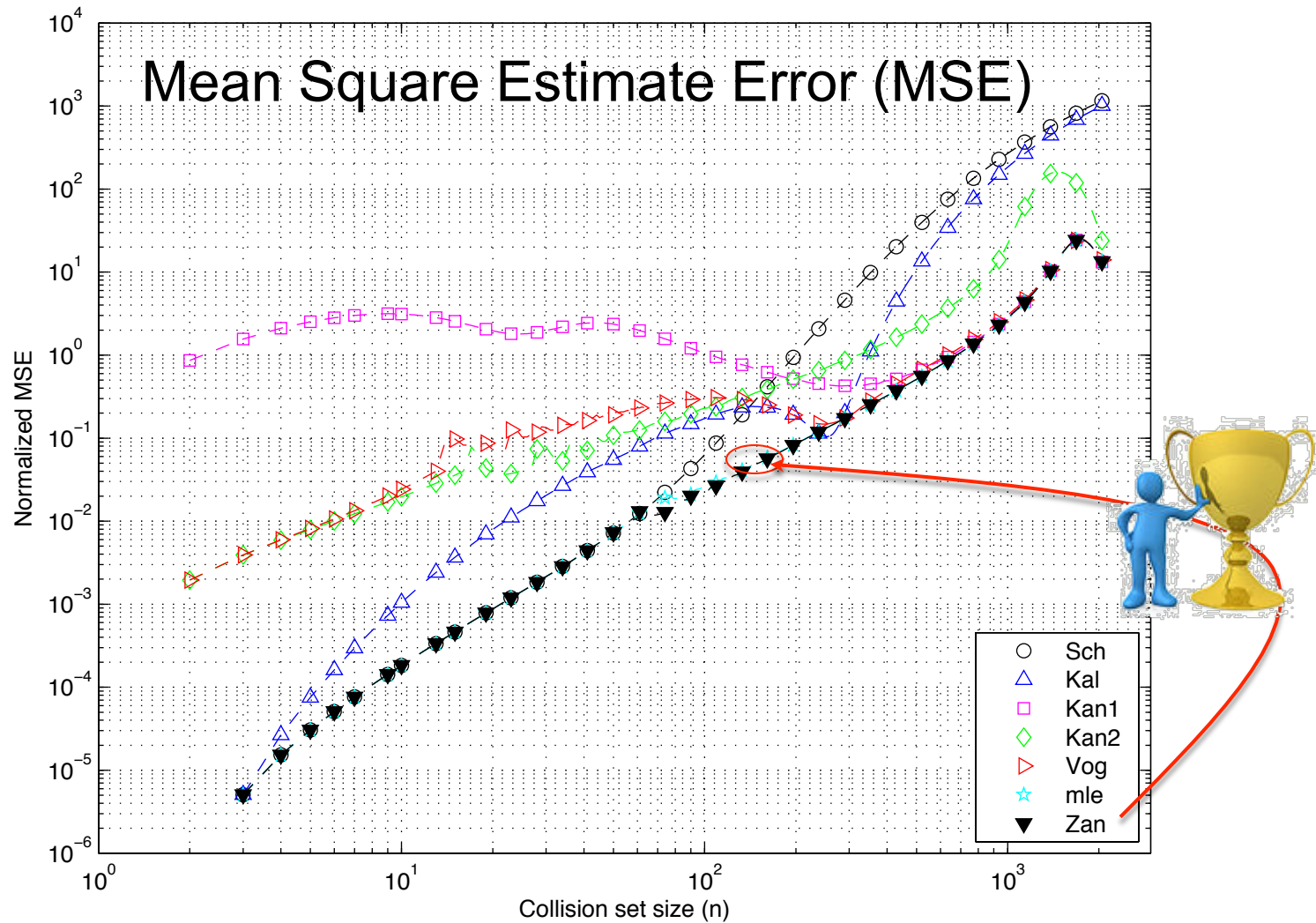
□ [Vogt02]:  $\hat{n} = \arg \min_n \left\{ \left( s - E[s|n] \right)^2 + \left( c - E[c|n] \right)^2 + \left( i - E[i|n] \right)^2 \right\}$

□ [Khandelwat06]:  $\hat{n} = \arg \min_n \left\{ \left( s - E[s|n] \right)^2 \right\}$   $\hat{n} = \arg \min_n \left\{ \left( i - E[i|n] \right)^2 \right\}$

□ [Kodialam06]:  $\hat{n} = w \log\left(\frac{1}{i}\right)$   $\hat{n} = \left\{ w\mu : (1 + \mu)e^{-\mu} = \frac{w - c}{w} \right\}$

□ [Zanella12]:  $\hat{n} = \left\{ w\mu : \frac{\mu w - s}{c} = \frac{\mu(e^\mu - 1)}{e^\mu - 1 - \mu} \right\}$

# A glance at BSEFs performance





# Start up phase

- What frame size  $w_0$  shall be used at the very first cycle?
  - ▣  $w_0$  shall be small, to have a first estimate of  $n$  as soon as possible
  - ▣  $w_0$  shall be large to avoid many collisions
- Solution: *Probabilistic Framed Slotted Aloha*
  - ▣ Nodes transmit with probability  $p$
- Issue:  $p, w_0$  shall be set to strike a balance between estimate accuracy & performance loss



# Setting $p$ and $w_0$

- Let  $m_n$  be the mean batch size  $n$ 
  - ▣ if unknown, arbitrarily assumed equal to  $N_{\max}/2$
- Set  $p$  such that  $w_0$  is optimal for a batch size equal

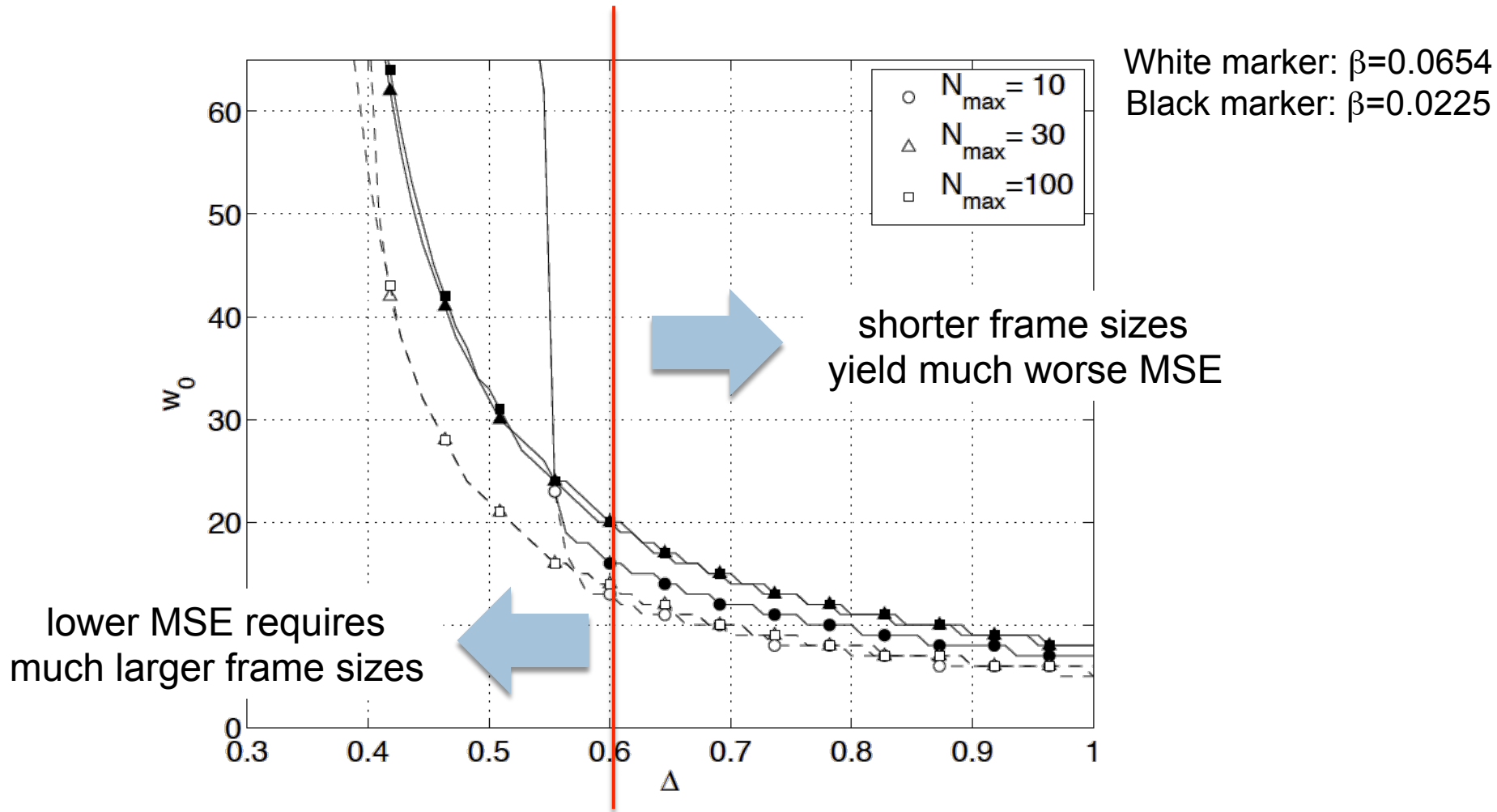
to  $pm_n$ :  $\frac{pm_n}{w_0} = \mu_\infty \rightarrow p = \frac{w_0}{m_n} \mu_\infty$

- Set  $w_0$  such that the mean square estimate error,  $MSE(p)$  is lower than  $\Delta m_n$ , where  $\Delta$  is a design parameter

$$w_0 = \min \left\{ w : E \left[ \left( n - \frac{\hat{n}}{p} \right)^2 \middle| w \right] \leq \Delta m_n \right\}$$





# Initial frame size vs $\Delta$

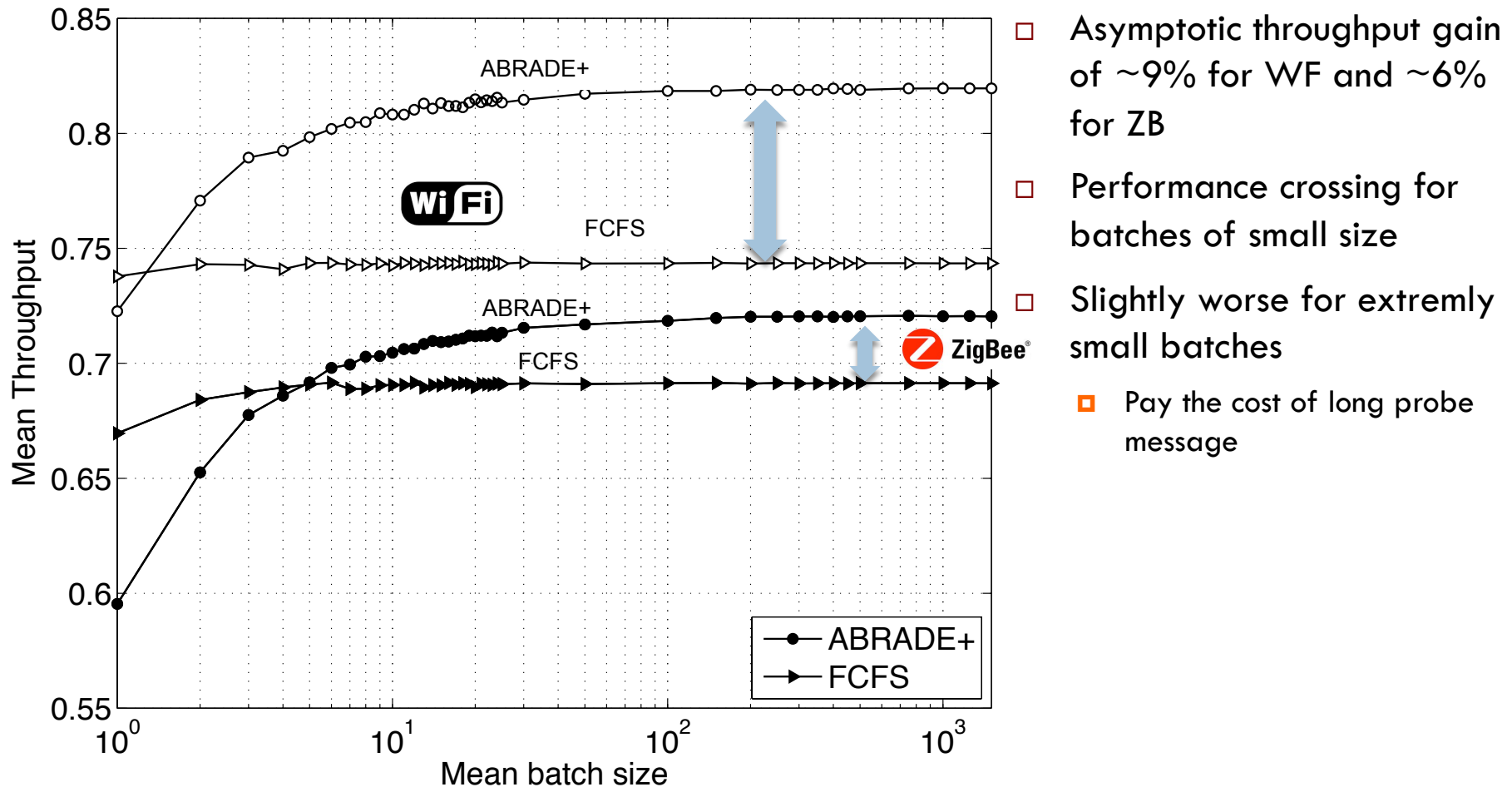


# Case study

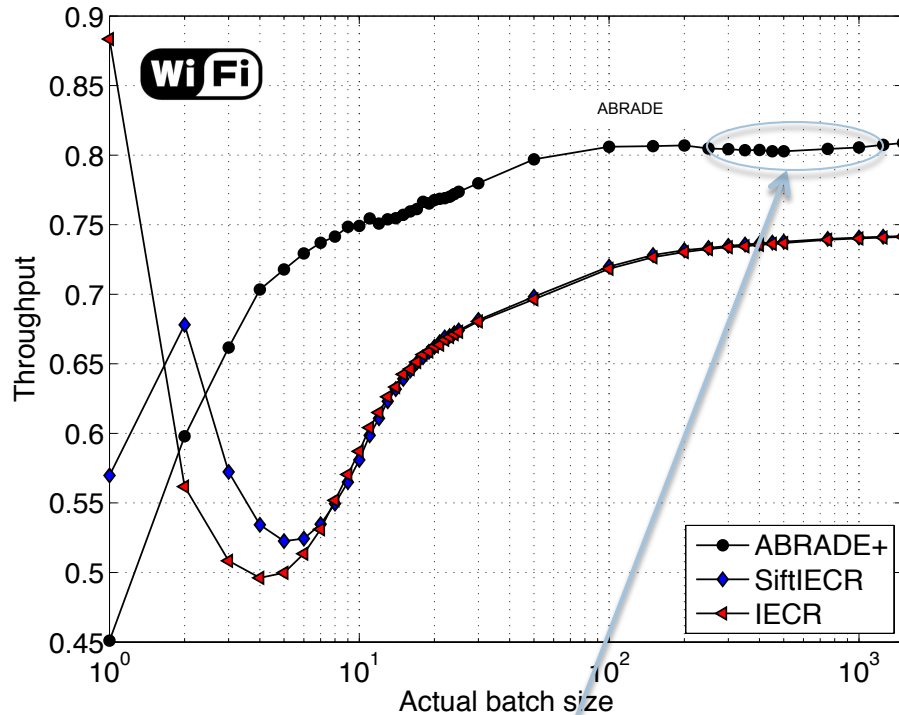
- Parameters set according to WiFi (WF) & ZigBee (ZB) specifications
  1. Batch size  $n$  with Poisson distribution of known mean  $N$
  2. Batch size totally unknown to the algorithm
    - $m_n$  arbitrarily set to 100
    - $\Delta=0.6$

	$T_{data}$ [ms]	$b$	$b_s$	$b_c$	$b_p = w/L_{max}$
 WiFi	0.399	0.0225	0.1319	0.1319	$w/18496$
 ZigBee®	4.896	0.0654	0.1111	0.0458	$w/944$

# 1. Poisson: Throughput

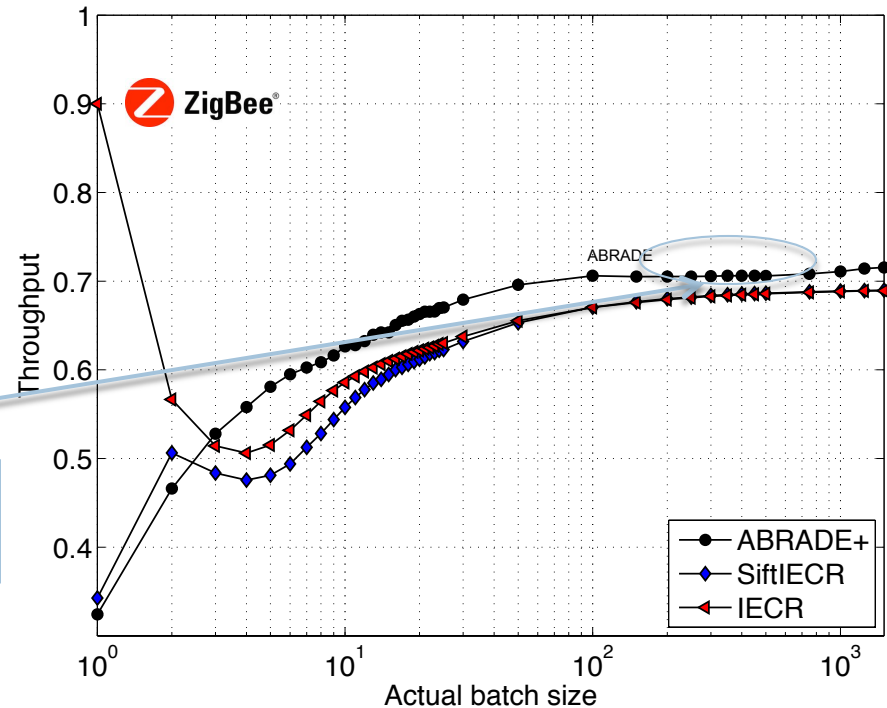


# 2. Unknown: Throughput



Performance contraction due to initial parameter setting ( $m_n=50$ )

- Throughput gain is maintained also in the case of unknown batch size, as long as the batch is larger than few units
- Gain is more evident for WF than ZB





# Summarizing

- The batch resolution problem is still challenging
  - Count and identify RTags
  - Bunch of sensors replying to probes
- Analysis shall carefully consider all protocol layer aspects
  - Initialization & feedback may have dramatic impact
- Large literature, but still much to be done!



# Main bibliography

- J. Capetanakis, "Tree algorithms for packet broadcast channels," IEEE Trans. on Information Theory, vol. 25, no. 5, pp. 505–515, Sep. 1979.
- R. Gallager, "Conflict resolution in random access broadcast networks," in AFOSR Workshop Commun. Theory Appl., Sep. 1978, pp. 74–76.
- Zanella, A., "Adaptive Batch Resolution Algorithm with Deferred Feedback for Wireless Systems," Wireless Communications, IEEE Transactions on , vol. 11, no.10, pp.3528,3539, October 2012
- A. Zanella "Estimating Collision Set Size in Framed Slotted Aloha Wireless Networks and RFID Systems". IEEE Communications Letters, Volume 16, Issue 3, pp: 300-303, March 2012



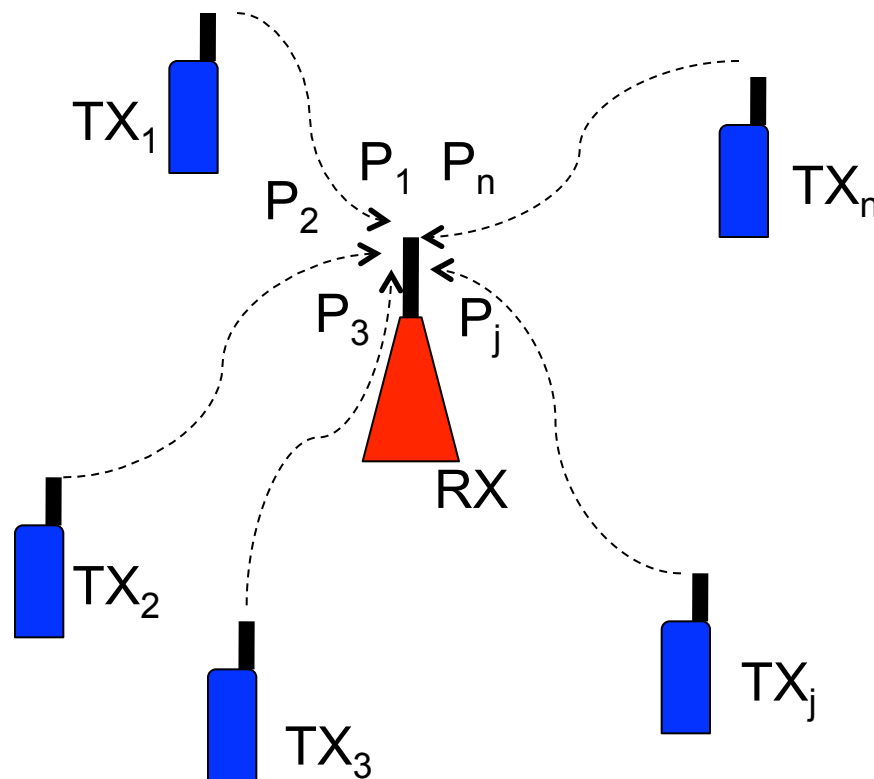
# Improving M2M cell capacity

Exploiting multipacket reception capabilities and SIC



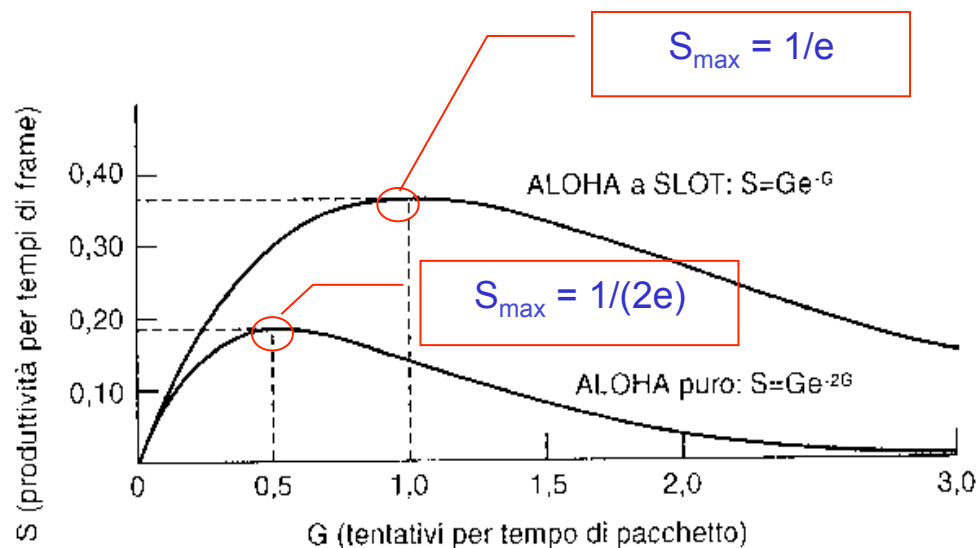
# The problem

- Reference scenario:
  - ▣ one base station, multiple transmitters, uplink channel
- The problem:
  - ▣ How many transmitters can be served?
  - ▣ What's the maximum *cell capacity*?
  - ▣ How can we get it?



# The literature

- Classic Aloha model [Kleinrock75]:
  - ▣ Destructive interference: one single transmission at a time
  - ▣ Max throughput:  $S = 1/(2e)$
  - ▣ Slotted version:  $S = 1/e$



- **Capture phenomenon** [LauLeung, TCOM92]
  - When the various signals are received with **significantly different powers** → *capture effect* may take place
  - the strongest signals may survive the collision and be correctly decoded
- Generalization: “Capture” occurs when *one or more* of the overlapping signals are successfully decoded by the receiver despite the interference



# Capture models

- Statistical geometry model [Roberts, ComRev75]
  - Intended signal is captured when the **strongest interferer** is sufficiently far apart from the receiver
  - Doesn't account for actual signal propagation phenomena nor does it consider cumulative effect of multiple "weak" interferers
- Arrival Time model [Davis&Gronemeyer, TCOM80]
  - Intended signal is captured when arrival instants of the first and second signals are sufficiently apart
  - Doesn't account for signal strengths, nor for simultaneous transmissions

# Other capture models

- Counting model [Wieselthier et al, TCOM89]
  - ▣ Capture occurs with probability that depends on the number of overlapping signals
  - ▣ Doesn't account for actual signal powers
  - ▣ At most one capture per transmission attempt
- Power model [Ephremides&Luo, TIT02]
  - ▣ Capture if no other signal with higher power overlaps in time
  - ▣ Doesn't account for cumulative effect of multiple “weak” interferers

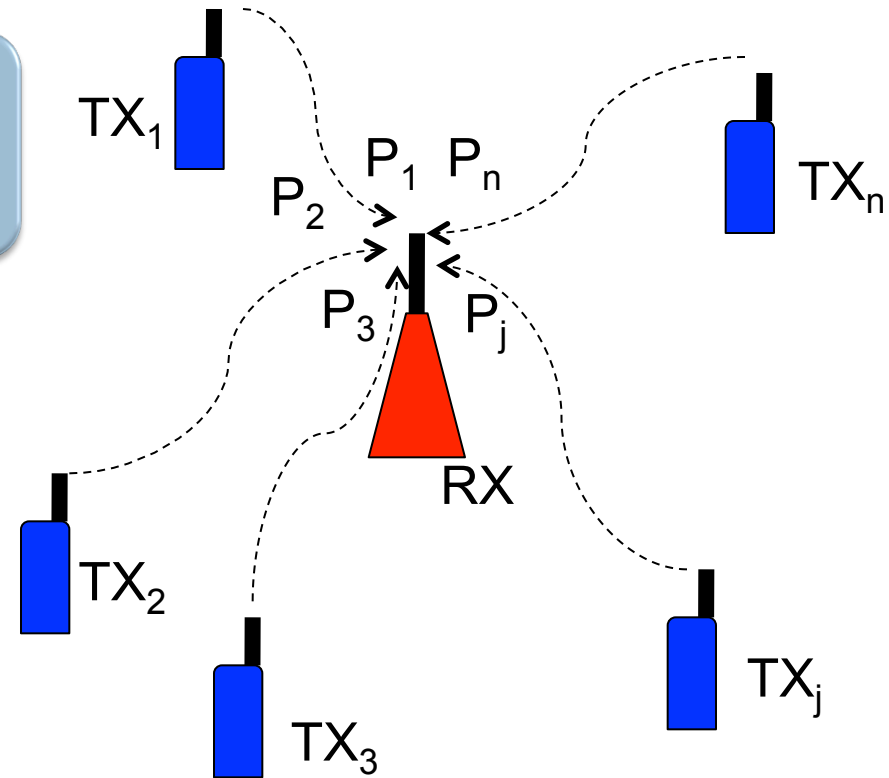
# MPR and Stability

- MPR & Stability [Ghez, Verdu, Schwartz, ITAC88]
  - ▣ They assume a given pmd for the number  $r$  of captured signals out of a collision of  $n$  overlapping signals
  - ▣ Show that MPR can stabilize ALOHA and that max throughput is  $S_{\max} = E[r]$
  - ▣ Don't give an expression for the pmd of  $r$  given  $n$

# Physical capture model

## Decoding model: SINR threshold

- Use of strong coding to achieve Shannon capacity
- $P_j$ : power of the  $j$ -th signal at the receiver
- $N_0$ : noise power (neglected)
- $\gamma_j$ : SINR of the  $j$ -th signal
- $b$ : capture threshold



Aggregate interference

$$\gamma_j = \frac{P_j}{I + N_0}$$

$\gamma_j > b \rightarrow j$ -th signal is correctly decoded (capture)  
 $\gamma_j < b \rightarrow j$ -th signal is collided (missed)

# Multi Packet Reception

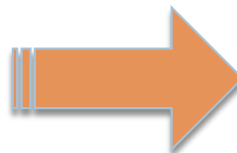
- MPR can be enabled by means of
  - *Signal spreading (DSSS)*
    - $b < 1 \rightarrow$  multiple signals (up to  $1/b$ ) can be captured at a time
  - *Successive interference cancellation (SIC)*
    1. Capture signal  $j$  with SINR  $\gamma_j > b$
    2. Reconstruct and cancel signal  $j$  from the overall received signal
      - Cancellation leaves a fraction  $z$  of residual interference power
    3. Repeat iteratively



# Open questions

## System parameters

- Number of simultaneous transmissions ( $n$ )
- Statistical distribution of the receiver signal powers ( $P_i$ )
- Capture threshold ( $b$ )
- Max number of SIC iterations ( $K$ )
- Interference cancellation ratio ( $z$ )



**Capture  
probability?**  
**System  
throughput?**



# The answer

- Capture probability
  - $C_n(r;K) = \Pr[r \text{ signals out of } n \text{ are captured within at most } K \text{ SIC cycles}]$
- Computing  $C_n(r;K)$  is difficult because the **SINRs** are all **coupled!!!**
  - E.g.  $\gamma_1 = \frac{P_1}{P_2} > b \Rightarrow \gamma_2 = \frac{P_2}{P_1} < \frac{1}{b}$
  - Computation of  $C_n(r;k)$  becomes more and more complex as the number  $n$  of signals increases
- SIC makes things even worse

# State of the art

- **Narrowband ( $b > 1$ ), No SIC ( $K = 0$ )**
  - [Zorzi&Rao,JSAC1994,TVT1997] derive the probability  $C_n(1;0)$  that **one signal** is captured
    - MPR and SIC are not considered
- **Wideband ( $b < 1$ ), No SIC ( $K = 0$ )**
  - Can capture multiple signals in one reception cycle
  - [Nguyen&Ephremides&Wieselthier,ISIT06,ISIT07] derive the probability  $1 - C_n(0;0)$  that **at least one signal** is captured
    - Expression involves  $n$  folded integrals, does not scale with  $n$
- **Wideband ( $b < 1$ )+SIC ( $K > 0$ )**
  - [ViterbiJSAC90] shows that SIC can achieve Shannon capacity in AWGN channels
    - Requires suitable received signals power allocation
  - [Narasimhan, ISIT07] studies outage rate regions in presence of Rayleigh fading
    - Eqs can be computed only for few users
  - [Weber et al, TIT07] study SIC in ad hoc wireless networks
    - Derive **bounds** on the transmission capacity based on stochastic geometry arguments

# State of the art

- **Wideband ( $b < 1$ ) + SIC ( $K > 0$ ) (cont)**
  - [ZanellaZorzi, TCOM2012] provide a scalable method for the numerical evaluation of the capture probability distribution  $C_n(r;K)$ 
    - Investigate capture distribution & system throughput when varying system parameters  $\{n, b, K, z\}$
  - Provide approximate expressions for the capture probability and the MPR throughput



# CAPTURE PROBABILITY FOR PURE MPR CASE



No SIC ( $K=0$ )

# Problem statement

- The capture condition can be expressed as

$$\gamma_j = \frac{P_j}{\Lambda - P_j} > b \Rightarrow P_j > b(\Lambda - P_j) \Rightarrow P_j > \Lambda \frac{b}{b+1} = \Lambda b'$$

- $b'$  is termed *modified capture threshold*
  - $b'\Lambda$  is named *absolute capture threshold*
- We aim at determining the expression of
 
$$C_n(r) = \Pr[r \text{ signals out of } n \text{ are captured}]$$

# Capture distribution

- Because of the problem symmetry we have

$$C_n(r) = \binom{n}{r} \underbrace{\Pr(P_{1:r} > \Lambda b')}_{\text{r captured signals}} \underbrace{\Pr(P_{r+1:n} \leq \Lambda b')}_{\text{n-r missed signals}}$$

$c_n(r)$

Conditioning on  $\Lambda=x$  we get...

$$c_n(r) = \int_0^\infty \Pr(P_{1:r} > xb', P_{r+1:n} \leq xb' | \Lambda = x) f_\Lambda(x) dx$$

and applying Bayes' rule:  $P[A | B]P[B] = P[B | A]P[A]$

$$c_n(r) = \int_0^\infty f(\Lambda = x | P_{1:r} > xb', P_{r+1:n} \leq xb') \underbrace{\Pr(P_{1:r} > xb', P_{r+1:n} \leq xb')}_{\text{iid}} dx$$

$$(1 - F_P(xb'))^r F_P(xb')^{n-r}$$

# Conditioned aggregate received power $\Lambda_r$

- The issue is now to compute the PDF

$$f_{\Lambda_r}(x) = \Pr(\Lambda \cong x \mid P_{1:r} > xb', P_{r+1:n} \leq xb')$$

- We introduce the auxiliary r.v.

$$\tilde{\Lambda}_r(u) = \sum_{h=1}^r \alpha_h(u) + \sum_{k=1}^{n-r} \beta_k(u)$$

iid rvs with PDF  
 $f_{\alpha(u)}(x) = f_P(x \mid P > u)$

iid rvs with PDF  
 $f_{\beta(u)}(x) = f_P(x \mid P \leq u)$

- $\tilde{\Lambda}_r(u)$  gives the aggregate power **given that**  $r$  signals are above  $\mathbf{u}$ , and  $n-r$  are below  $\mathbf{u}$  → setting  $\mathbf{u} = \mathbf{x}b'$  we get

$$f_{\Lambda_r}(x) = f_{\tilde{\Lambda}_r(\mathbf{x}b')}(x)$$



# Auxiliary random variable

- Since  $\alpha_h(u)$  and  $\beta_h(u)$  are independent, we get

$$f_{\tilde{\Lambda}_r(u)}(x) = \left( f_{\alpha(u)} \otimes f_{\alpha(u)} \otimes \cdots \otimes f_{\alpha(u)} \otimes f_{\beta(u)} \otimes f_{\beta(u)} \otimes \cdots \otimes f_{\beta(u)} \right)(x)$$

*Fourier Transform*

$$f_{\tilde{\Lambda}_r(u)}(x) = \int_{-\infty}^{+\infty} \left[ \Psi_{\alpha(u)}(\xi) \right]^r \left[ \Psi_{\beta(u)}(\xi) \right]^{n-r} e^{j2\pi x \xi} d\xi$$

*Inverse Fourier Transform*

- Putting all the pieces together we get the final expression

$$C_n(r) = \binom{n}{r} \int_0^\infty (1 - F_P(xb'))^r F_P(xb')^{n-r} \times$$

$$\left( \int_{-\infty}^{+\infty} \left[ \Psi_{\alpha(xb')}(\xi) \right]^r \left[ \Psi_{\beta(xb')}(\xi) \right]^{n-r} e^{j2\pi x \xi} d\xi \right) dx$$

# Capture distribution: approximate expression

- If  $n \gg r$  &  $r=0$  or  $r=n$  or  $r \approx n/2$  the central limit theorem applies

$$\tilde{\Lambda}_r(u) = \sum_{h=1}^r \alpha_h(u) + \sum_{k=1}^{n-r} \beta_k(u)$$

$$N\left(rm_{\alpha(u)}, r\sigma_{\alpha(u)}^2\right) \quad N\left((n-r)m_{\beta(u)}, (n-r)\sigma_{\beta(u)}^2\right)$$

- from which we get the following approximate expression of  $C_n(r)$ :

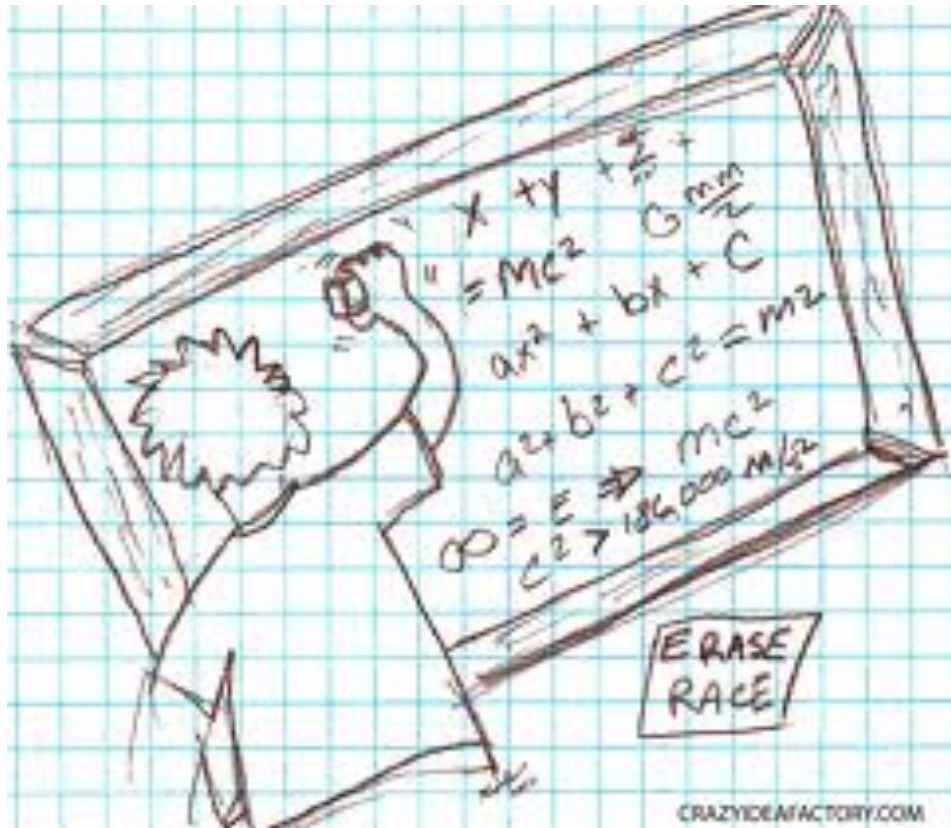
$$\tilde{C}_n(r) = \binom{n}{r} \int_{n P_m}^{n P_M} \frac{\exp\left(-\frac{(x - rm_{\alpha(xb')}) - (n-r)m_{\beta(xb')}}{2(r\sigma_{\alpha(xb')}^2 + (n-r)\sigma_{\beta(xb')}^2)}\right)^2}{\sqrt{2\pi [r\sigma_{\alpha(xb')}^2 + (n-r)\sigma_{\beta(xb')}^2]}} \times (1 - F_P(b'x))^r F_P(b'x)^{n-r} dx$$

# Throughput

- $k$ : reception capability
  - ▣ max number of signals that can be simultaneously decoded
- $S_n(k)$ : average number of signals captured by a system with reception capacity  $k$  and a collision size  $n$

$$S_n(k) = \sum_{r=1}^{k-1} r C_n(r) + k \sum_{r=k}^n C_n(r)$$

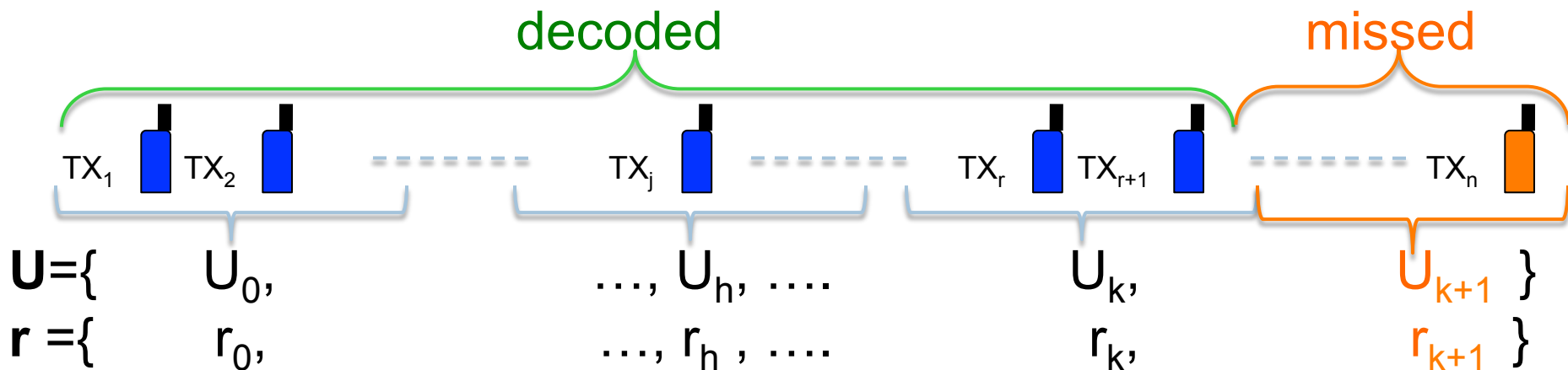
- Note: previous literature focused on  $S_n(1)=1-C_n(0)$  only!



# ANALYSIS OF THE SIC CASE

# Notation: reception set and vector

- $n$  : number of overlapping signals
- $r$  : overall number of decoded signals
- $h = \{0, 1, \dots, K\}$ : SIC iteration
- $U_h$ : set of signals decoded at the  $h$ -th SIC iteration
  - $U_{k+1}$ : set of missed signals at the end of the reception process
- $\mathbf{r} = [r_0, r_1, \dots, r_k, r_{k+1}]$ : reception vector
  - $r_h = |U_h|$ ,  $r_{k+1} = |U_{k+1}| = n - r$



# Notation: aggregate power

- Set of signal powers for users in  $U_h$

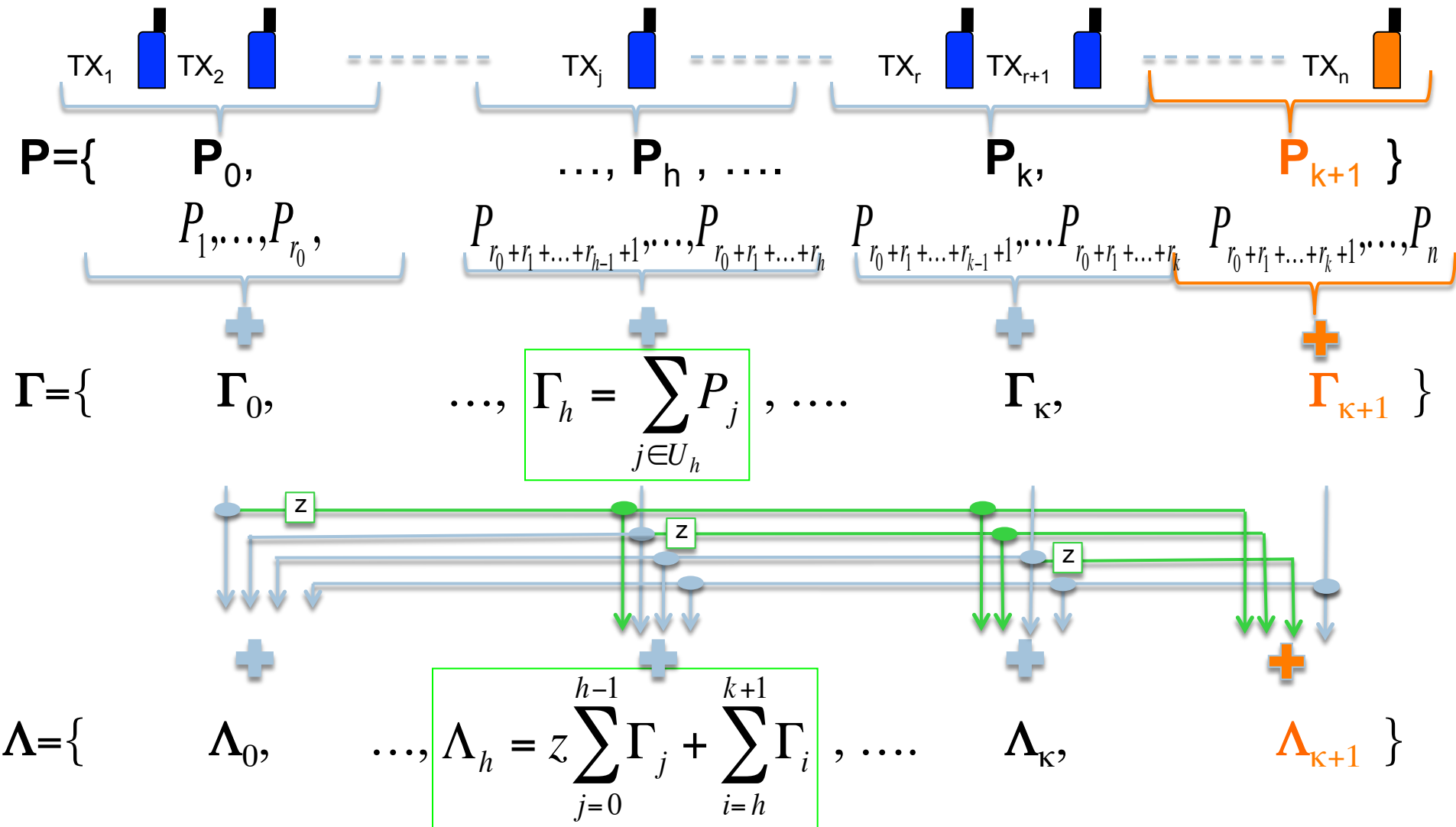
$$\mathbf{P}_h = \left\{ P_{r_0+r_1+\dots+r_{h-1}+1}, \dots, P_{r_0+r_1+\dots+r_h} \right\}$$

- Aggregate power of users in  $U_h$

$$\Gamma_h = \sum_{j \in U_h} P_j$$

- Overall sign. power at the  $h$ -th decoding cycle

$$\Lambda_h = z \sum_{j=0}^{h-1} \Gamma_j + \sum_{i=h}^{k+1} \Gamma_i$$





# Step 1: a bit of combinatorial analysis

$\Pr[r \text{ signals are decoded in at most } K \text{ iterations}]$

$$C_n(r; K) = \sum_{k=0}^K \sum_{\mathbf{r}} A(\mathbf{r}) c(\mathbf{r})$$

Combinatorial coefficient

Ordered probability distribution

$\Pr$ 

- first  $r_0$  signals are decoded at iteration 0
- successive  $r_1$  signals are decoded at iteration 1
- $\vdots$
- successive  $r_h$  signals are decoded at iteration  $h$
- $\vdots$
- last  $r_{k+1}$  signals are undecoded after  $k$  iterations



# Step 2: express decoding probability in terms of $P_j$

- Signals in  $U_h$  are decoded at the  $h$ -th SIC iteration if
  1. were not decoded at previous iterations
  2. verify capture condition after  $h$  SIC iterations

- Mathematically  $\forall j \in U_h$ ,

$$1. \quad \gamma_j = \frac{P_j}{\Lambda_{h-1} - P_j} \leq b \Rightarrow P_j \leq b' \Lambda_{h-1}$$

$$\text{where} \quad b' = \frac{b}{b+1}$$

$$2. \quad \gamma_j = \frac{P_j}{\Lambda_h - P_j} > b \Rightarrow P_j > b' \Lambda_h$$

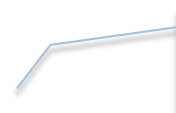
- Considering all  $k$  SIC iterations...

$$c(\mathbf{r}) = \Pr[\mathbf{P}_0 > b' \Lambda_0 \geq \mathbf{P}_1 > b' \Lambda_1, \dots, b' \Lambda_{k-1} \geq \mathbf{P}_k > b' \Lambda_k, \mathbf{P}_{k+1} \leq b' \Lambda_{k+1}]$$

# Step 3: let's start conditioning

- The capture threshold at each SIC iteration are

$$b' \Lambda_h = b' \left( z \sum_{j=0}^{h-1} \Gamma_j + \sum_{i=h}^{k+1} \Gamma_i \right)$$



Aggregate power of  
signals in  $U_i$

- Conditioning on  $\{\Gamma_h = g_h\}$  the capture thresholds becomes deterministic

$$\lambda_h(\mathbf{g}) = b' \left( z \sum_{j=0}^{h-1} g_j + \sum_{i=h}^{k+1} g_i \right)$$

- Then, we can write (we omit  $\mathbf{g}$  in the argument of  $\lambda_h$ )

$$c(\mathbf{r}) = \iiint \Pr[\mathbf{P}_0 > \lambda_0 \geq \mathbf{P}_1 > \lambda_1, \dots, \lambda_{k-1} \geq \mathbf{P}_k > \lambda_k, \mathbf{P}_{k+1} \leq \lambda_{k+1} | \Gamma = \mathbf{g}] \Pr[\Gamma \cong \mathbf{g}] d\mathbf{g}$$

k+2 nested integrals

PDF of the random vector  $\Gamma$   
evaluated in  $\mathbf{g} = [g_0, \dots, g_{k+1}]$

# Step 4: swap terms

- Applying Bayes rule we get

$$c(\mathbf{r}) = \iiint \Pr[\Gamma \cong \mathbf{g} | \mathbf{P}_0 > \lambda_0 \geq \mathbf{P}_1 > \lambda_1, \dots, \lambda_{k-1} \geq \mathbf{P}_k > \lambda_k, \mathbf{P}_{k+1} \leq \lambda_{k+1}]$$

$$\Pr[\mathbf{P}_0 > \lambda_0 \geq \mathbf{P}_1 > \lambda_1, \dots, \lambda_{k-1} \geq \mathbf{P}_k > \lambda_k, \mathbf{P}_{k+1} \leq \lambda_{k+1}] d\mathbf{g}$$

$$\stackrel{\text{iid}}{\prod_{h=0}^k [F_P(\lambda_h) - F_P(\lambda_{h-1})]^{r_h}}$$

- The issue now is to compute this conditional probability

$$\stackrel{\text{iid}}{\Pr[\Gamma \cong \mathbf{g} | \mathbf{P}_h \in (\lambda_h, \lambda_{h-1}]_{h=0}^k, \mathbf{P}_{k+1} \leq \lambda_{k+1}]} =$$

$$\lambda_{-1} = -\infty \quad \Pr[\Gamma_{k+1} \cong g_{k+1} | \mathbf{P}_{k+1} \in (0, \lambda_{k+1}]] \prod_{h=0}^k \Pr[\Gamma_h \cong g_h | \mathbf{P}_h \in (\lambda_h, \lambda_{h-1}]]$$



# Step 5: aux variables help decoupling terms

- Each  $\Gamma_h$  is the aggregate power of the signals in  $U_h$  **given that** they are in the interval  $(\lambda_{h-1}, \lambda_h]$
- We then define

$$\tilde{\Gamma}_h(u, v) = \sum_{i=1}^{r_h} \alpha_{h,i}(u, v)$$

- where  $\alpha_{h,i}(u, v)$  are iid with PDF  $f_{\alpha(u,v)}(x) = f_P(x|P \in (u, v])$
- Hence, for any given  $\mathbf{g}$ , we have

$$\Pr[\Gamma_h \cong g_h | \mathbf{P}_h \in (\lambda_h, \lambda_{h-1}]] =$$

$$\Pr[\tilde{\Gamma}_h(\lambda_h, \lambda_{h-1}) \cong g_h] = \left[ \bigotimes_{i=1}^{r_h} f_{\alpha(\lambda_h, \lambda_{h-1})} \right](g_h) \stackrel{\text{Fourier Transform}}{=} \int_{-\infty}^{+\infty} \left[ \Psi_{\alpha(\lambda_h, \lambda_{h-1})}(\xi) \right]^{r_h} e^{i2\pi\xi g_h} d\xi$$

# Step 6: put all pieces together

$$c(\mathbf{r}) = \iiint F_P(\lambda_{k+1})^{r_{k+1}} \left[ \prod_{h=0}^k [F_P(\lambda_h) - F_P(\lambda_{h-1})]^{r_h} \right] \\ \left[ \int_{-\infty}^{+\infty} \prod_{h=0}^k [\Psi_{\alpha(\lambda_h, \lambda_{h-1})}(\xi)]^{r_h} e^{i2\pi\xi g_h} d\xi \right]^{r_h} \int_{-\infty}^{+\infty} \Psi_{\alpha(0, \lambda_{k+1})}(\xi) e^{i2\pi\xi g_{k+1}} d\xi dg$$

- Number of nested integrals grows linearly with number K of SIC iterations, not with  $n$ 
  - ▣ Equation can be computed for large values of  $n$ , provided that the number of SIC iterations remains reasonable (5÷6)
- Central limit theorem can be invoked for sufficiently large  $r_h$

$$\int_{-\infty}^{+\infty} \Psi_{\alpha(u,v)}(\xi)^r e^{i2\pi\xi g} d\xi \approx \exp\left(\frac{\left(g - rm_{\alpha(u,v)}\right)^2}{2r\sigma_{\alpha(u,v)}^2}\right) / \sqrt{2\pi r\sigma_{\alpha(u,v)}^2}$$

# Throughput

- $S_n(k)$ : average number of signals captured by a system with collision size  $n$  and at most  $K$  SIC iterations

- Exact expression:

$$S_n(K) = \sum_{r=1}^K r C_n(r; K)$$

- Approximate (iterative) expression

$$\tilde{S}_n(K) = \sum_{h=0}^K \tilde{r}_h$$

- Where  $\tilde{r}_h$  is the approximate mean number of signals decoded at the  $h$ -th SIC iteration



# Approximate mean number of captures: first reception

□ Iteration  $h=0$ : number of undecoded signals  $n_0=n$

□ Compute capture threshold

$$I_0 = b(n-1)E[P] = b(n-1)m_{\alpha(0,\infty)}$$

□ Approximate capture condition

$$\Pr[P > I_0] = 1 - F_P(I_0)$$

□ Mean number of decoded signals

$$\tilde{r}_0 = n(1 - F_P(I_0))$$

□ Residual interference power

$$R_0 = z\tilde{r}_0 E[P|P > I_0] = z\tilde{r}_0 m_{\alpha(I_0,\infty)}$$

# Approximate mean number of captures: first reception

- Iteration  $h > 0$ : number of undecoded signals:  $n_h = n - \sum_{i=0}^{h-1} \tilde{r}_i$

Residual interf.

Interf. from undecoded signals

- Compute capture threshold 
$$I_h = b \left( \sum_{i=0}^{h-1} R_i + (n_h - 1) E[P | P \leq I_{h-1}] \right)$$

- Approximate capture condition

$$\Pr[P > I_h | P \leq I_{h-1}] = 1 - F_{\alpha(0, I_{h-1})}(I_h)$$

- Mean number of decoded signals

$$\tilde{r}_h = n_h \left( 1 - F_{\alpha(0, I_{h-1})}(I_h) \right)$$

- Residual interference power

$$R_h = z \tilde{r}_h E[P | I_{h-1} \geq P > I_h] = z \tilde{r}_h m_{\alpha(I_h, I_{h-1})}$$





# CASE STUDY

# Case study

## Pure Path Loss (PL)

- TXs uniformly distributed within a circle of radius  $R$  centered at RX

## Rayleigh Fading (RF)

- TXs at equal distance from RX (or **long-term power control**) but signals affected by multi-path fading

## PathLoss & Rayleigh fading (PLRF)

- TXs uniformly scattered around RX, within a disk of radius  $R$  with signals affected by independent Rayleigh fading

## LogNormal (LN)

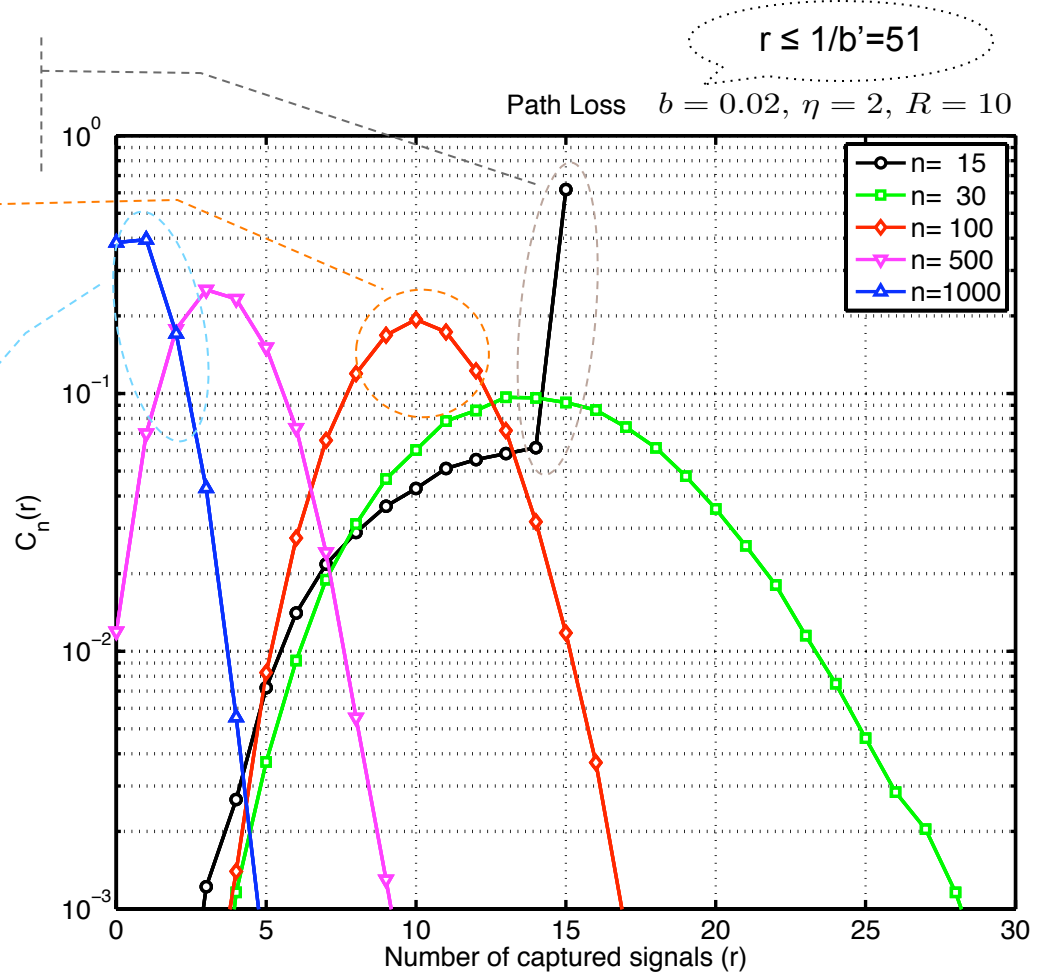
- TXs at equal distance from RX (or **short-term power control**) but nominal signals power affected by small Gaussian noise [dBm]

# $C_n(r)$ in PL scenario

When  $n \ll 1/b'$ , all signals are captured with high probability

When  $n > 1/b'$ , the distribution of the number of signals captured is bell-shaped

When  $n \gg 1/b'$ , fewer and fewer signals are captured



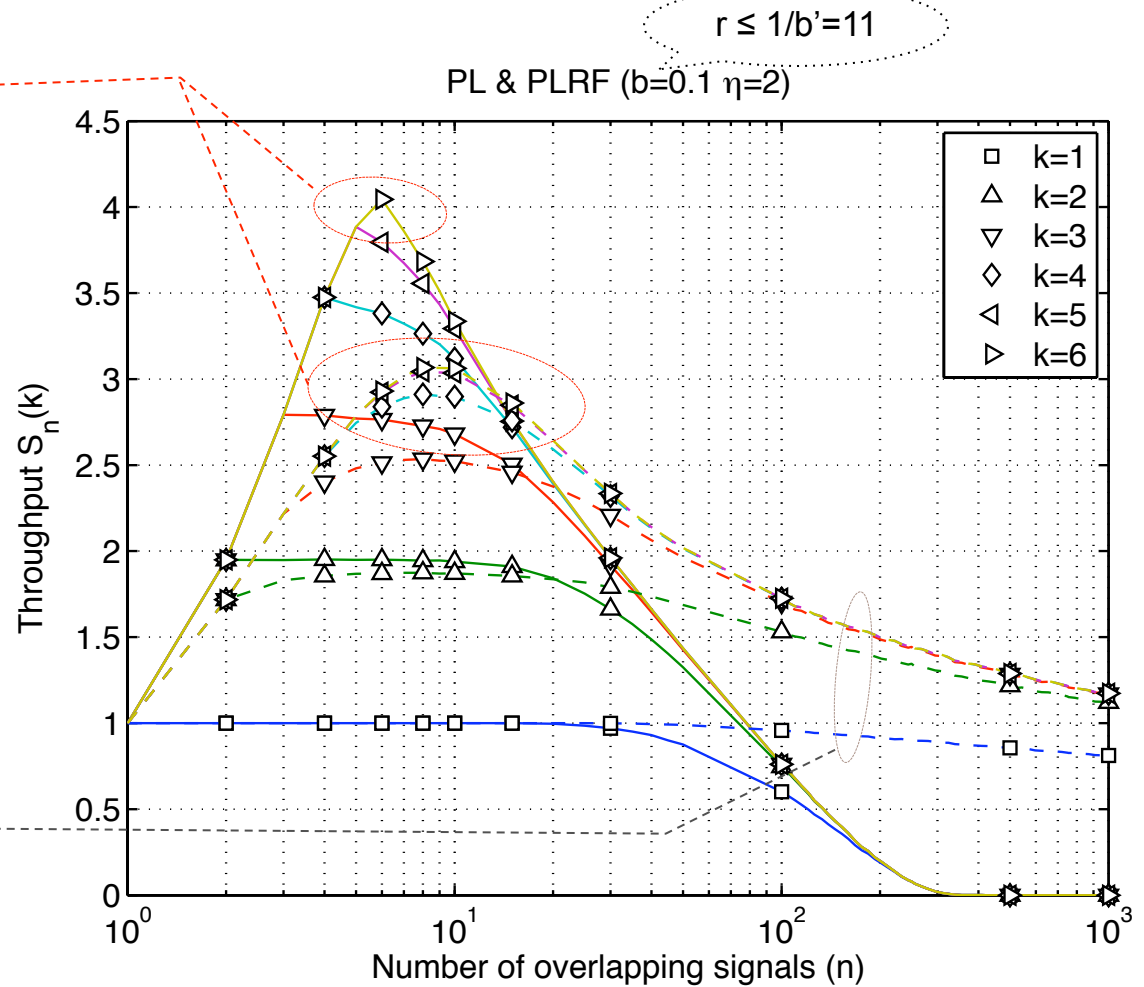
# Throughput in PL & PLRF

$$S(6) \cong S(11)$$



Max performance are closely approached even with partial reception capability.

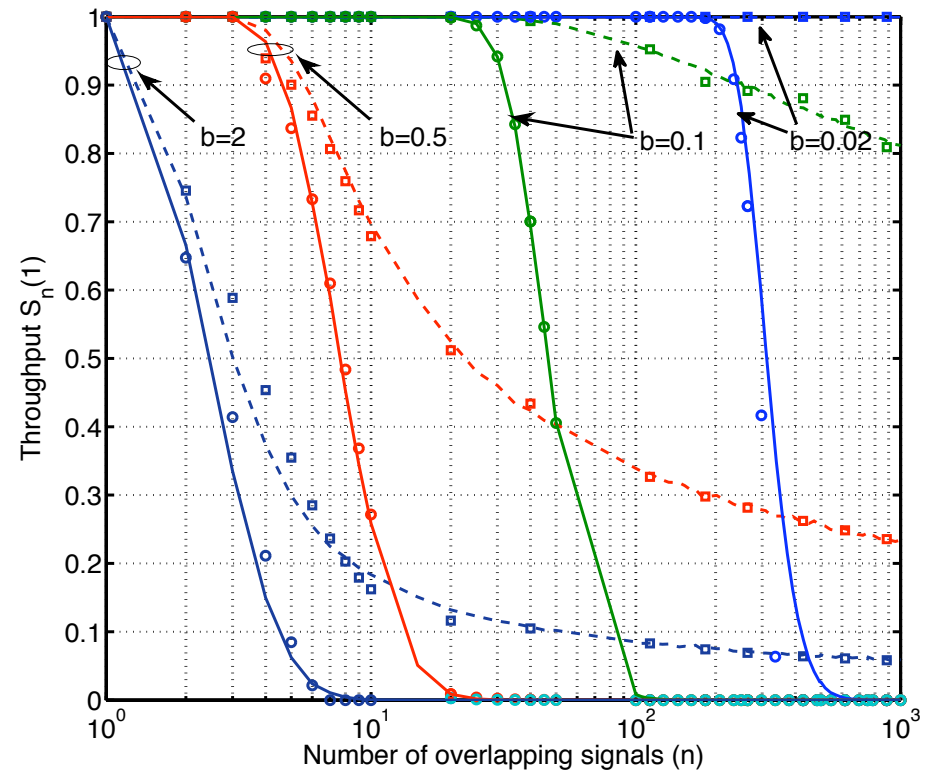
Rayleigh fading augments diversity of received signal strength & increases capture probabilities for large values of  $n$



# Approximate vs exact expressions

- $S_n(1) = 1 - C_n(0) = \Pr[\text{capturing at least one signal}]$ 
  - ▣ metric considered in most of the previous literature
- The accuracy of the approx. of  $C_n(0)$  is **very good** in all the considered cases
- The approximation of  $C_n(r)$  is **not very good** when either  $r$  or  $n-r$  are positive but small (not shown here)

	RF	PLRF
Exact	————	-----
Approximate	▣	○





# PERFORMANCE WITH SIC

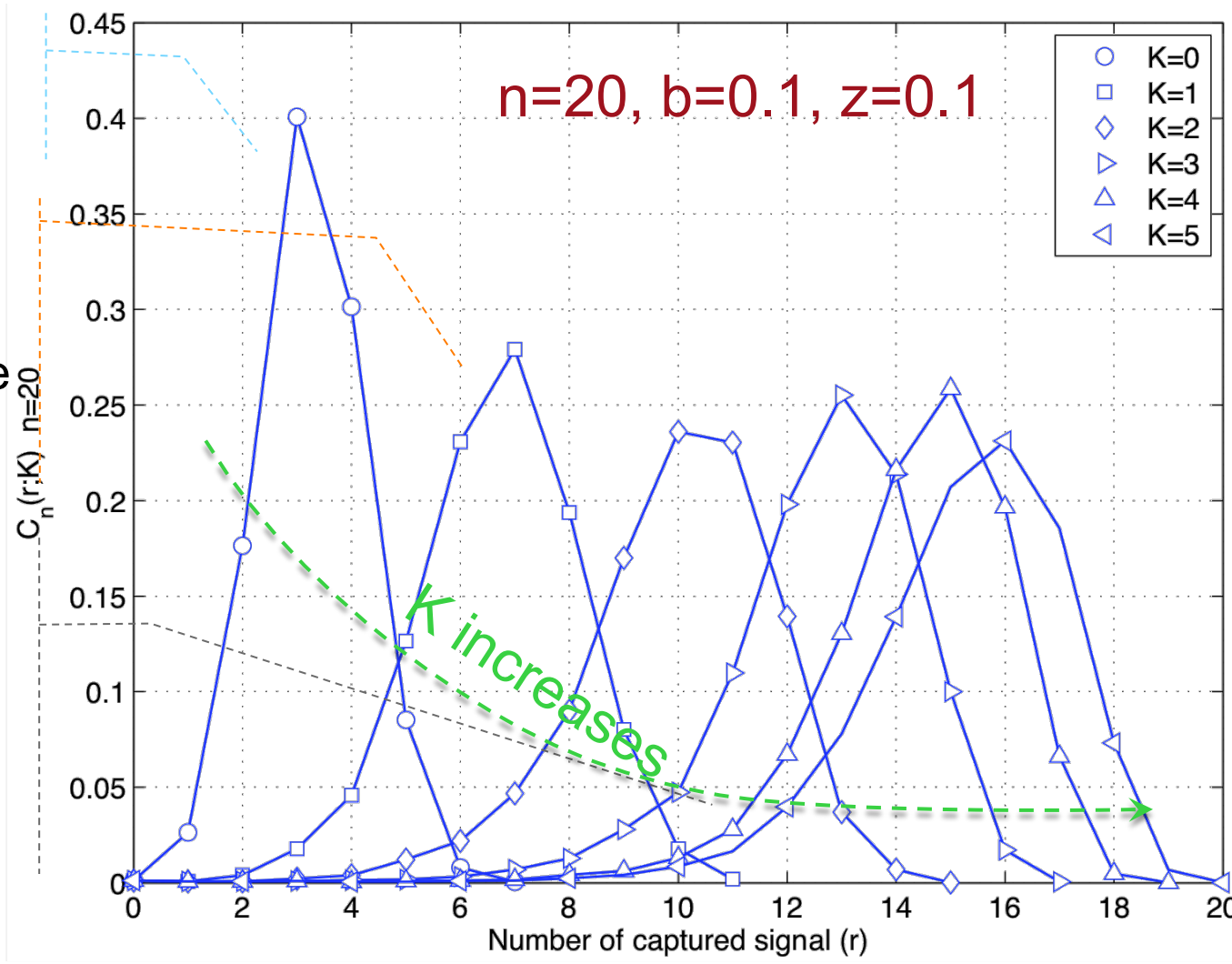
Rayleigh channel only

# Capture probability distribution

No SIC (K=0): likely to decode 2÷5 signals

One SIC (K=1): likely to decode 4÷10 signals, double capture capabilities!!!

Multiple SIC (K>1): capture probability keeps improving, but gain reduces



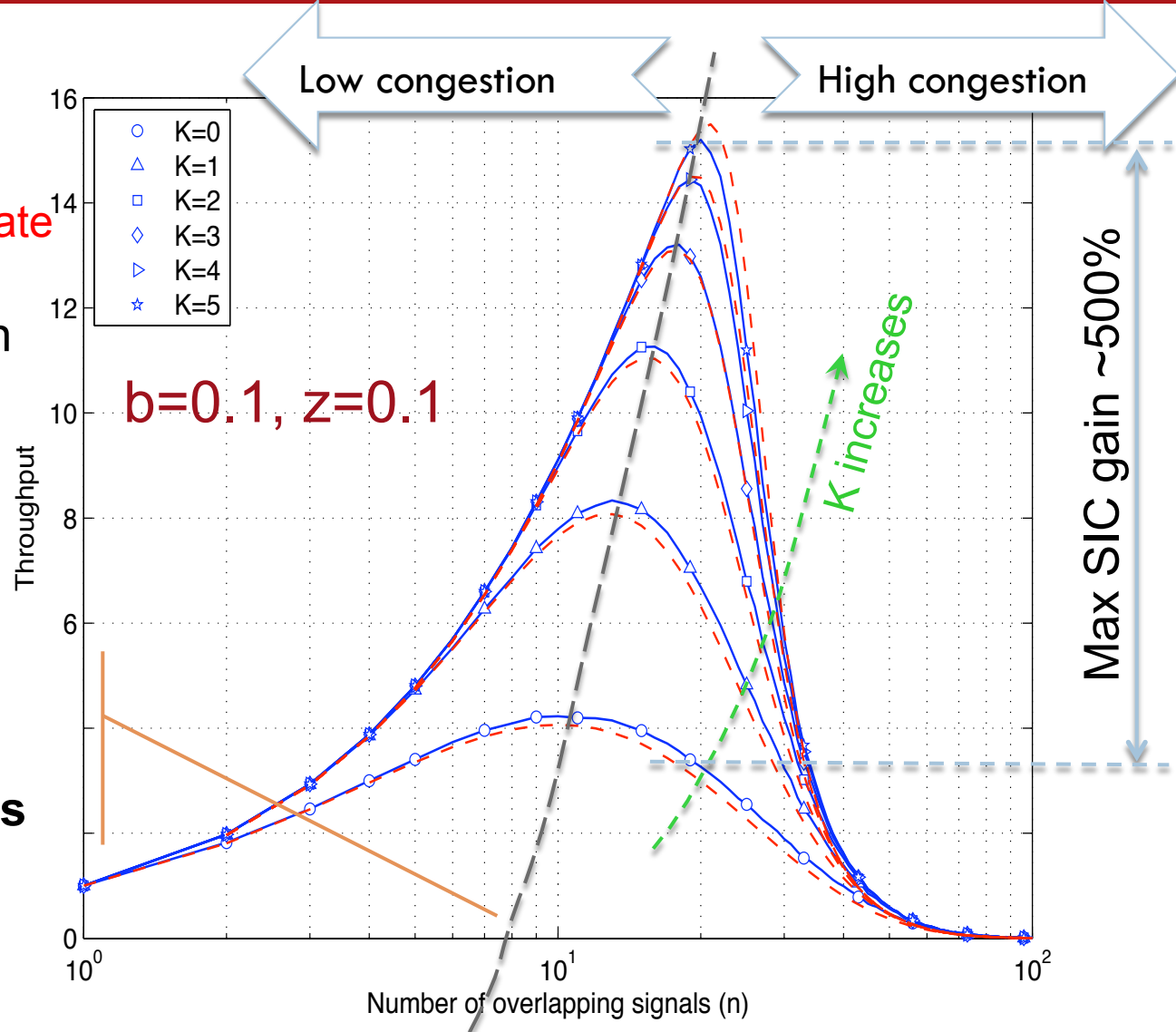
# Throughput vs n

— exact  
 - - - approximate

Approximation is quite good!

optimal # of concurrent transmissions

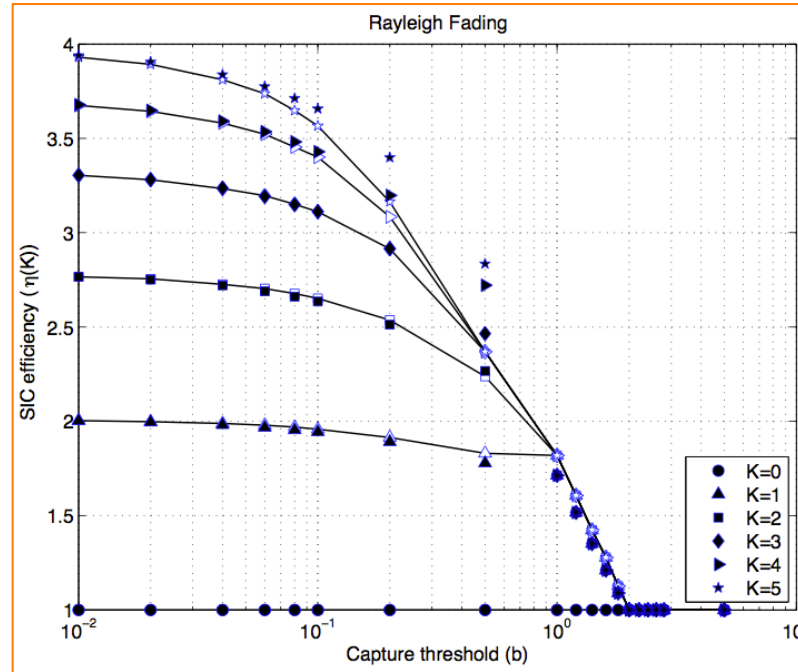
$b=0.1, z=0.1$



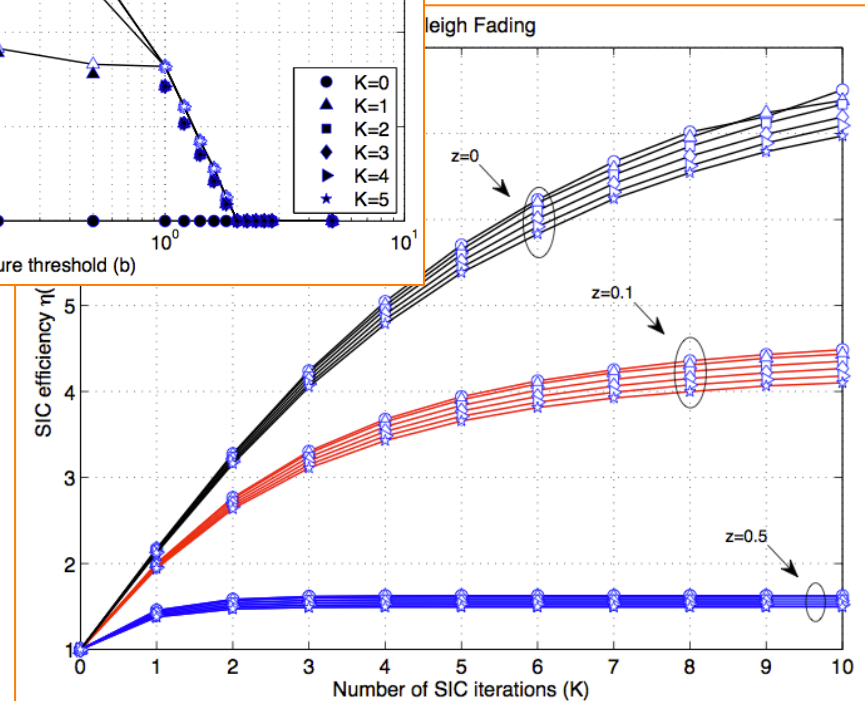


# Max SIC gain analysis

- SIC is more effective with small  $b$
- The less residual interference, the larger the SIC gain
- For  $K > 1/z$ , SIC gain is negligible
  - Empirical conjecture



$$\eta(K) = \frac{\max_n \{S_n^{(s)}(K)\}}{\max_n \{S_n^{(s)}(0)\}}$$

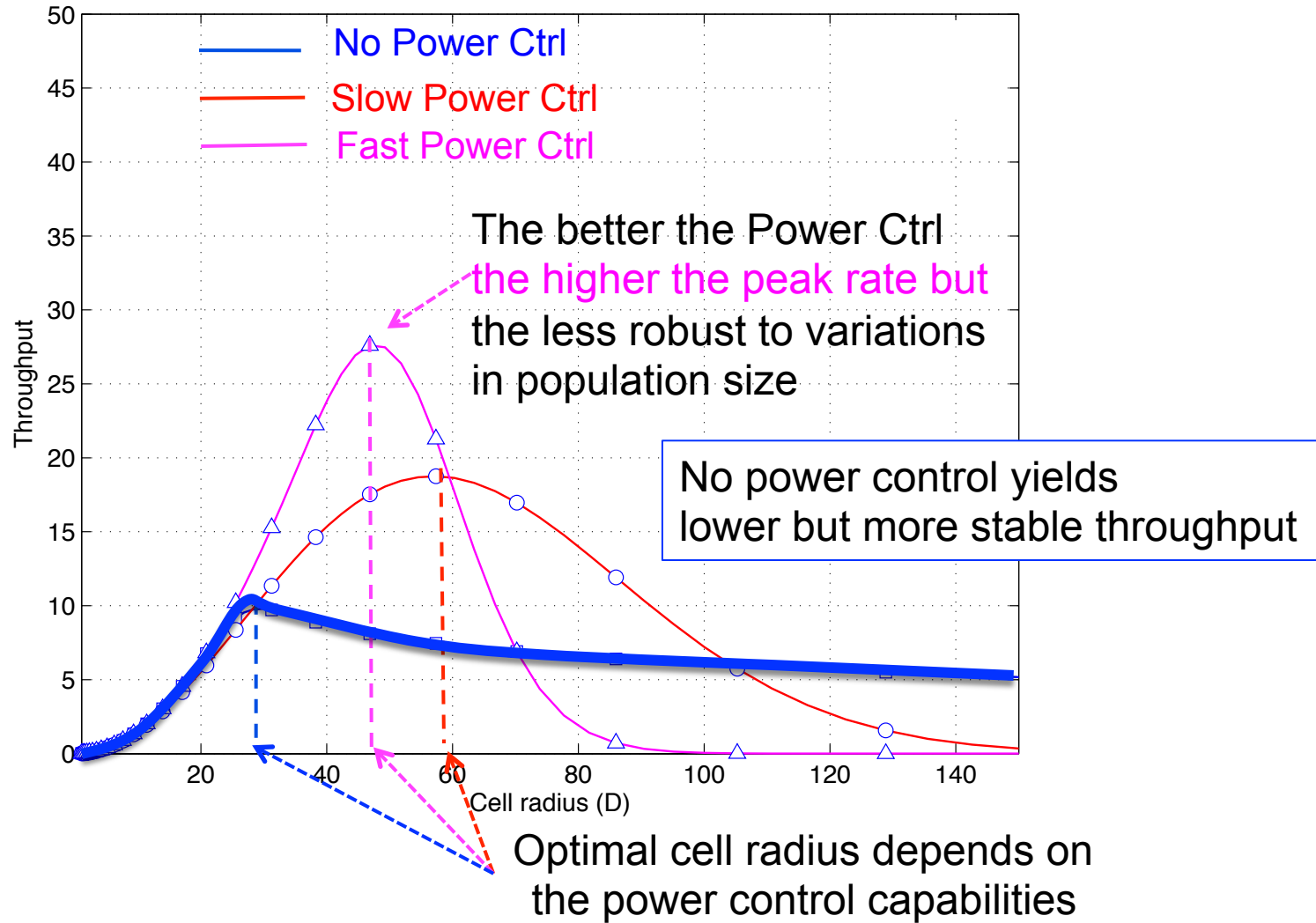




# An example of application to network planning

- Goal: dimensioning an MPR access point
  - Given parameters:
    - ▣ Users' spatial distribution: Poisson process with density  $\delta$
  - Performance metric: average throughput  $S_D(K) = E[S_n(K)]$
  - Knobs
    - ▣ Cell radius:  $D$
    - ▣ Capture threshold:  $b$
    - ▣ MPR capability:  $R$
    - ▣ SIC capability:  $K$
    - ▣ Power control capabilities
      - NoPC: No power control
      - SPC: Slow power control
      - FPC: Fast power control
- Path Loss propagation model (PL)  
 → Rayleigh Fading (RF)  
 → LogNormal (shadowing) fading (LN)

# 1<sup>st</sup> step: determine throughput when varying cell radius with infinite MPR and no SIC

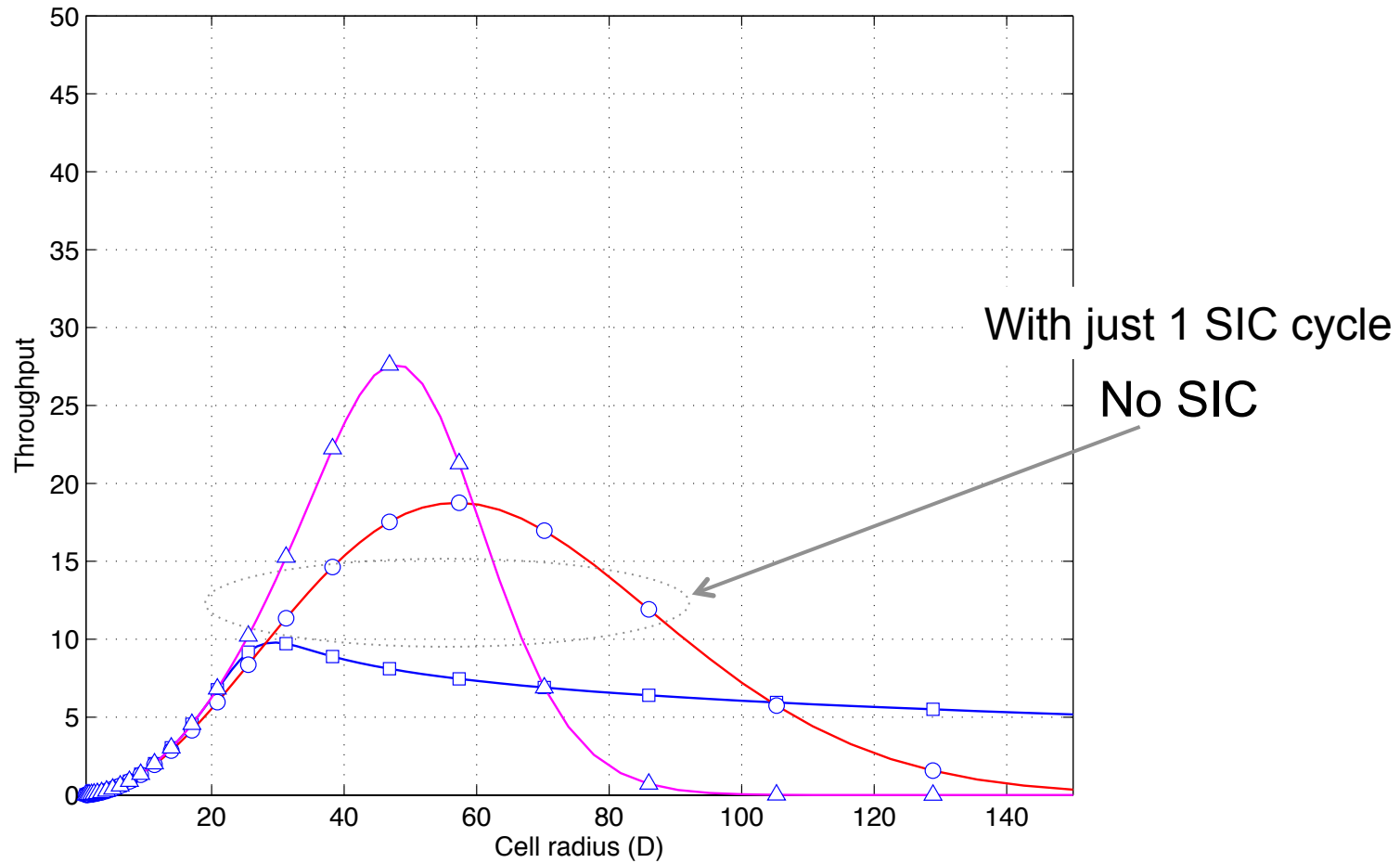




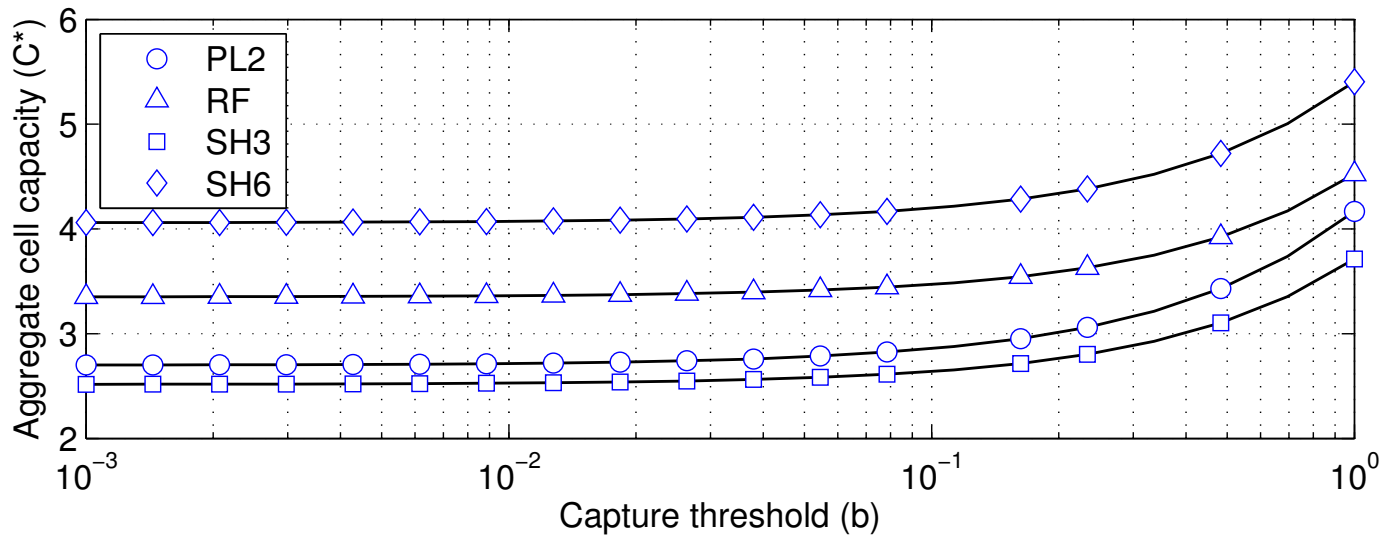
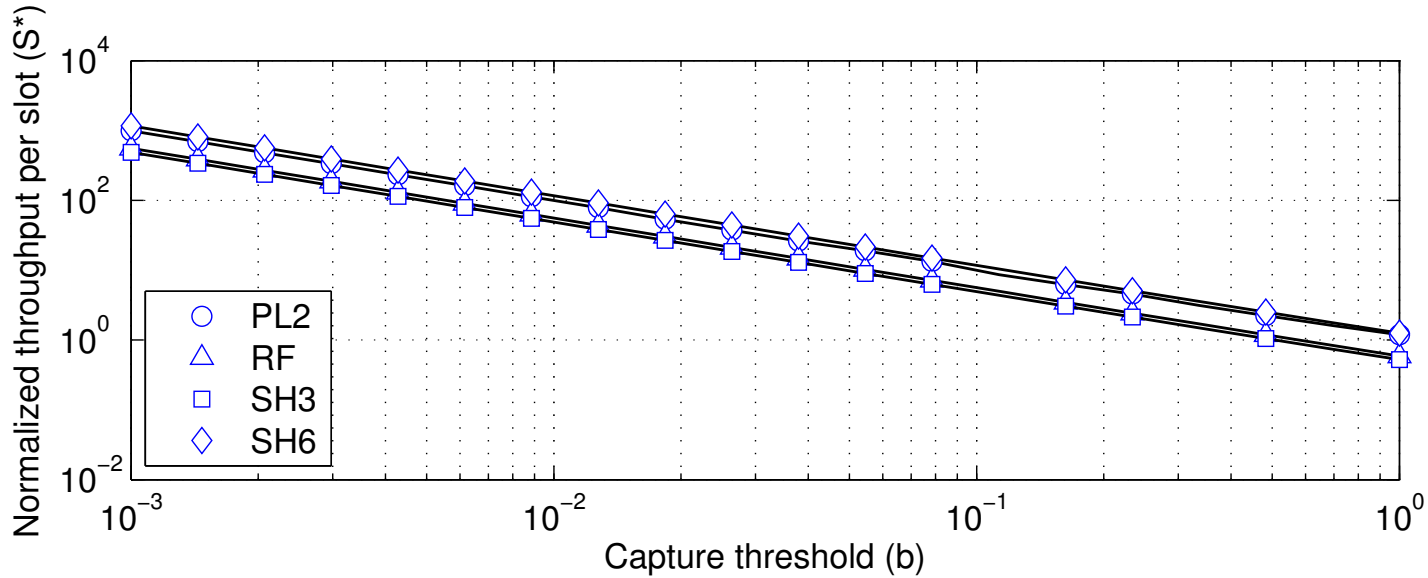
## 2<sup>nd</sup> step: determine the number of MPR cycles

- Focusing on the No Power Control case we set the cell radius to  $D=50\text{m}$
- Using our equations we find the minimum MPR capability  $R_m$  of the receiver beyond which the performance gain becomes negligible
  - we can set  $R_m = \arg \min_R \{S_n(R)/S_n(\infty) \geq 1 - \rho\}$  where  $\rho$  is the maximum acceptable performance loss
  - In our example,  $R_m=15$  yields less than 10% of throughput loss

# 3<sup>rd</sup> step: introduce SIC



# Asymptotic performance



Zanella et al. "M2M massive wireless access: challenges, research issues, and ways forward" – Globecom 2013

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- A. Zanella, and M. Zorzi "Theoretical Analysis of the Capture Probability in Wireless Systems with Multiple Packet Reception Capabilities" IEEE Transactions on Communications, vol.60, no.4, pp.1058-1071, April 2012
- Zanella et al. "M2M massive wireless access: challenges, research issues, and ways forward" – Globecom 2013



# Spare slides



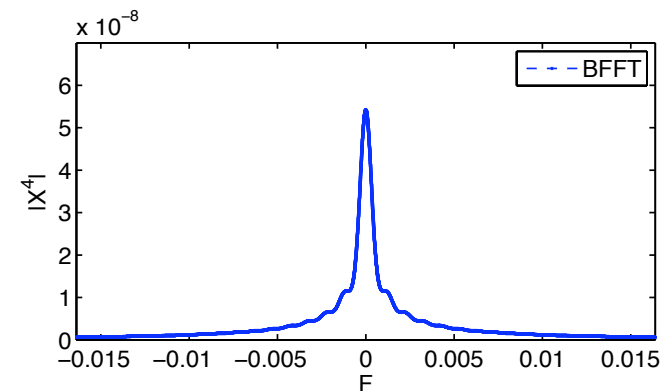
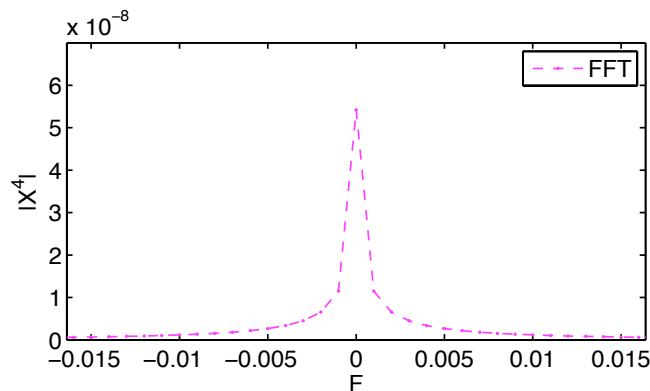
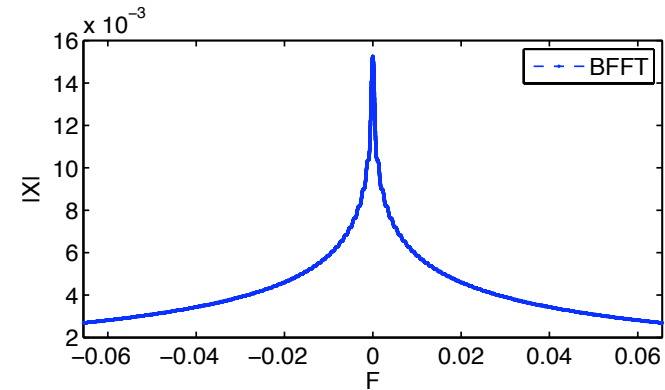
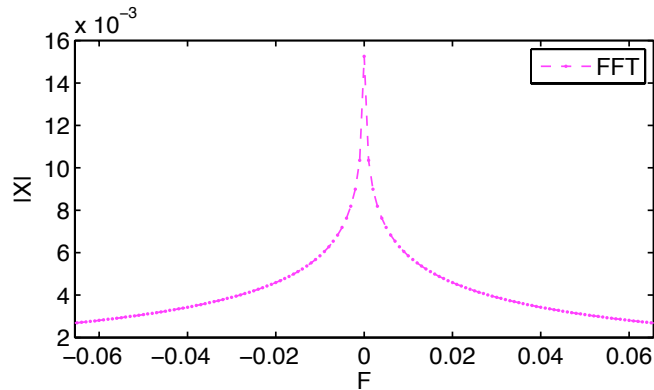




# Break!!!! We all need it!



# Bluestein FFT algorithm



- FFT samples are equally spaced over the entire signal bandwidth
- When raising the FTs to a power  $>1$  most of such samples reduce to zero!
- Bluestein's FFT algorithm (BFFT) "squeezes" the samples into a fraction of the original bandwidth, so that samples are still significant after power raising

# Rayleigh fading

- Exponential distribution of the received power  $P_i$

$$f_P(x) = \exp(-x)1(x); \quad F_P(x) = [1 - \exp(-x)]1(x);$$

- Fourier Transform of the auxiliary rv  $\alpha(u,v)$

$$\Psi_{\alpha(u,v)}(\xi) = \frac{e^{-u(i2\pi\xi + 1)} - e^{-v(i2\pi\xi + 1)}}{(1 + i2\pi\xi)(e^{-u} - e^{-v})}$$

- Mean value of  $\alpha(u,v)$

$$m_{\alpha(u,v)} = \frac{(u+1)e^{-u} - (v+1)e^{-v}}{e^{-u} - e^{-v}}$$

# Appendix: asymptotic throughput

$$\begin{aligned}
 \mathcal{T}^*(n) &\simeq \frac{n}{\Lambda}. & \frac{n}{\Lambda} &\simeq \mathbb{E} \left[ y_{w_n^*, n} \right] + \sum_{s=0}^{\infty} \frac{n-s}{\Lambda} p_{w_n^*, n}(s) \\
 & & &\simeq \mathbb{E} \left[ y_{w_n^*, n} \right] + \frac{n}{\Lambda} - \frac{\mathbb{E}[S]}{\Lambda},
 \end{aligned}$$

$$\Lambda \simeq \frac{\mathbb{E}[S]}{\mathbb{E} \left[ y_{w_n^*, n} \right]}.$$

$$\Lambda \simeq \frac{w_n^* n q_n^* (1 - q_n^*)^{n-1}}{\beta_p + w_n^* [\beta_c + (1 - q_n^*)^n (\beta - \beta_c) + n q_n^* (1 - q_n^*)^{n-1} (1 - \beta_c)]},$$

# Appendix: asymptotic throughput

$$\mu_n = nq_n^*, \quad \mu_\infty = \lim_{n \rightarrow \infty} \mu_n.$$

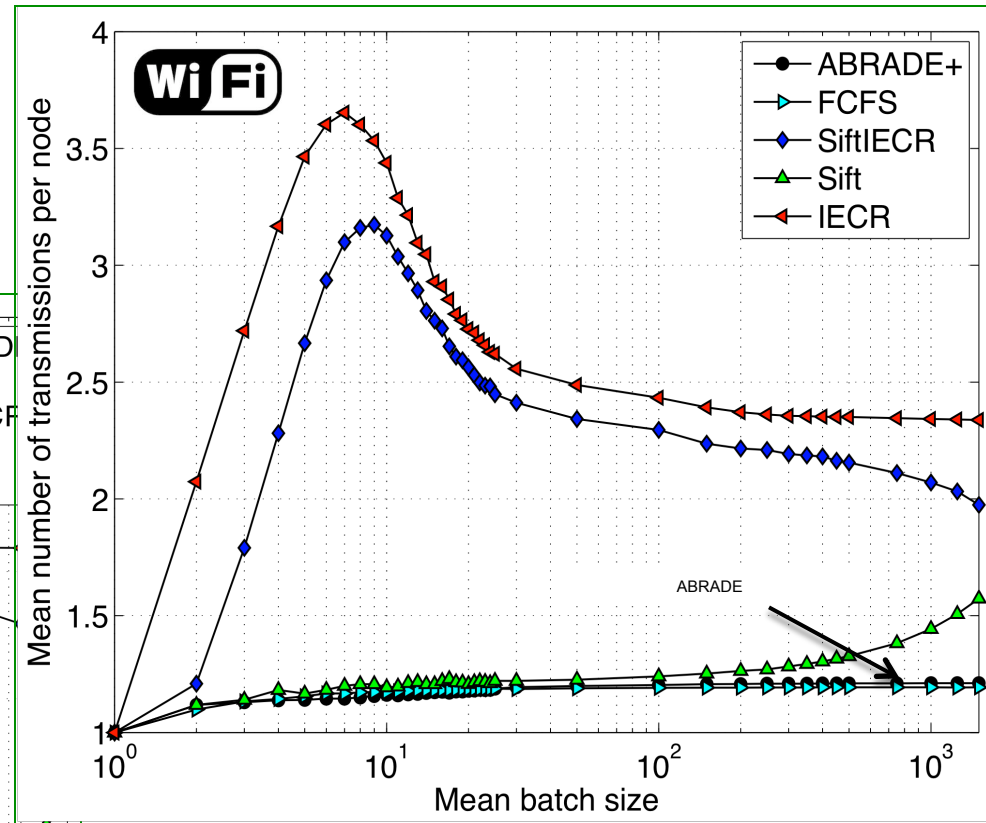
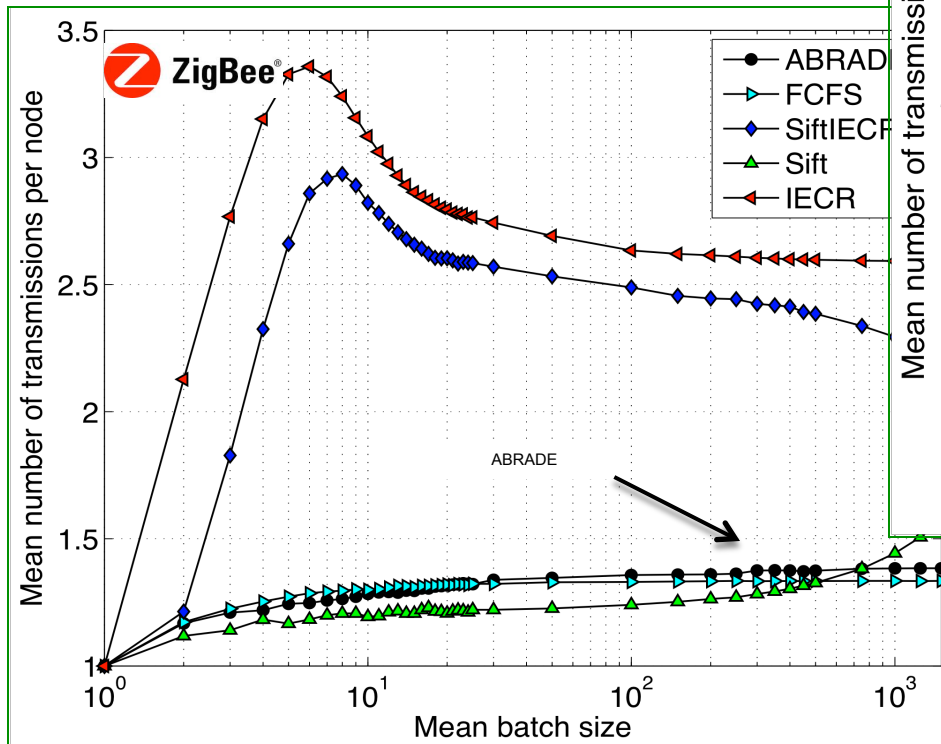
$$\Lambda = \frac{\mu_\infty e^{-\mu_\infty}}{b_p + \beta_c + e^{-\mu_\infty}(\beta - \beta_c) + \mu_\infty e^{-\mu_\infty}(1 - \beta_c)}$$

Taking the derivative wrt  $\mu_\infty$   $\mu_\infty = 1 + \frac{\beta - \beta_c}{b_p + \beta_c} \exp(-\mu_\infty),$

$$\Lambda = \frac{e^{-\mu_\infty}}{b_p + \beta_c + e^{-\mu_\infty}(1 - \beta_c)},$$

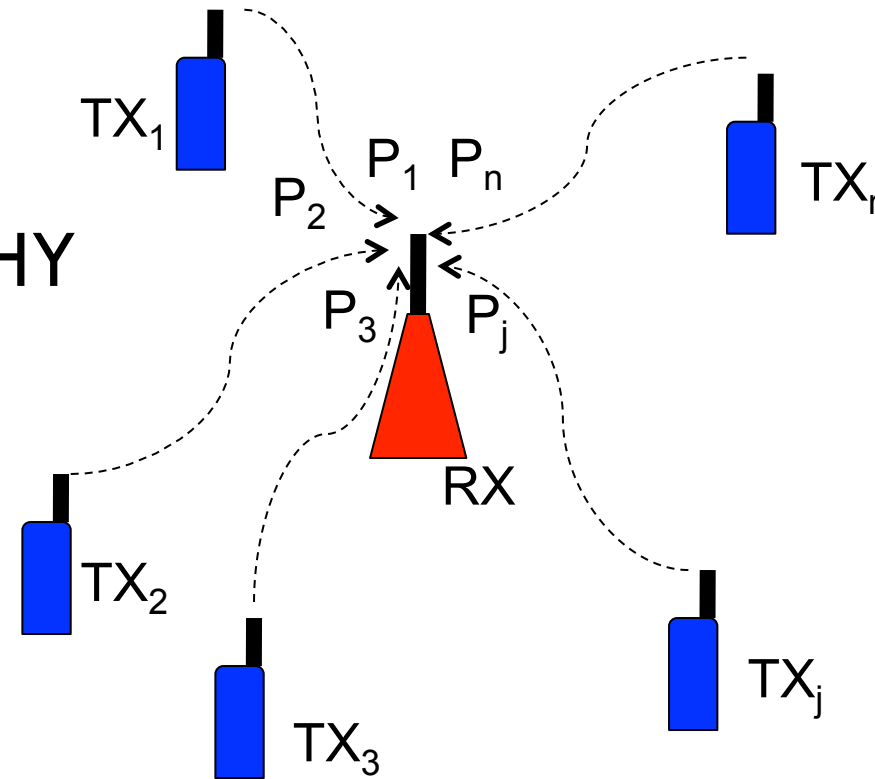
# 1. Poisson: Energy efficiency

- Mean number of tx per slot (proportional to energy consumption) comparable to the best performing algorithms



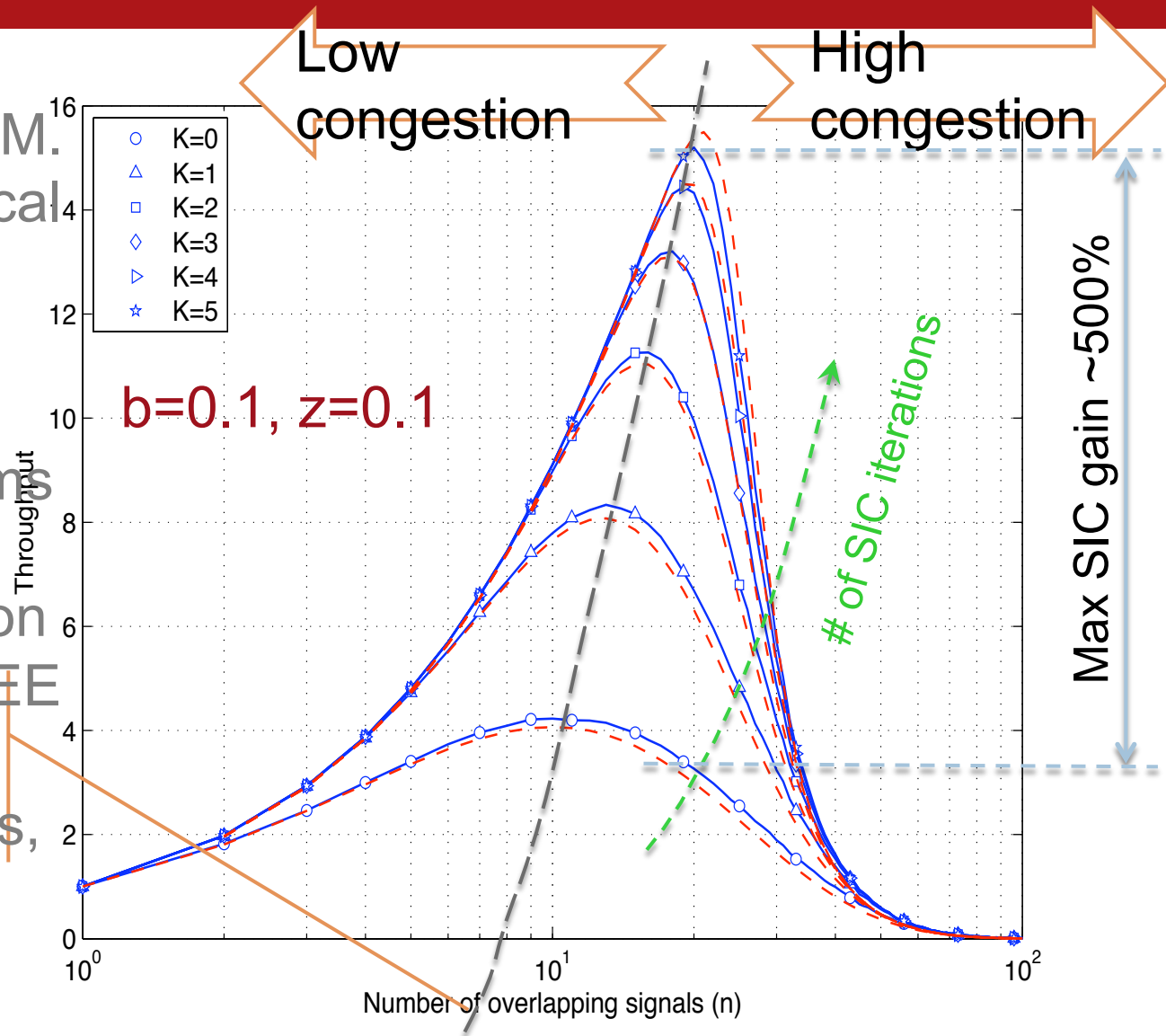
# massive asynchronous access

- approach
  - ▣ move complexity to BS
  - ▣ use advanced MAC/PHY
    - MPR: multi packet reception
    - SIC: successive interference cancellation

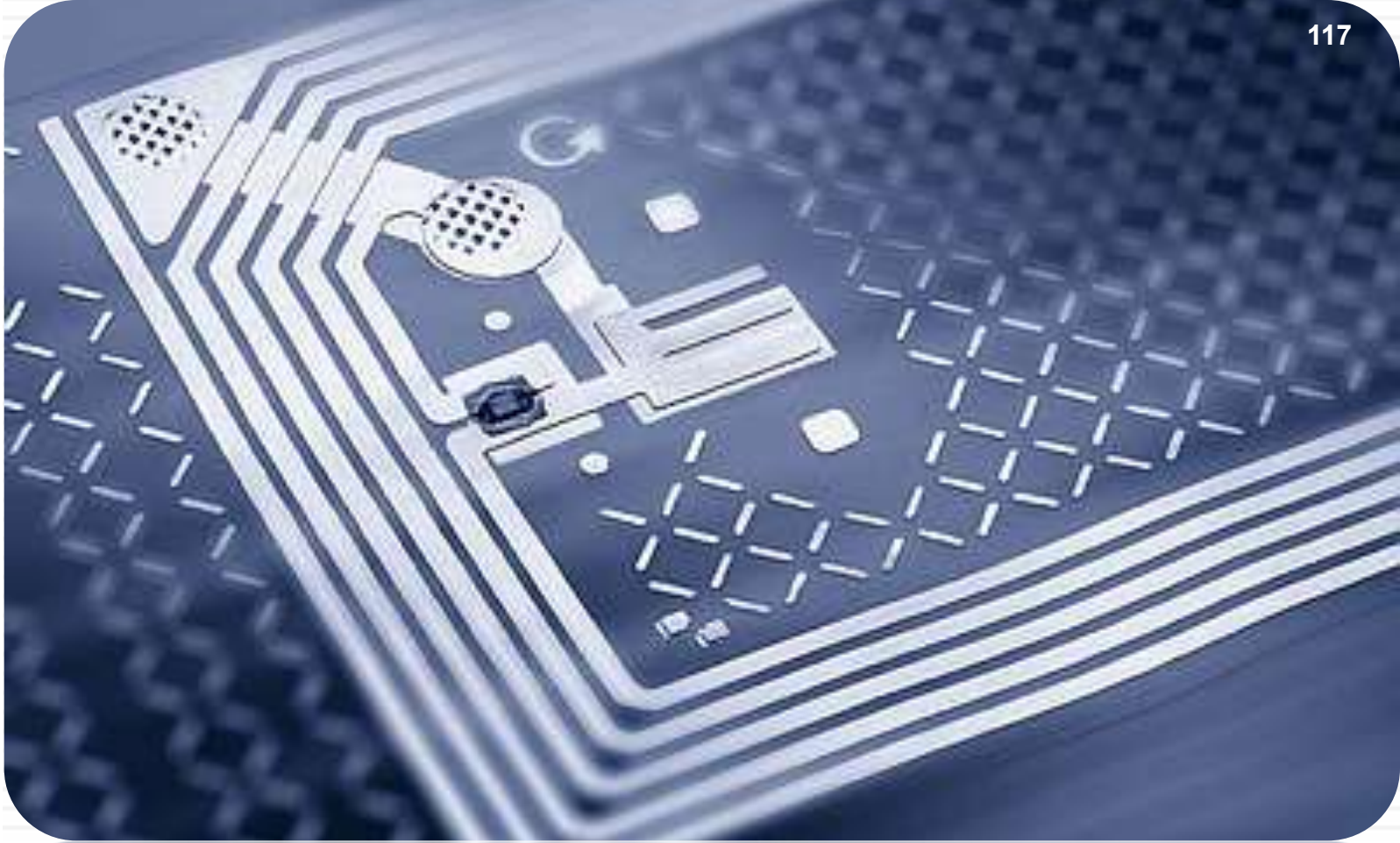


# SIC+MPR throughput

A. Zanella, and M. Zorzi, "Theoretical Analysis of the Capture Probability in Wireless Systems with Multiple Packet Reception Capabilities" IEEE Transactions on Communications, 2012



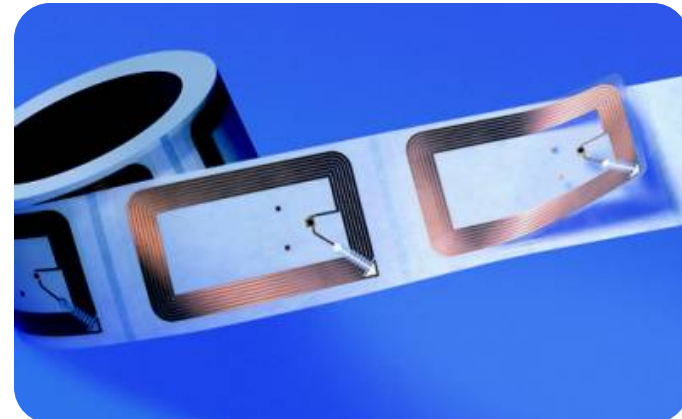
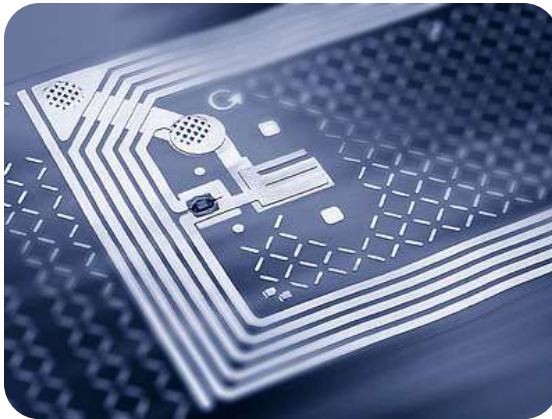




## What is RFID

# What is RFID

- RFID = **R**adio **F**requency **I**dentification

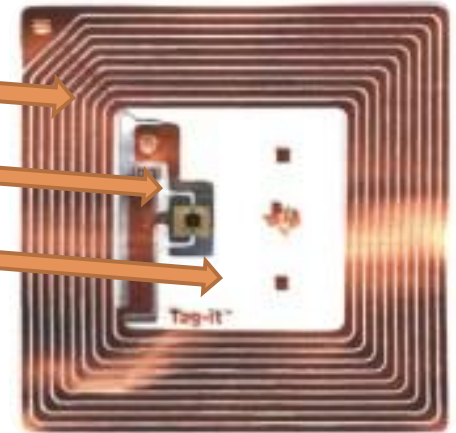


- An RFID tag is an object that can be applied to or incorporated into a product, animal, or person for the purpose of identification using radiowaves
- Some tags can be read from several meters away and beyond the line of sight of the reader

Wikipedia  
definition of RFID

# Components and types of RFID tag

- ❑ Antenna: for receiving and transmitting the signal
- ❑ Integrated Chip
- ❑ Plastic Inlay
- ❑ Maybe sensor, battery, external memory...



## TYPES

- ❑ **Passive:** no battery, the electrical current induced in the antenna by the incoming radio frequency signal provides just enough power in the tag to power up and transmit a response
- ❑ **Active:** internal power source, which is used to power the integrated circuits and broadcast the signal to the reader
- ❑ **Semipassive:** similar to active tags in that they have their own power source, but the battery only powers the microchip and does not broadcast a signal.

From Wikipedia

125/134 kHz  
13,56 MHz  
868/915 MHz  
>2,4 GHz

international standard for  
RFID: Epc Gen2 *Electronic  
Product Code Generation 2:*

# Communication in passive tags

Checksum		EPC Code (or User ID Code)								Lock	PC	
Byte	0	1	0	1	2	3	4	5	6	7	0	0
Bit	0-7	8-15	0-7	8-15	16-23	24-31	32-39	40-47	48-55	56-63	0-7	0-7

*Class I Tag Memory Structure*

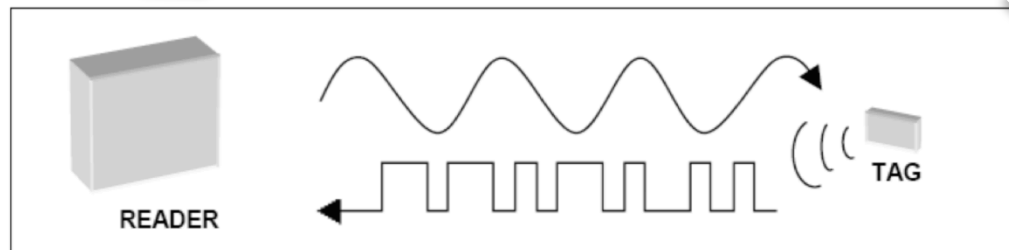
Image from Alien Guide

Reader transmits a wave signal

The tag's antenna gets the power

This power is sufficient to read the data saved in the chip

And transmit the answer to the reader



*The reader transmits a continuous wave signal. The tag breaks up (modulates) that signal into patterns of ones and zeroes that convey its data to the reader.*

Image from Alien Guide



# RFID vs BAR CODE

## RFID

- ❑ Is possible to attach a tag on many surfaces
- ❑ No line-of-sight
- ❑ Many informations and/or applications
- ❑ Can be reprogrammed in the field to reflect current information
- ❑ Cheap: 0,20 \$

## BAR CODE

- ❑ Now everything has a bar code
- ❑ Requires line-of-sight
- ❑ Only ID information
- ❑ Data is fixed at the moment the label is printed
- ❑ Cost free

..but RFID are not only for identification scope..