Practical schemes for Secret Key Agreement as applied to QKD

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Outline

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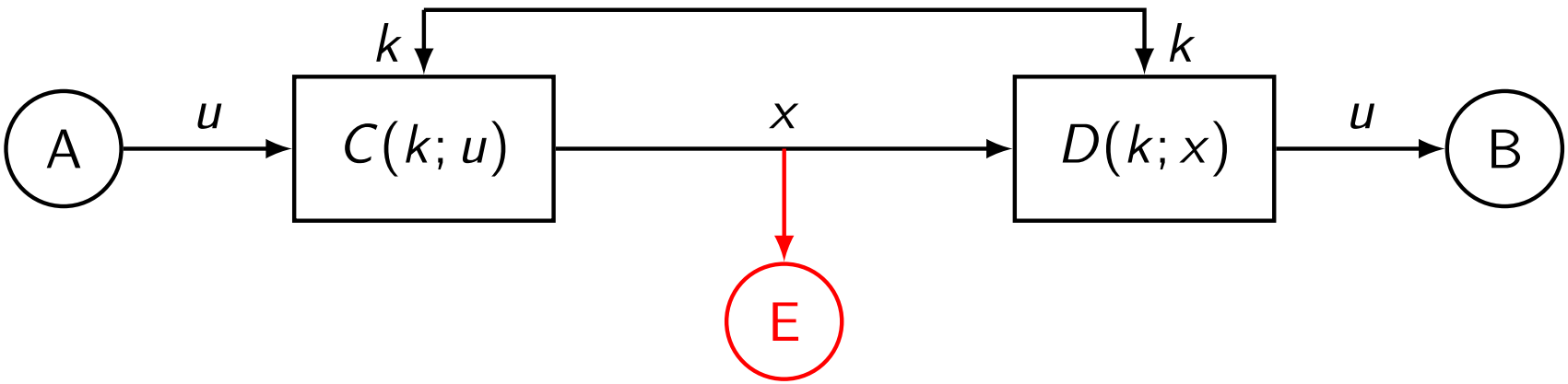
Security in a communication system



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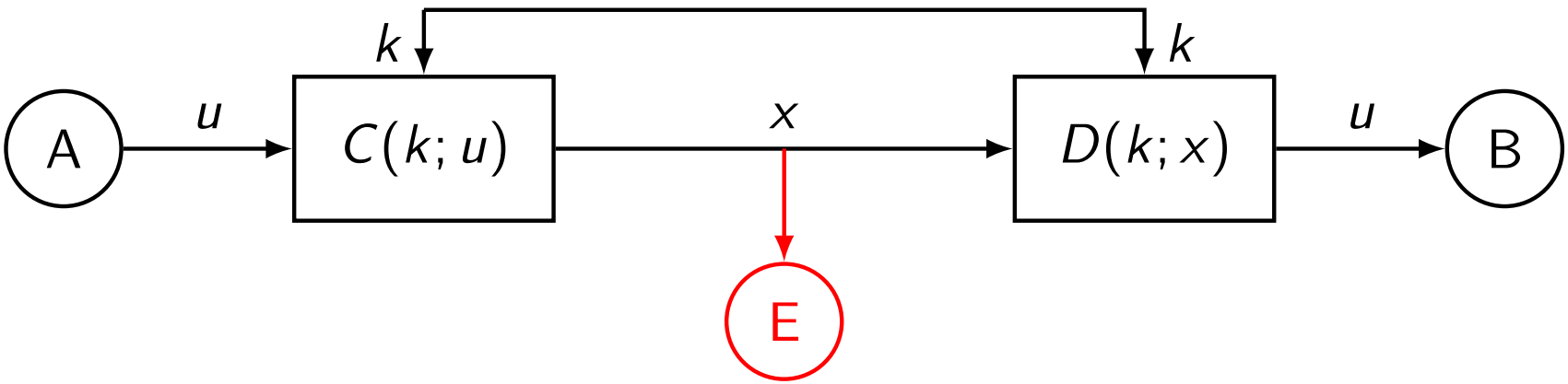
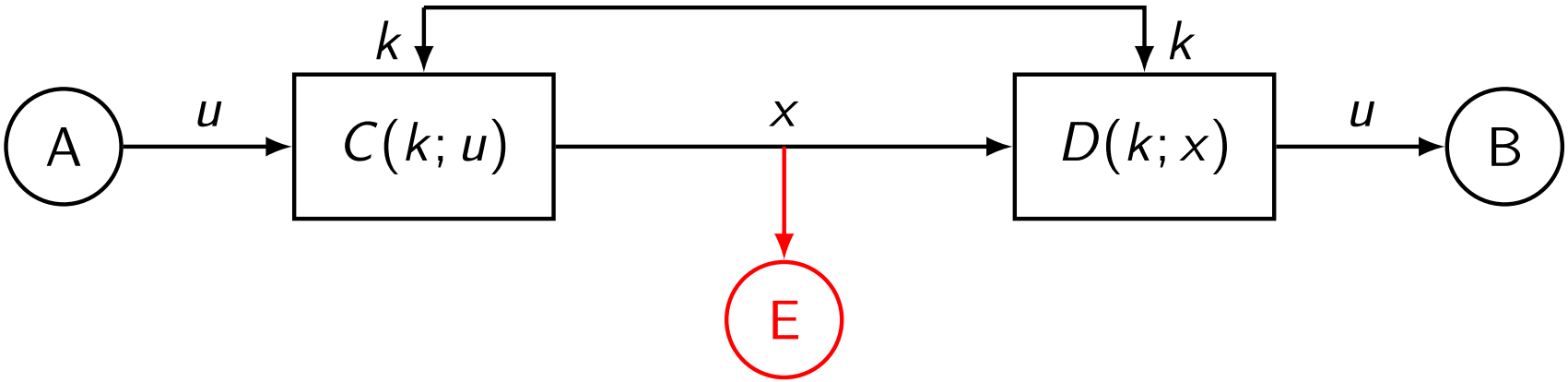


Kerchoffs's Principle

A cryptosystem should be secure even if everything about the system, *except the key*, is public knowledge.

Computational Vs. Information-Theoretic Security



- *Computational security*

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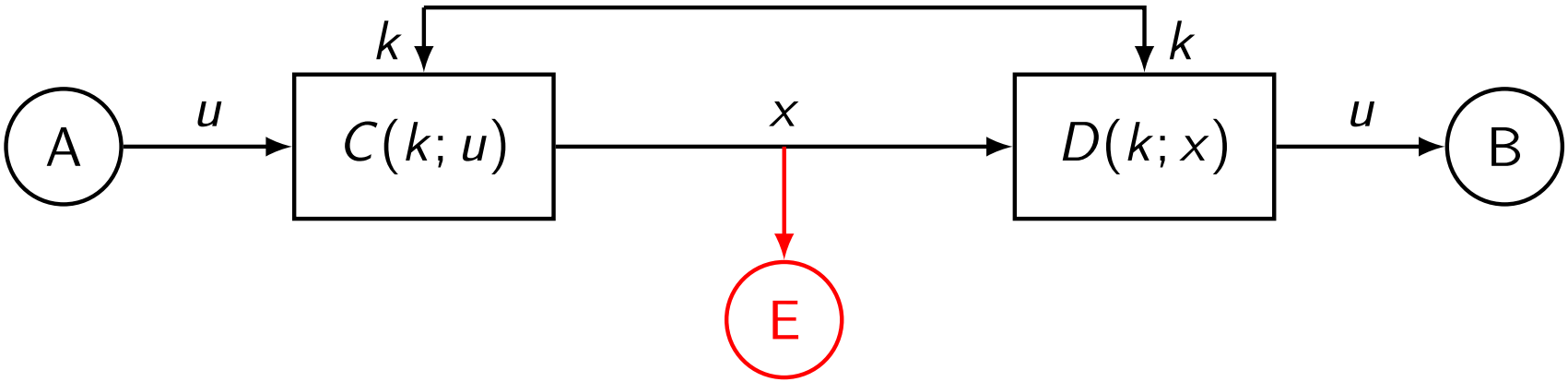

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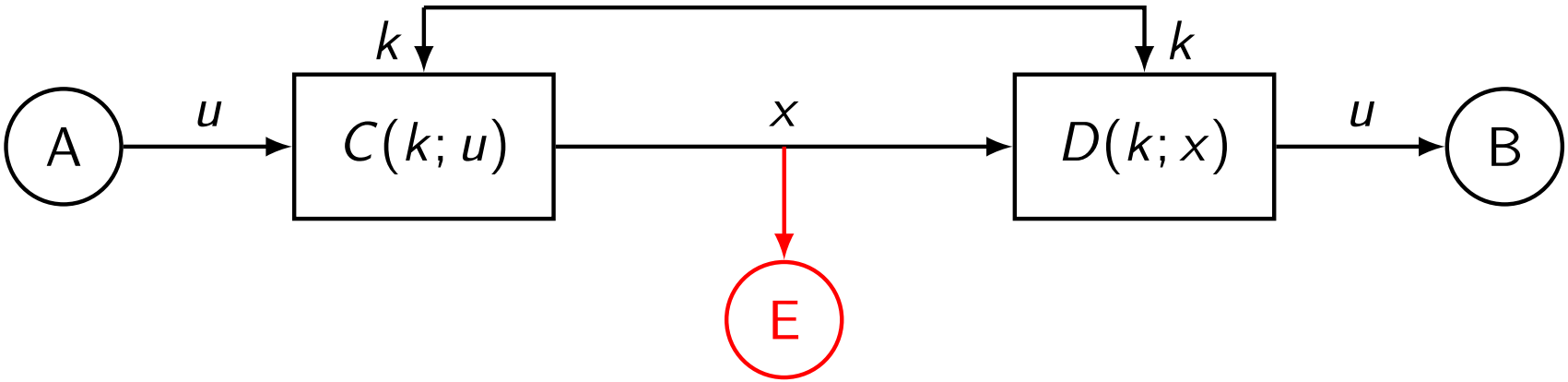

- *Computational security*
  - computing u is *computationally infeasible* given x but not k
  - based on the assumed, yet unproven, hardness of a certain problem (e.g., factoring large integers)
- Information-Theoretic Security
  - $I(u; x) \to 0$
  - based on information theory, it is the strongest notion of security, no assumptions on the attacker's computing power

Quantum tools for I-T security

- Physical laws of quantum mechanics can be exploited while looking for I-T security

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     - no perturbation $\Rightarrow$ no measurement $\Rightarrow$ no eavesdropping

Quantum tools for I-T security

- Physical laws of quantum mechanics can be exploited while looking for I-T security
  1. *Eavesdropping detection*: in quantum systems, one cannot take a measurement without perturbing the system itself.
     - passive attacks can be detected
     - no perturbation $\Rightarrow$ no measurement $\Rightarrow$ no eavesdropping
  2. *No-cloning theorem*: perfect copying is impossible in the quantum domain.
     - replay and man-in-the-middle attacks are more difficult to deploy

Quantum Key Distribution

- Eavesdropping detection + no-cloning theorem

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$$\Downarrow$$

Quantum Key Distribution
(QKD)

QKD at UNIPD: the QuantumFuture project

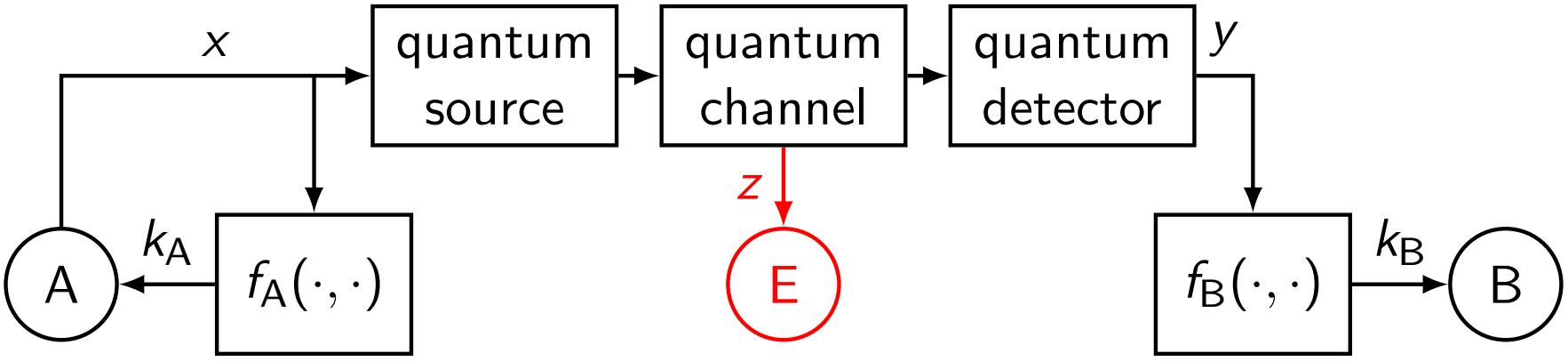
QuantumFuture

- 3-year research project at UNIPD
- 1.4 M€, funded by the University of Padova
- 4 RUs: Telecom, Controls, Optics, Astronomy
- Main focus on free-space QKD

More information available at:
http://quantumfuture.dei.unipd.it

Outline

QKD system model (I)



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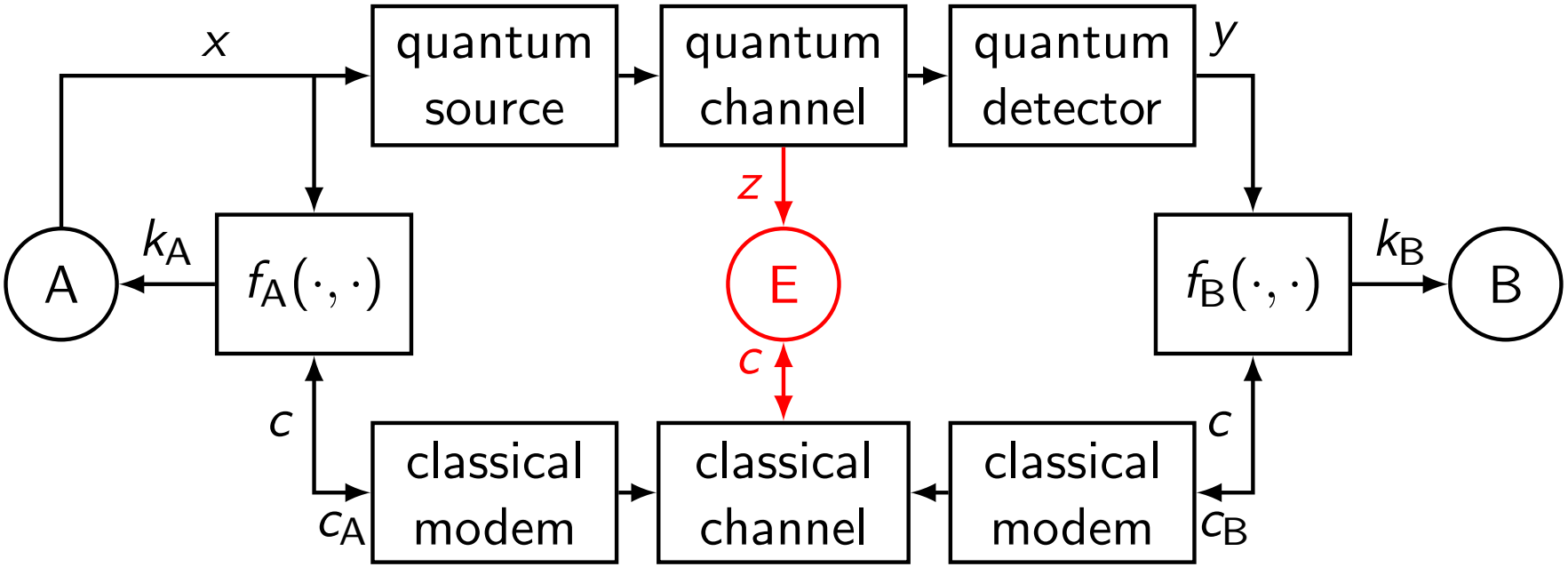

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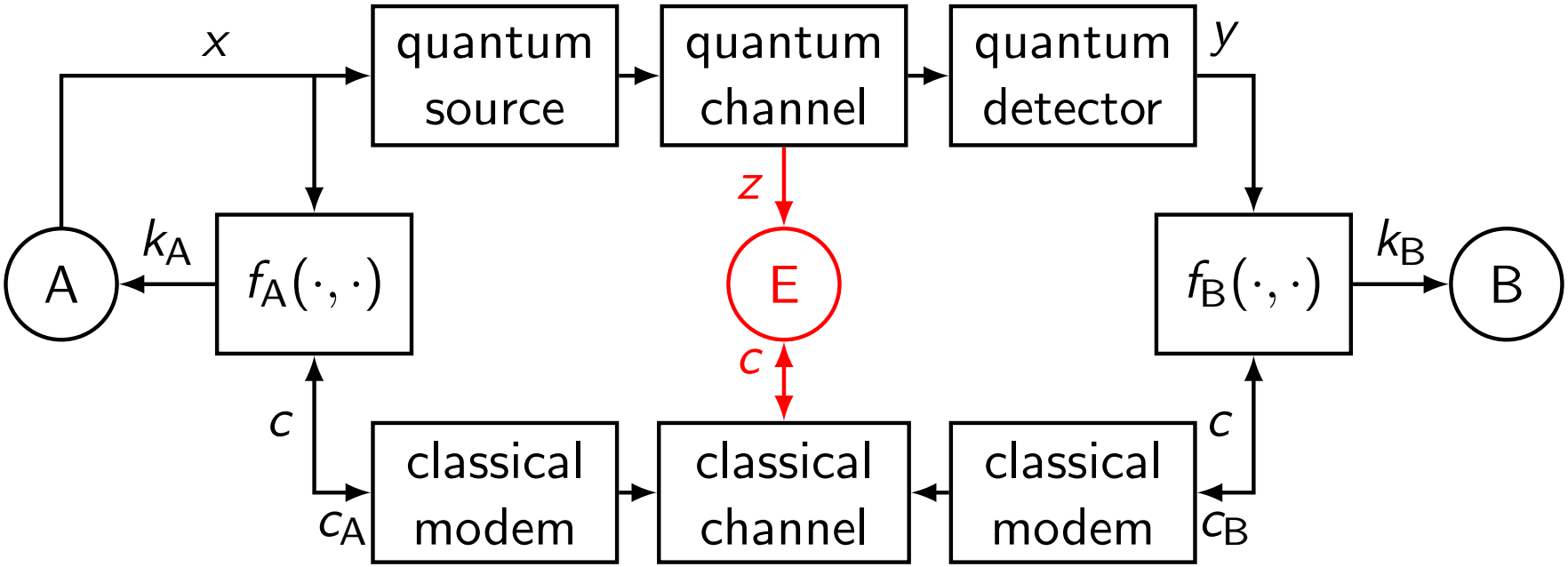
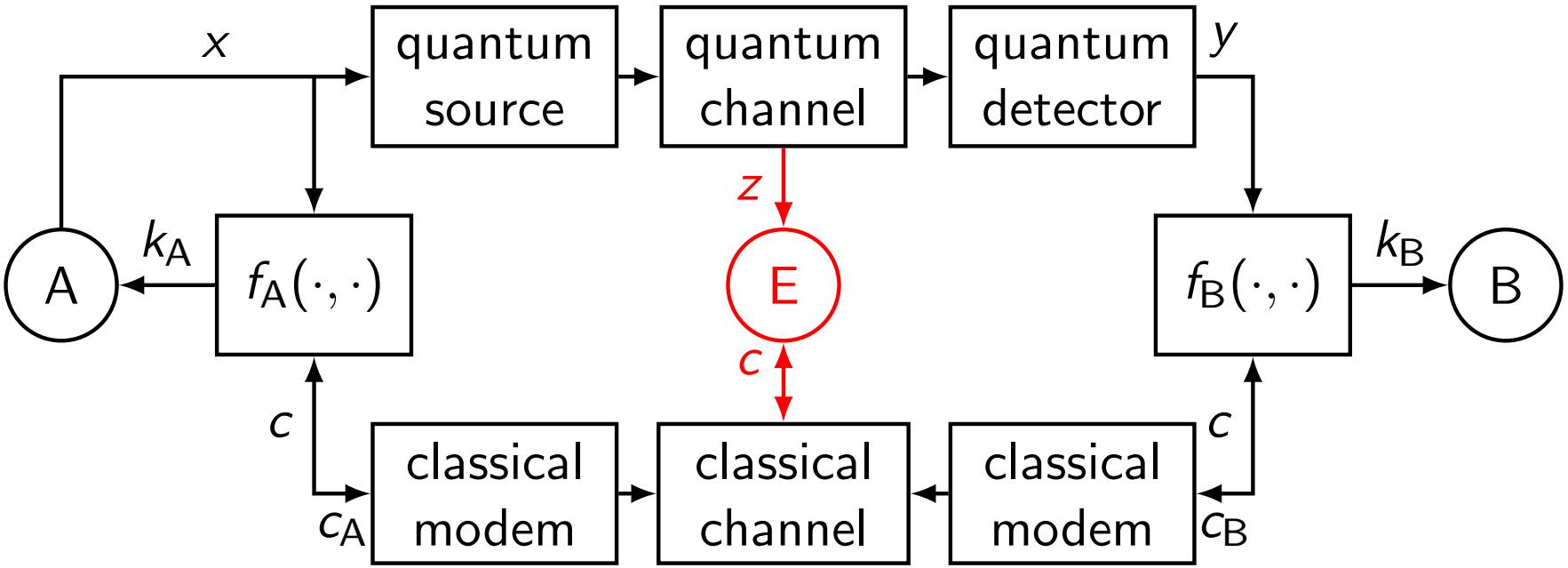


Channel characteristics

Quantum Ch.	Classical Ch.
private	public, authenticated
low rate	high rate
unreliable	reliable

QKD system model (II)



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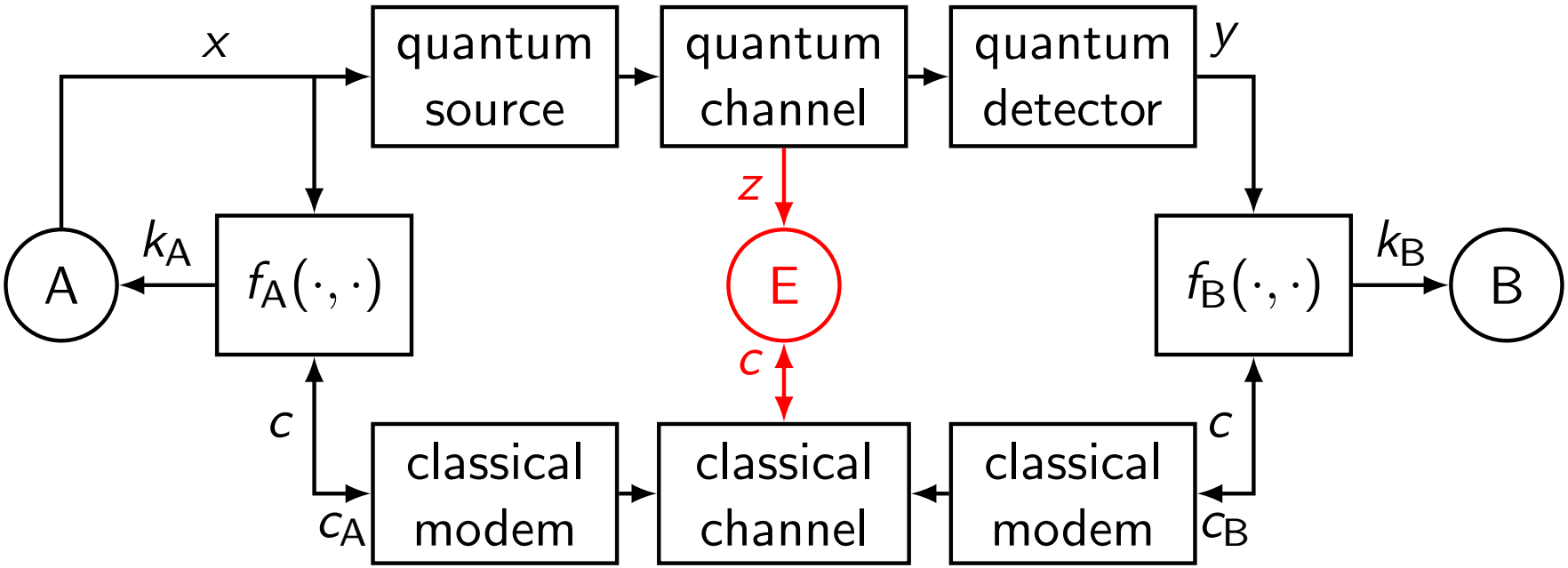

Objective

$$\max_{f_a, f_B, x} H(k_A) \quad \text{subject to:}$$

correctness: $P[k_A = k_B] < \varepsilon$

secrecy: $I(k_A, k_B; z, c) < \varepsilon'$

uniformity: $L - H(K_A) < \varepsilon''$

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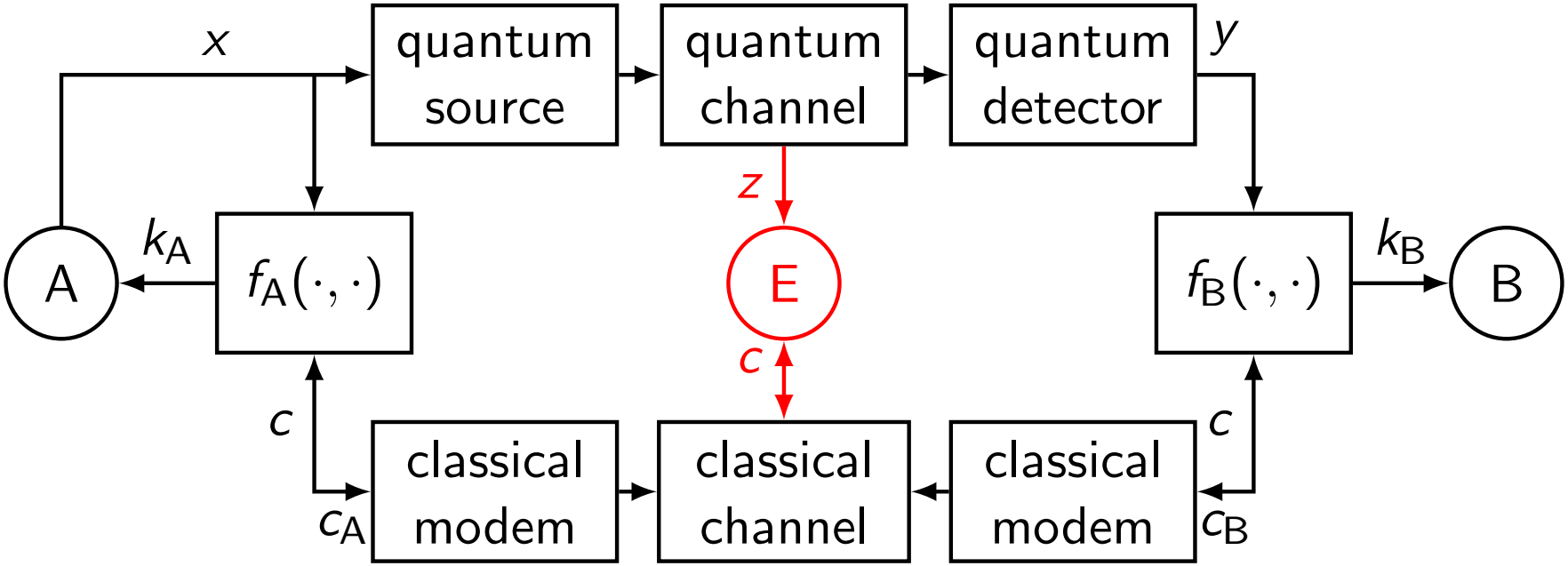

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Upper bound

For $\varepsilon, \varepsilon', \varepsilon'' \to 0$

$$\max_{f_A, f_B, x} H(k_A) \leq \max_{x} I(x; y|z)$$

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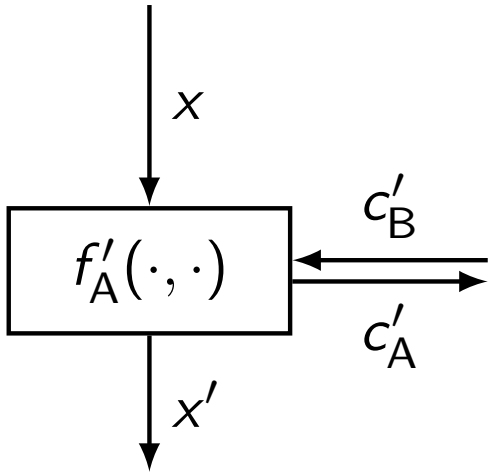

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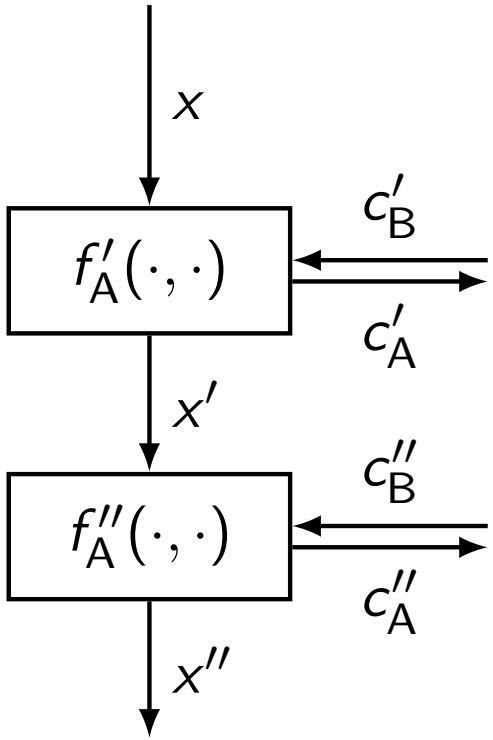

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3. Privacy amplification $\rightarrow$ secrecy

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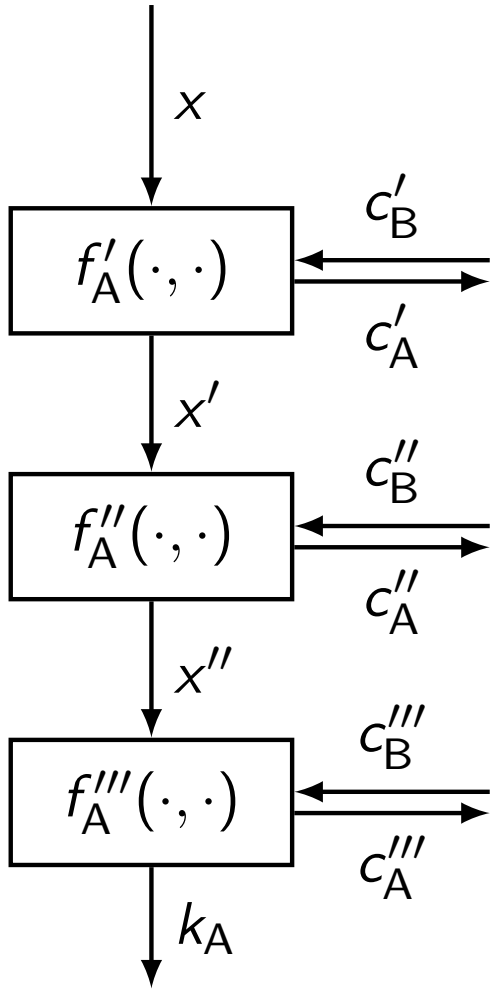

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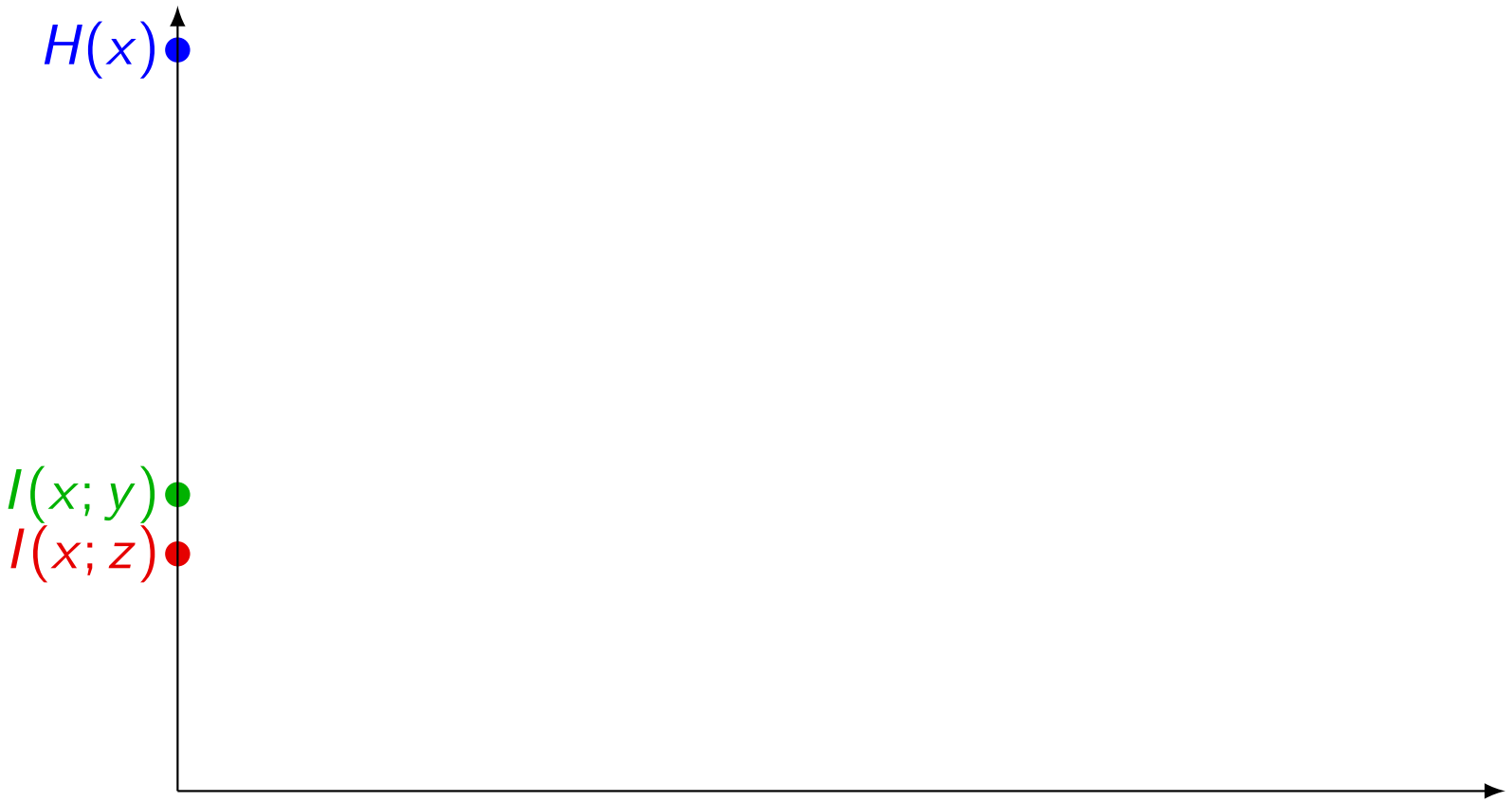

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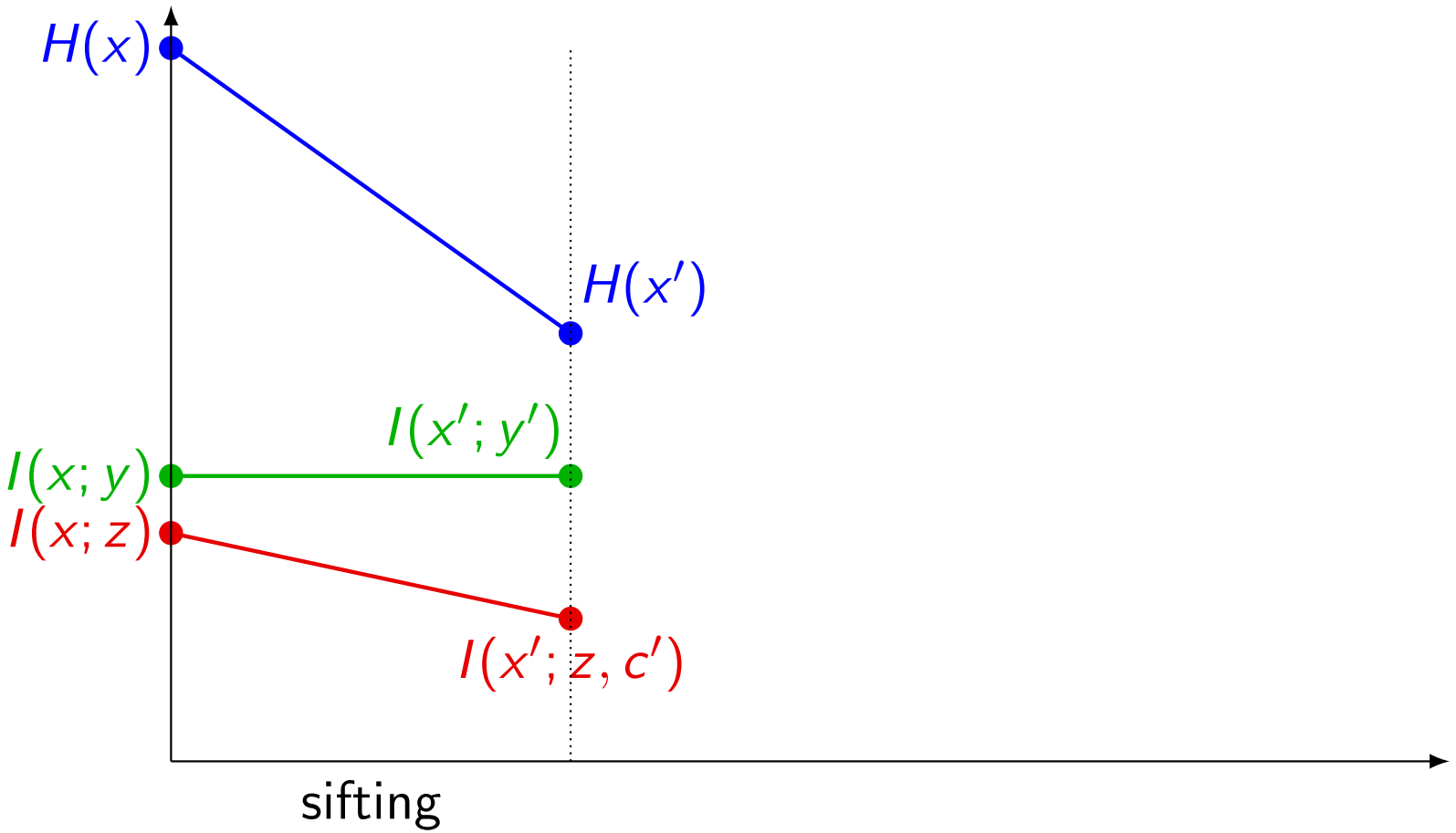

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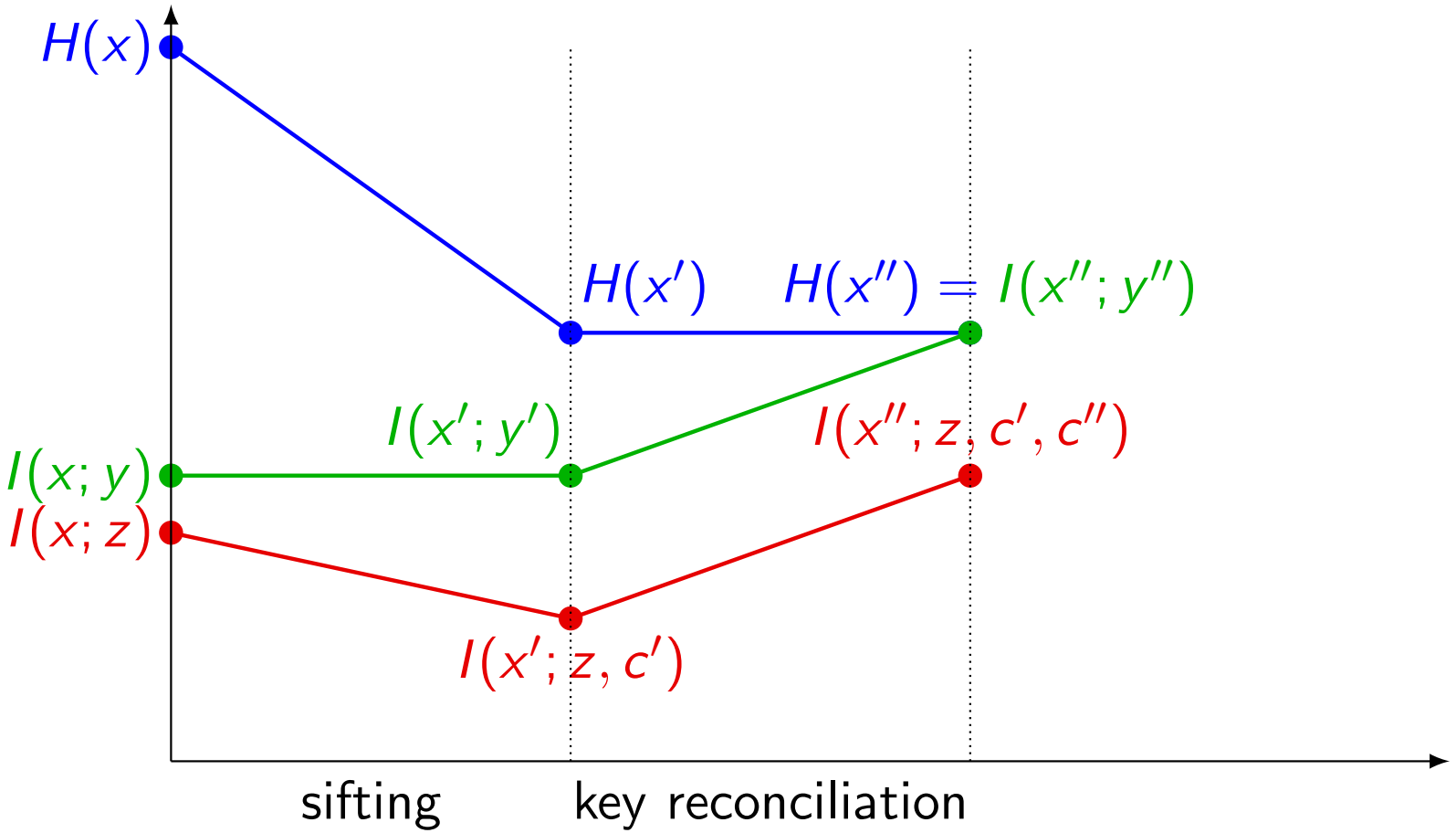

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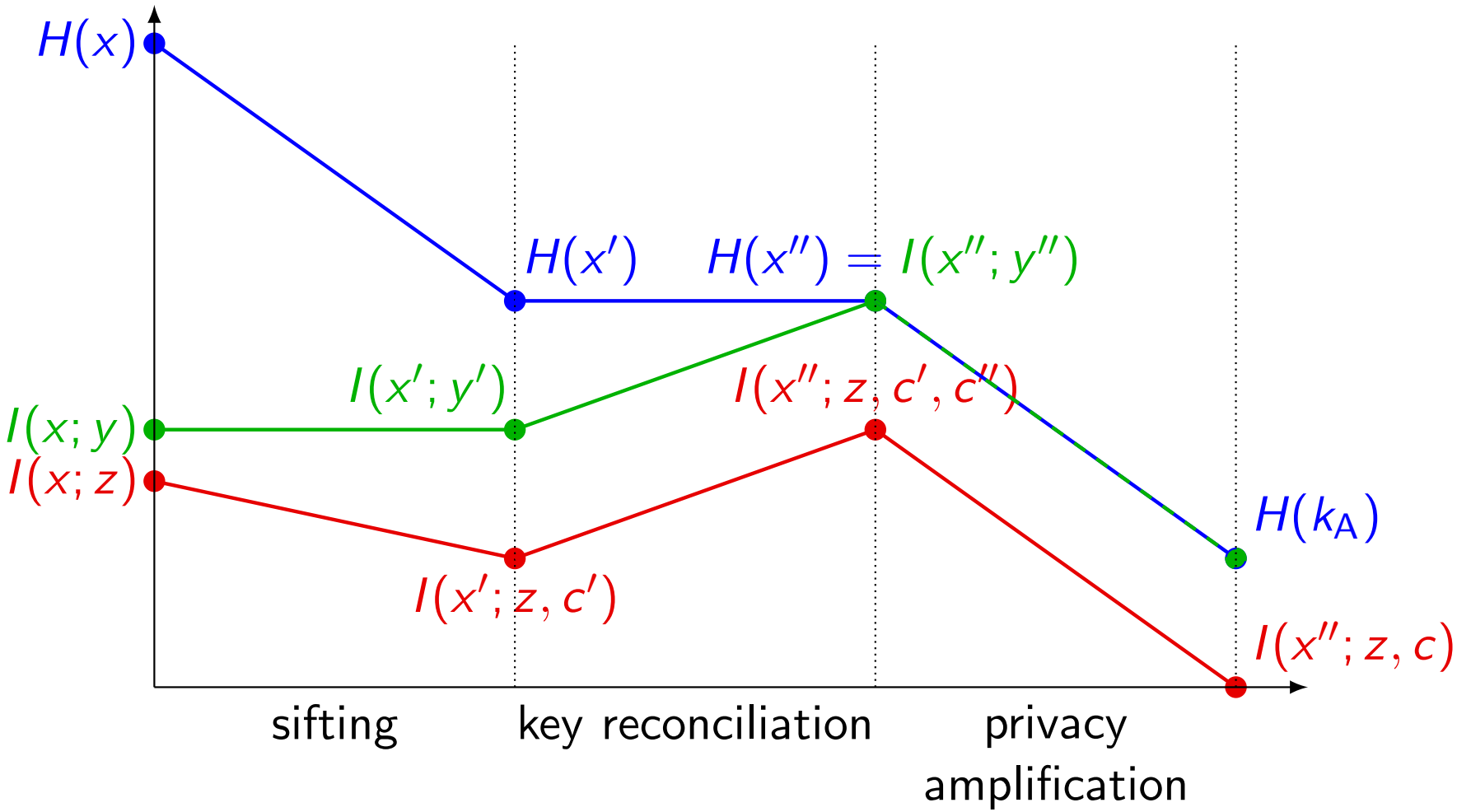
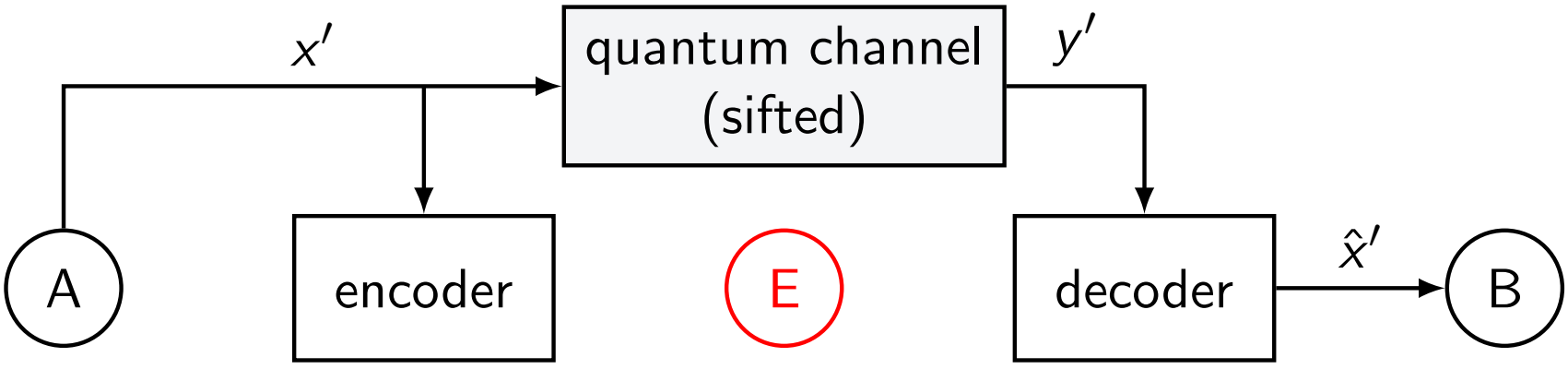


Outline

Key reconciliation



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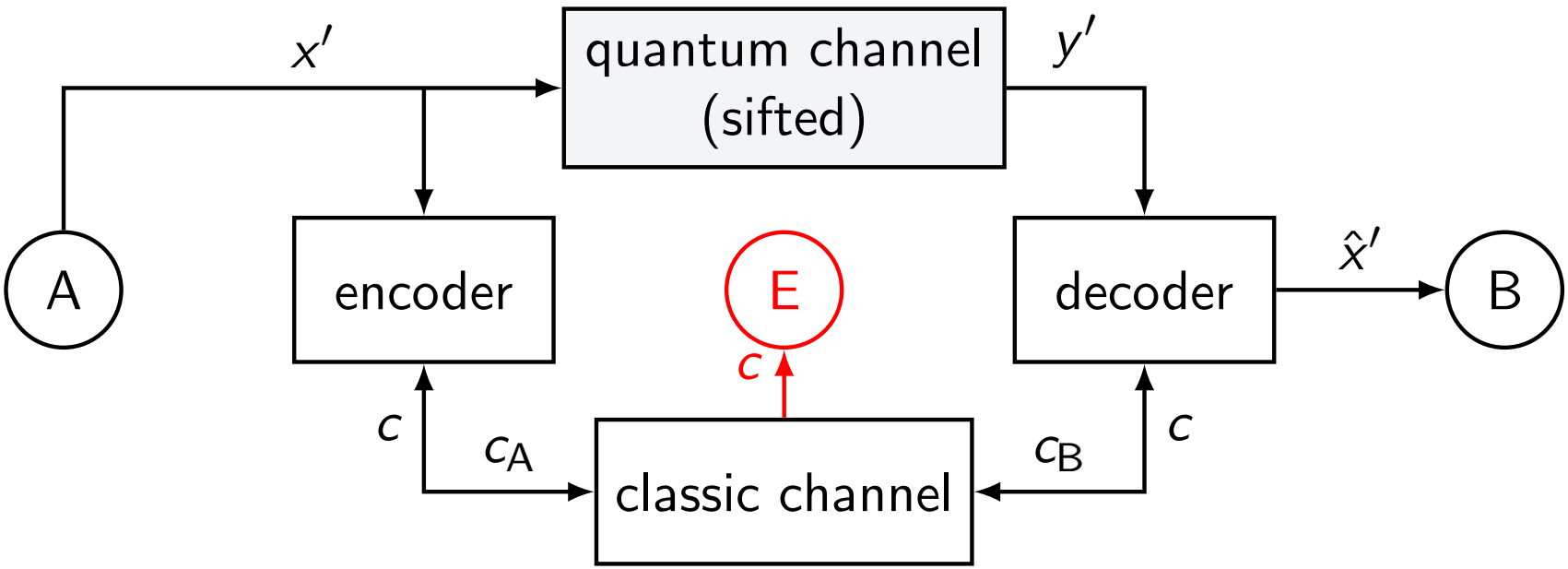

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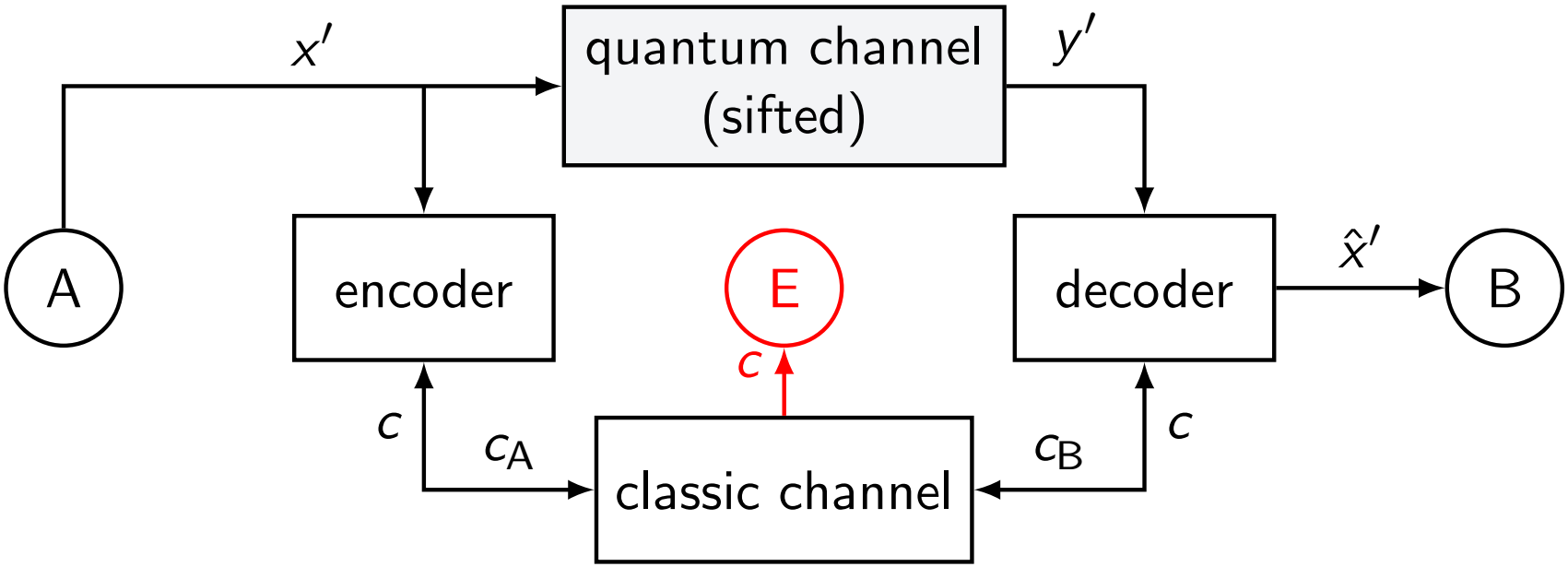

Goals

1. Correctness: $P[x' = \hat{x}'] \approx 1$
2. Minimum information leakage: $I(x'; c) \to 0$

Key reconciliation approaches (I)

1. *Interactive* - keys are interactively reconciled by means of a binary error search based on multiple, subsequent public communications [Brassard-Salvail, '93].

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2. *Systematic* - given a $(n + r, n)$ generating matrix $\mathbf{G} = \begin{bmatrix} \mathbf{I}_n \\ \mathbf{A} \end{bmatrix}$:

   Alice transmits $\mathbf{c} = \mathbf{A}\mathbf{x}'$
   Bob chooses $\hat{\mathbf{x}}' = \arg\min_{\mathbf{a} \in \mathcal{C}} d(\mathbf{a}, \mathbf{y})$
   Examples: LDPC [Mondin *et al.*, '10]
   BCH [Traisilanun *et al.*, '07]

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   Examples: LDPC [Mondin *et al.*, '10]
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3. *Hashing* - given a $(n, n-r)$ parity check matrix $\mathbf{H}$:
   Alice transmits $\mathbf{c} = \mathbf{H}\mathbf{x}'$
   Bob chooses $\hat{\mathbf{x}}' = \arg\min_{\mathbf{a}:\mathbf{H}\mathbf{a}=\mathbf{c}} d(\mathbf{a}, \mathbf{y})$
   Examples: Winnow [Buttler *et al.*, '03]
   LDPC [Elkouss *et al.*, '09]

Key reconciliation approaches (II)

The choice of the coding technique for reconciliation depends on the model for the classical channel

Layer	Ch. type	Condition	Delays	Codes used
Physical	AWGN	high SNR	none	systematic (soft)
Data link	binary	low BER	low	systematic (hard)
Net & up	packet	error free	long	interactive, hashing

Case study: Winnow

- Hashing-based key reconciliation protocol
- Ingredients
  - parity check
  - Hamming codes
  - syndrome decoding

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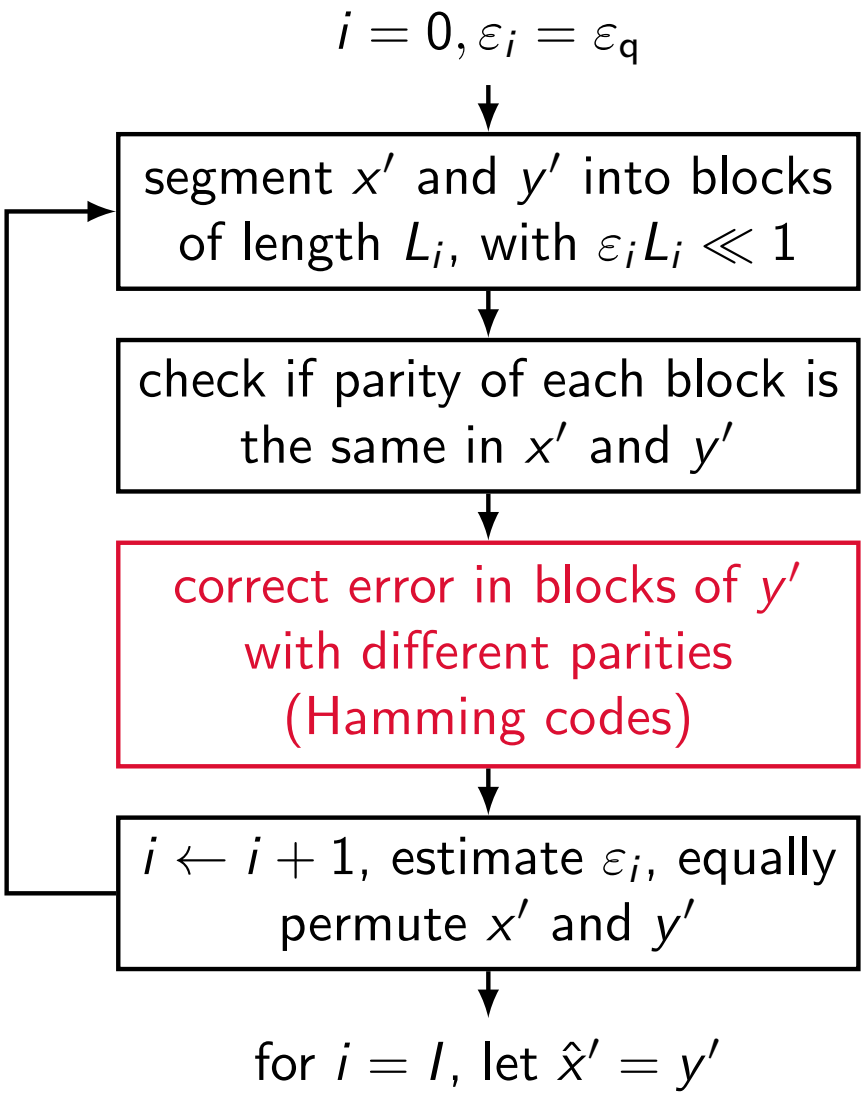

- Hashing-based key reconciliation protocol
- Ingredients
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- the condition $\varepsilon_i L_i \ll 1$ ensures that multiple errors in a block are unlikely
- the block parities and the syndromes need to be exchanged (c_A, c_B)
- it can correct a single error per block

Case study: Winnow optimization (I)

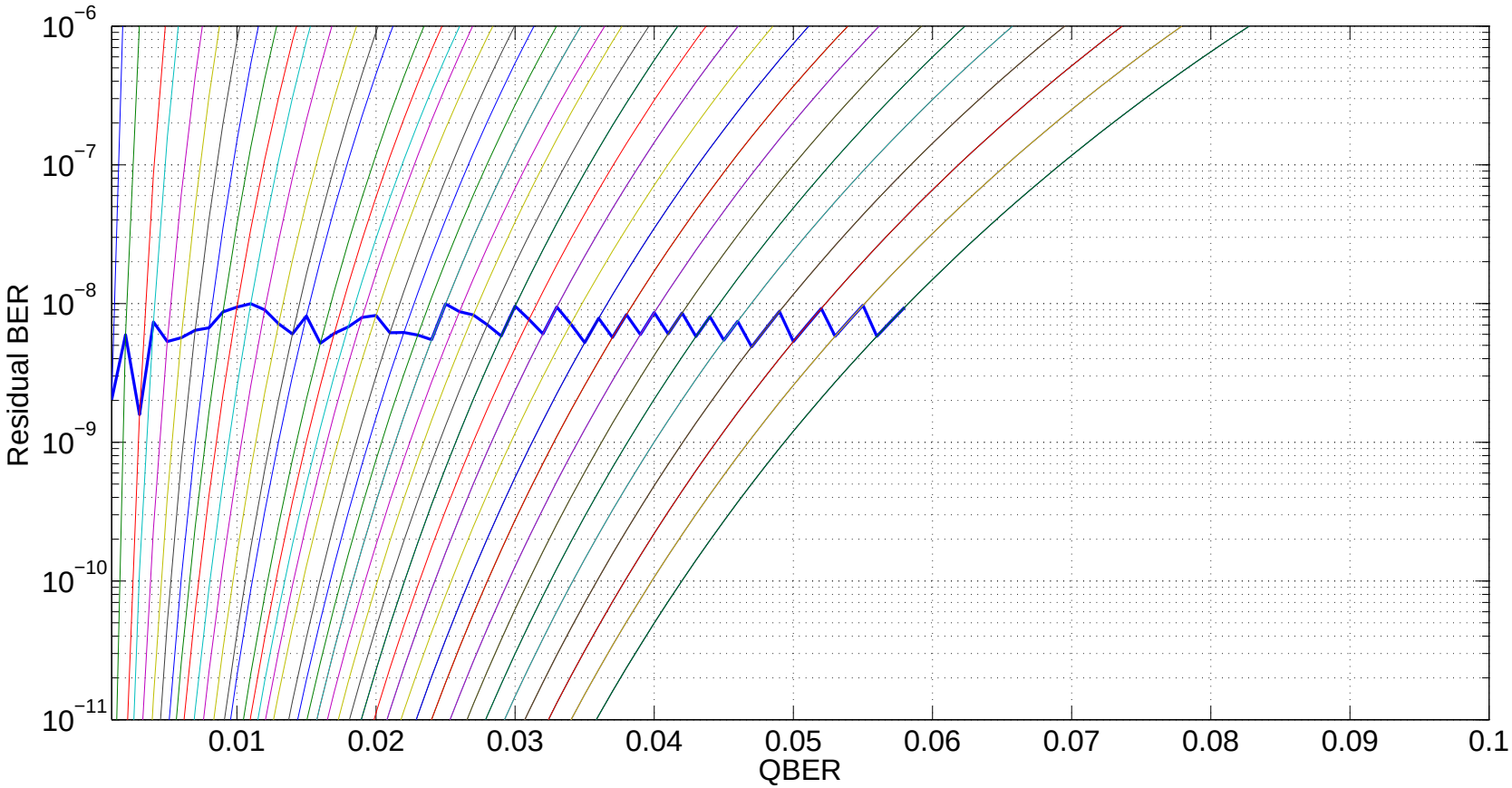

Figure: Winnow block size optimization: residual BER with target BER equal to 10^{-8}, max 4 iterations, max block size 512 bit.

Case study: Winnow optimization (II)

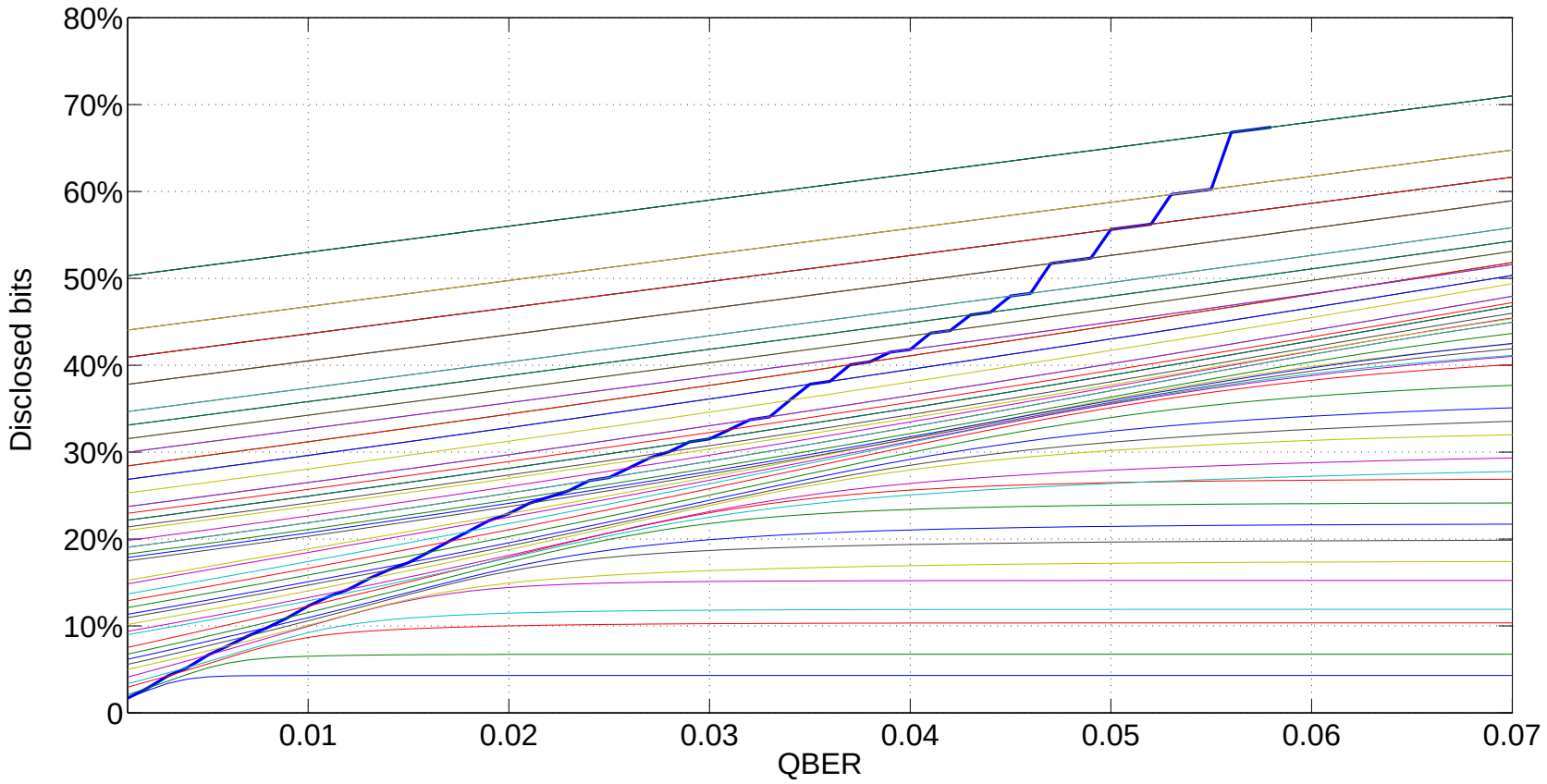

Figure: Winnow block size optimization: fraction of disclosed bits with target BER equal to 10^{-8}, max 4 iterations, max block size 512 bit.

Case study: Winnow optimization (III)

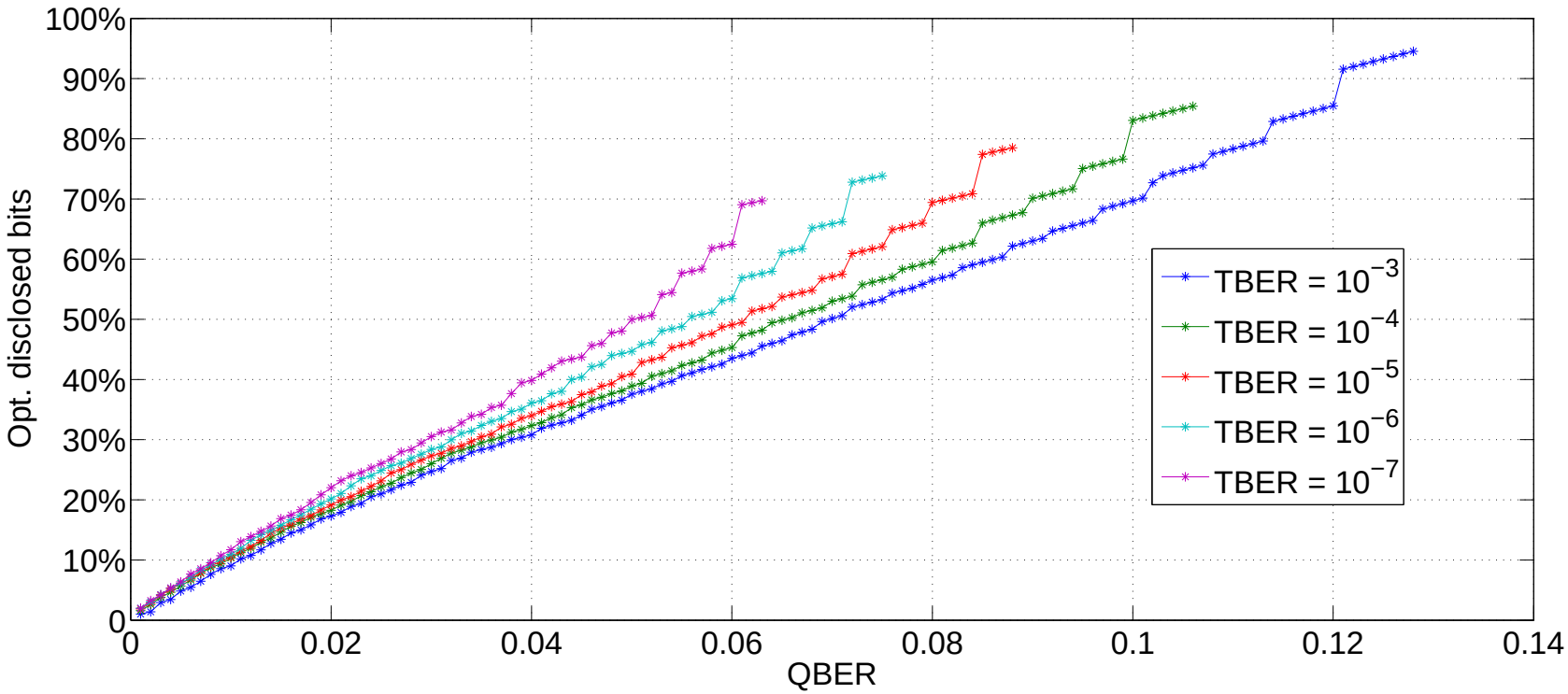

Figure: Winnow, optimal percentage of disclosed bits for different target bit error rates, max 4 iterations, max block size 512 bit.

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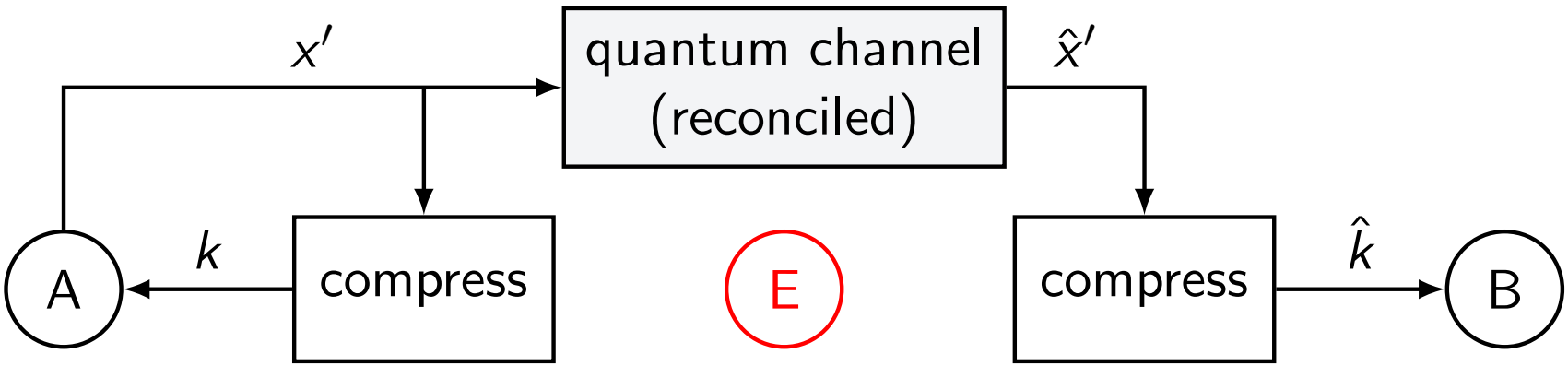
Privacy amplification



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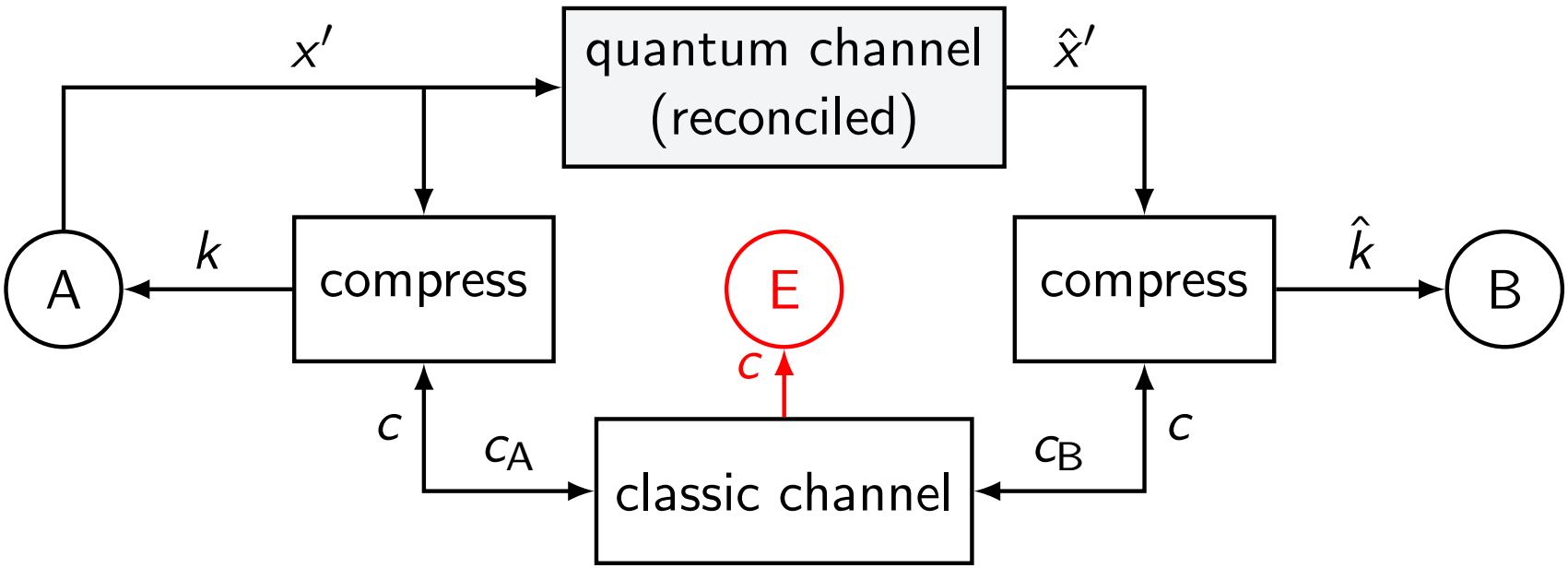

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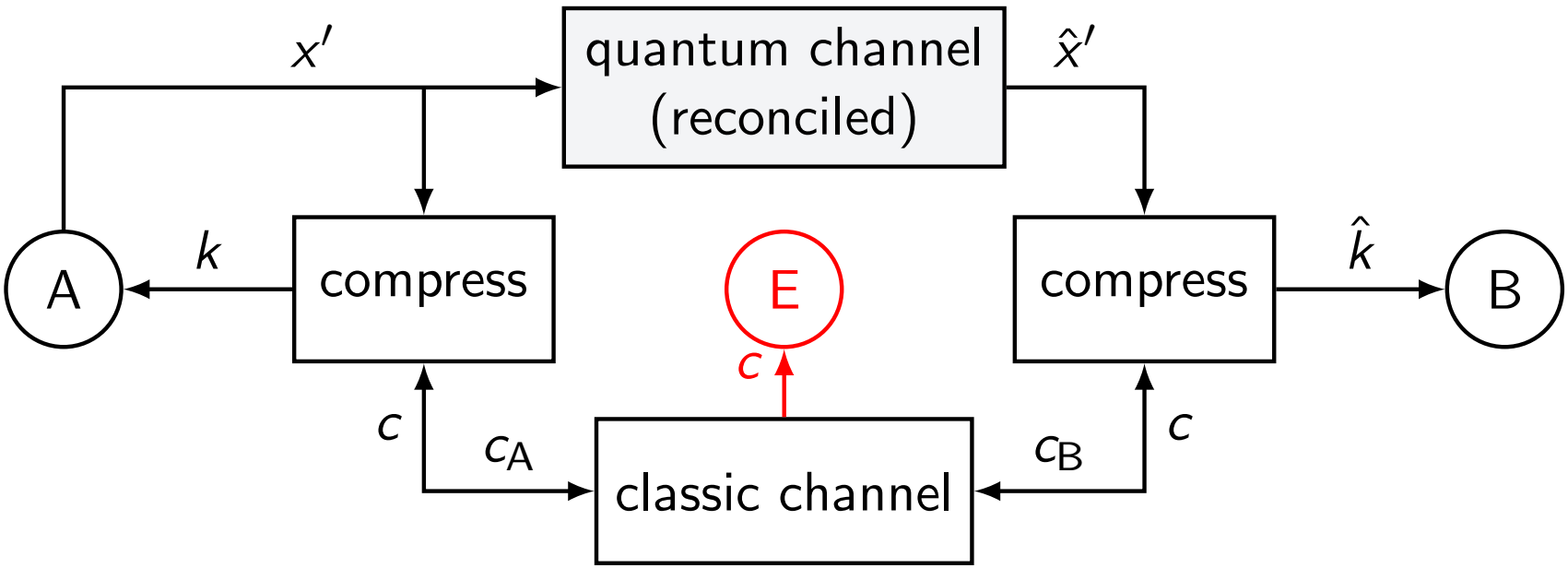

Goals

1. Maximum privacy: $I(\mathbf{k}; \mathbf{z}, \mathbf{c}) < \varepsilon''$
2. Minimum compression: $\max H(\mathbf{k})$

Choosing a compression function

How to choose a compression function?

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Assume we know that Eve has observed some t-bit linear function of the reconciled key

$$\mathbf{z} = \mathbf{M}\mathbf{x}' \quad , \quad \text{with } \mathbf{M} \in \{0,1\}^{t \times n}$$

(include $\mathbf{c}$ observed during reconciliation)

Choosing a compression function: known results

Theorem (Universal hashing functions [Bennett *et al.*, '95])

If the compressing function $\mathbf{A}$ *is chosen uniformly from a class of universal hashing* $s \times n$ *matrices, then on average (over* $\mathbf{M}$ *and* $\mathbf{A}$*)*

$$I(\mathbf{k}; \mathbf{z}, \mathbf{A}) \leq \frac{2^{n-s-t}}{\ln 2}$$

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Theorem (Point-wise bound [Gilbert *et al.*, '01])

In order to achieve a failure probability $P_{fail} \leq s'$ *and a mutual information* $I(\mathbf{k}; \mathbf{z} = z^*, \mathbf{A}) \leq \frac{2^{-s''}}{\ln 2}$*, the privacy amplification compression factor should be at least* $s = s' + s''$*.*

Choosing a compression function: open problems

Open problems

- Bennett's theorem gives only an *average* bound on the value of $I(\mathbf{k}; \mathbf{z}, \mathbf{A})$ over random choices of $\mathbf{A}$.

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Idea

Find a tool for establishing a point-wise bound on the mutual information, i.e., $I(\mathbf{k}; \mathbf{z}, \mathbf{A} = \mathbf{a}^*)$.

Choosing a compression function

Once we choose a hashing matrix $\mathbf{A}$, we would like to obtain

1. $H(\mathbf{k}) = s$ (perfect uniformity)
2. $I(\mathbf{k}; \mathbf{z}) = 0$ (perfect secrecy)

Lemma 1

If $\text{rank}(\mathbf{A}) = s$ and $\mathbf{x}'$ is uniform over $\{0,1\}^n$, then $\mathbf{k}$ is uniform over $\{0,1\}^s$

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Example: binary Toeplitz matrices

- $\mathbf{A}$ is uniquely specified by $n + s - 1$ bits $\mathbf{a} = [a_{-r+1}, \ldots, a_{n-1}]$
- If $\mathbf{a}$ is uniform in $\{0,1\}^{n+s-1}$, $\mathrm{P}\left[\text{rank}(\mathbf{A}) < s\right] = 1/2^{n-s+1}$

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Lemma 2

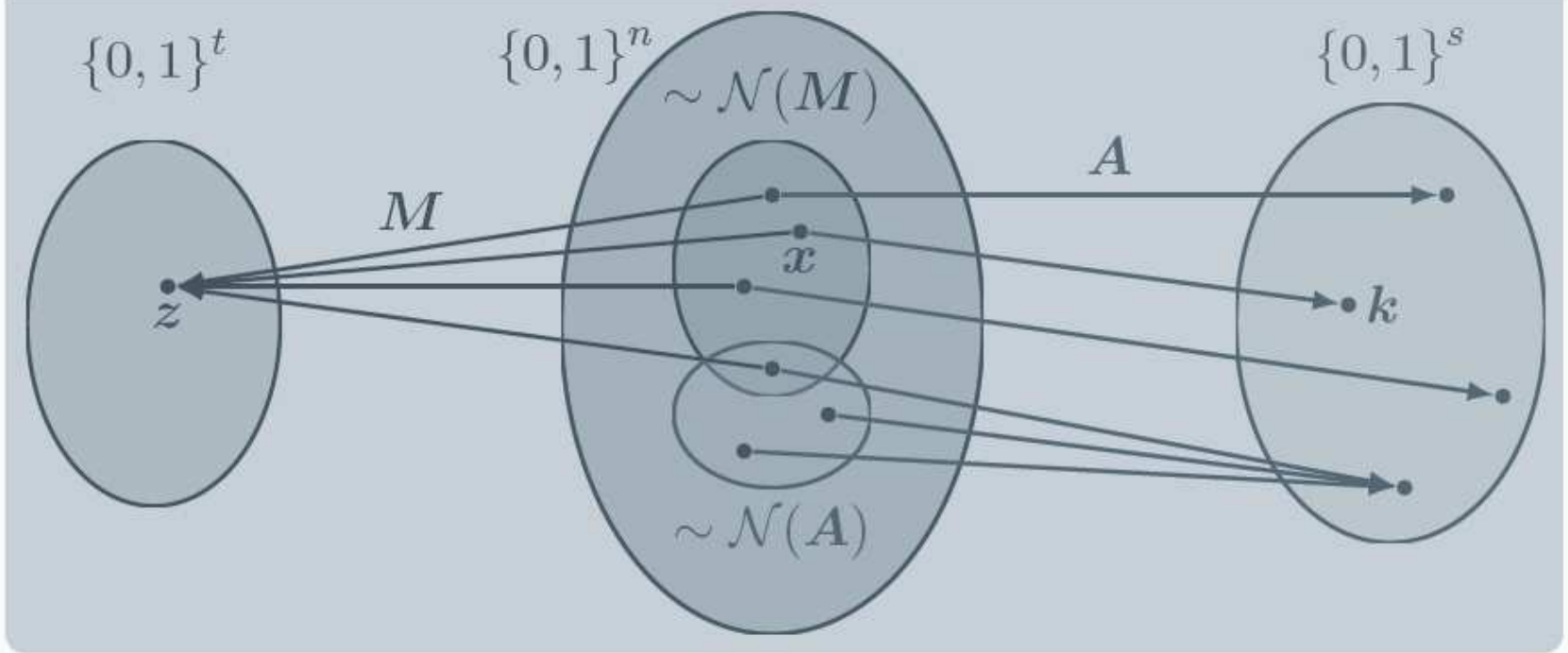
If $\dim \mathcal{N}(\mathbf{M}) - \dim\left(\mathcal{N}(\mathbf{M}) \cap \mathcal{N}(\mathbf{A})\right) = \text{rank}(\mathbf{A})$ and $\mathbf{x}'$ is uniform over $\{0,1\}^n$, then $I(\mathbf{k}; \mathbf{z}) = 0$

Choosing a compression function

Theorem

If $\dim \mathcal{N}(\mathbf{M}) - \dim\left(\mathcal{N}(\mathbf{M}) \cap \mathcal{N}(\mathbf{A})\right) = s$ *and* $\mathbf{x}'$ *is uniform over* $\{0,1\}^n$*, then* $\mathbf{k}$ *is uniform and perfectly secret.*

Illustration



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... do we perfectly know **M**, that is, the matrix summarizing Eve's information?

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Possible solution (future work): assume a specific, though probabilistic attack model (e.g., selective intercept and resend) and bound the mutual information given its statistical description.

Summary

- Achievements
  - extended analysis and optimization of the Winnow protocol
  - deployment of a framework for the design of the optimal privacy-amplification function
- Future works
  - investigation of different techniques for key reconciliation (e.g., LDPC codes)
  - further development of the above framework

Questions

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