Graph-based Automatic Suggestion of Relationships among Images of Illuminated Manuscripts

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ABSTRACT

Scientific research on illuminated manuscripts includes the disclosure of relationships among images belonging to different manuscripts. Relationships can be modeled as typed links, which induce an hypertext over the archive. In this paper we present a formal model for annotations, which is the basis to build methods for automatically processing existing relationships among link types and exploiting the properties of the graph which models the hypertext. The result of this processing is twofold: new relationships can be suggested to help users in their research work, and the existing ones can be semantically validated to check for inconsistencies.

Categories and Subject Descriptors

H.3.7 [Information Storage and Retrieval]: Digital Libraries

General Terms

Algorithms, Design, Human Factors

Keywords

annotation, digital images, user requirements, education

1. INTRODUCTION

This paper reports on the development of automatic tools for research users of multimedia digital archives. Archives of specific interest are those constituted by images taken from original *illuminated manuscripts*, which are books, usually handwritten, that include illustrations. A digital archive, which stores historical images from manuscripts and manages the related information, usually has two goals. The first goal represents the preservation of cultural heritage. The second one represents the dissemination of the historical material, because the content of a digital archive may be accessed by a wider community of users than a physical one. There is a third aspect that plays an important role

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in preservation and dissemination of cultural heritage. Illuminated manuscripts are the subject of scientific research in different areas, namely art history and history of science, and all the disciplines that are related to their content. To this aim, a digital archive of illuminated manuscripts needs to be enriched by a set of tools and services that help researchers studying the development of scientific illustrations over the centuries.

Our work is based on a previous study carried out with a user centered approach [1] where final users were researchers and scholars in the different areas related to illuminated manuscripts. According to reached results, the use of annotations has been proposed as a useful way of accessing a digital archive, sharing knowledge in a collaborative environment of researchers, and disseminating research results to students [2]. In this paper we formalize the concept of annotations to propose tools for the automatic analysis of user's annotations, with the aim of highlighting inconsistencies and suggesting new relationships among the images of the digital archive. The paper is structured as follows. The formal model for annotations of the content of a digital archive is presented in Section 2, and the methods exploited for automatic suggestions are presented in Section 3. Conclusions are drawn in Section 4.

2. A FORMAL MODEL OF ANNOTATIONS OF DIGITAL CONTENT

This section presents the formal model for describing annotations; this model builds on the one proposed in [3].

Digital Object Sets

An archive of illuminated manuscripts has to deal with different kinds of *Digital Objects (DOs)*. A preliminary user study highlighted that the objects that are studied by researchers are of three kinds: *manuscripts, pages* within a given manuscript, and *details* of pages, which usually are hand drawn images. We call them *Digital Contents (DCs)*, because they carry the information content that is the subject of scientific research.

The user study highlighted that a fourth DO has to be added to the digital archive: the *annotation* on digital content. Annotations are authored by researchers, and they may be either a tool for studying the collection of manuscripts – e.g., a way to highlight some interesting relationships that need to be further investigated – or the results itself of scientific research – the disclosure of new information about the DCs in the archive. The following definition formalizes the different sets of DOs we need to deal with.

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Definition 1. Let us define the following sets:

- M is a set of manuscripts and $m \in M$ is a generic manuscript.
- P is a set of pages and $p \in P$ is a generic page. We define a function mp : $M \to 2^P$ which maps a manuscript to the pages contained in it. The following constraints must be adhered to:

 $\forall m \in M, \operatorname{mp}(m) \neq \emptyset$ $\forall m_1, m_2 \in M, \operatorname{mp}(m_1) \cap \operatorname{mp}(m_2) = \emptyset$

that is each manuscript must contain, at least, one page and pages cannot be shared among manuscripts.

 D is a set of details and d ∈ D is a generic detail. We define a function pd : P → 2^D which maps a page to the details contained in it. The following constraint must be adhered to:

$$\forall p_1, p_2 \in P, \operatorname{pd}(p_1) \cap \operatorname{pd}(p_2) = \emptyset$$

that is details cannot be shared among pages.

- $DC = M \cup P \cup D$ is a set of digital contents and $dc \in DC$ is a generic digital content.
- A is a set of annotations and $a \in A$ is a generic annotation.
- $DO = DC \cup A$ is a set of digital objects and $do \in DO$ is a generic digital object.

Note that DO (capital italic letters) is the set of defined digital objects, DO (capital letters) is the acronym for Digital Object and do (lowercase italic letters) is a digital object $do \in DO$. Similar considerations apply to digital contents and to annotations.

Each DO is uniquely identified by means of an handle.

Definition 2. *H* is a **set of handles** such that |H| = |DO|and $h \in H$ is a generic handle. We define a bijective function $h: H \to DO$ which maps a handle to the DO identified by it¹:

$$\forall do \in DO, \exists! h \in H \mid h(h) = do \Rightarrow h^{-1}(do) = h$$

We will explicitly indicate when a handle identifies an annotation with the notation h_a , for the generic handle, and with $H_a \subseteq H$ for the subset of annotations handles.

Author and Group of Authors

Each DO has an author who creates it. In the particular case of a digital archive of historical manuscripts, the author of a DC cannot interact with the present archive, because he lived centuries ago. On the other hand, nowadays researchers do not author manuscripts, pages, or details. For these reasons, we refer as *author* to only the users of the present archive that create annotations and that cannot create DCs. In our application scenario, groups of authors correspond to research groups, in which different researchers cooperate; a researcher may collaborate with different research groups. Definition 3. Let us define the following sets:

• AU is a **set of authors** and $au \in AU$ is a generic author. We define a function $au : AU \to 2^{H_a}$ which maps an author to the handles of the annotations authored by him. The following constraint must be adhered to:

$$\forall au \in AU, au(au) \neq \emptyset$$

that is each author in AU must author, at least, one annotation; an author may own more annotations, because au(au) is an element of the power set of H_a ;

• $GR \subseteq 2^{AU}$ is a set of groups of authors and $G \in GR$ is a generic group of authors. We define a function gr : $AU \rightarrow 2^{GR}$ which maps an author to groups of authors he belongs to. The following constraint must be adhered to:

$$\forall au \in AU, \operatorname{gr}(au) \neq \emptyset$$

that is each author in AU must belong to, at least, one group of authors.

Types of Annotation

The type of annotation represents part of the semantics of an annotation. The following definition formalizes the notion of type of annotation.

Definition 4. T is a set of types of annotation, and $t \in T$ is a generic type of annotation.

The **types graph** is a labeled directed graph (G_T, l_T) , where $G_T = (T, E_T \subseteq T \times T)$, T set of vertices, E_T set of edges, and $l_T : E_T \to L_T$ with L_T set of labels.

The goal of the types graph is to provide some sort of structure and hierarchy among the types of annotation in order to navigate and browse through them. E_T can be constrained in many ways to obtain the desired structure of types. The labeling function l_T can be further exploited to distinguish different kinds of edges in the set E_T to better explain the kind of relationship between two different meanings.

The general notion of *structure* in Digital Librarys (DLs) has been introduced in [4], and represented with a labeled directed graph, as a means of expressing different kinds of structure that could be needed in DLs, e.g. taxonomies and metadata. Thus, the types graph adheres to this definition of structure and it is a structure aimed at allowing the navigation through the different meanings of annotation. We assume that meanings represent a pre-existing knowledge on the application domain. The definition of the types needed for the particular task of annotating a digital archive of illuminated manuscripts has been the focus of previous work on user requirements [2]. Most relevant requirement has been the possibility to express a relationship between DCs in the archive through the use of a *linking annotation*. Linking annotations are divided in two groups, that is the types graph can be partitioned in two disjoint subgraphs, which express a hierarchical or a relatedness relationship respectively between DCs of the same set -M, P, or D.

Scope of Annotation

An annotation can have different scopes, i.e. it can be *private*, *shared*, or *public*.

¹ \exists ! is the *unique existential quantifier*, and it is read "there exists a unique ... such that ...".

Definition 5. Let $S = \{$ Private, Shared, Public $\}$ be a set of scopes and $s \in S$ is a scope. Let us define the following relations:

- equality relation =
 - $\{(s,s) \in S \times S \mid s \in S\} = \{(\text{Private}, \text{Private}), (\text{Shared}, \text{Shared}), (\text{Public}, \text{Public})\}$
- strict ordering relation \prec

{(Private, Shared), (Private, Public), (Shared, Public)}

• ordering relation \leq

$$\{(s_1, s_2) \in S \times S \mid s_1 = s_2 \lor s_1 \prec s_2\}$$

We assume that each annotation can have only one of the three scopes listed above. Note that (S, \leq) is a *totally or*dered set. The choice of three levels of scopes is motivated by the fact that, an annotation can either be: of general interest, that is the consolidated results of past scientific research (public); a tool for exchanging information on a research work carried out by a group of researchers (shared); a way to highlight an interesting aspect that needs further investigation before being submitted to other researchers (private).

Annotation

Now we can introduce a formal definition of annotation.

Definition 6. An **annotation** $a \in A$ is a tuple:

$$a = \left(h_a \in H_a, au_a \in AU, G_a \in 2^{GR}, s_a \in S, t_a \in T\right)$$

where:

- h_a is the unique handle of the annotation a, i.e. $h(h_a) = a$;
- au_a is the author of the annotation a, i.e. $h_a \in au(au_a)$;
- G_a are the groups of authors which can access the annotation, such that $G_a \subseteq \operatorname{gr}(au_a)$;
- s_a is the scope of the annotation a Private, Shared, or Public;
- t_a is the type of the annotation a.

According to this definition, the annotation is constrained to be authored by one and only one author. This may be alternatively considered as an additional constraint on the definition of author previously given, which may be expressed as $\forall au_1, au_2 \in AU \mid au(au_1) \cap au(au_2) = \emptyset$.

Annotation-based Hypertext

Given that each type of annotations that is taken into account expresses a relationship between two DCs in the form of a typed link, we consider that existing DCs and user's annotations constitute a hypertext.

Definition 7. The annotation-based hypertext is a labeled directed multigraph:

$$(\mathcal{H} = (DC, A), \text{annotate})$$

where:

- *DC* is the set of vertices;
- A is the set of edges;
- annotate : $A \rightarrow DC \times DC$ is the edge-function, which puts an edge between two DCs dc_1 and dc_2 if and only if there is a relationship between them, which is expressed by the annotation a.

The following constraints must be adhered to:

- 1. a $dc \in DC$ cannot be put in relationship with itself, that is $dc_1 \neq dc_2$;
- 2. the two DCs connected by an annotation must be of the same type, that is $dc_1, dc_2 \in M \lor dc_1, dc_2 \in P \lor dc_1, dc_2 \in D$.

The annotation-based hypertext is built by putting an edge between two DCs vertices, if an annotation between that two DCs exists. Note that edges can be put only between DCs and not between annotations: this means that an annotation cannot connect other annotations. The two constraints on the annotation-based hypertext are based on a study carried out on the user requirements of the researchers that will access and annotate the digital archive: annotations do not have to express a relationship of a DC with itself, or with DCs of different kind. Since there are no constraints on the number of annotations that connect a pair of vertices, we deal with a multigraph. The existence of multiple edges between the same pair of vertices allows us to express different kinds of relationships between two DCs. In this way, we take into account both the possibility of different interpretations of the same contents given by independent authors, and the partial results of a same author, who is studying a particular subset of the digital content and expresses alternative relationships that need further investigations. Users are not expected to access the whole annotation-based hypertext, because annotations have scopes that are related to user's access rights. Thus, the following definition introduces an operator suitable for choosing the subset of the annotation-based hypertext that can be accessed by a user.

Definition 8. Given an annotation-based hypertext \mathcal{H} , we introduce a **projection operator** that can have the forms:

• $\mathcal{H}^{\pi} = \pi (\mathcal{H}, AU_{\pi}, S_{\pi}, T_{\pi})$, with $AU_{\pi} \subseteq AU$, $S_{\pi} \subseteq S$, $T_{\pi} \subseteq T$, constructs a new annotation-based hypertext $\mathcal{H}^{\pi} \subseteq \mathcal{H}$ such that:

$$A^{\pi} = \{ a \in A \mid au_a \in AU_{\pi} \land s_a \in S_{\pi} \land t_a \in T_{\pi} \}$$
$$DC^{\pi} = DC$$

• $\mathcal{H}^{\pi} = \pi (\mathcal{H}, GR_{\pi}, S_{\pi}, T_{\pi})$, with $GR_{\pi} \subseteq GR, S_{\pi} \subseteq S$, $T_{\pi} \subseteq T$, constructs a new annotation-based hypertext $\mathcal{H}^{\pi} \subseteq \mathcal{H}$ such that:

$$\begin{cases} A^{\pi} = \{a \in A \mid G_a \in GR_{\pi} \land s_a \in S_{\pi} \land t_a \in T_{\pi} \} \\ DC^{\pi} = DC \end{cases}$$

Both operators have a generalized version, where the \star symbol can be replaced to an input parameter in order to express that the whole set has to be used.

For example $\pi(\mathcal{H}, AU_{\pi}, S_{\pi}, \star) = \pi(\mathcal{H}, AU_{\pi}, S_{\pi}, T).$

This operator provides us with a personalized view for the user of the annotation-based hypertext \mathcal{H} . The first form allows us to select edges on the basis of author(s), scope(s)and type(s) of the annotation, while the second form utilizes groups of authors instead of authors as selection criterion. This operator is quite flexible, if combined with the previous definitions. For example, if, given an author $au \in AU$, we want to extract the subgraph with all the public annotations (edges) inserted by authors that belong to the same groups of au, we can use $\mathcal{H}^{\pi} = \pi (\mathcal{H}, \operatorname{gr}(au), \operatorname{Public}, \star)$. Finally, the expressive power of this operator can be further enriched by using also the usual union, intersection, and difference set operators. The projection operator represents the standard way for a user to perceive the annotation-based hypertext, because a user is not allowed to access all the edges of the hypertext but he can access only the public ones, those belonging to him, and the ones shared with groups of authors the user belongs to.

Definition 9. Let us define the **annotation compatibil**ity set $C \subseteq A \times A \times [0,1]$ that expresses the degree of compatibility of the types of annotation among given pairs of annotations, where 0 means no compatibility at all and 1 means full compatibility. Let us define the **compatibil**ity score $c(C, a_1, a_2) = c \in [0, 1]$ between two annotations given an annotation compatibility set C, which returns the compatibility c between two annotations if $\exists (a_1, a_2, c) \in C$.

The actual value that c assumes for different pairs of annotation types is part of previous knowledge about the semantic of the annotation types and on their organization in the types graph. We assume this value is given by specialists in the field of illuminated manuscripts.

Definition 10. Given an annotation-based hypertext \mathcal{H} , a set T of annotation types, and a types graph G_T , we introduce a **pair-wise compatibility opera**tor $\xi(\mathcal{H}, T, G_T, dc_1, dc_2) = C_{\xi}$ that $\forall a_1, a_2 \in A \mid$ annotate $(a_1) = (dc_1, dc_2) =$ annotate (a_2) returns a compatibility score for the annotations connecting dc_1 and dc_2 .

Given an annotation-based hypertext \mathcal{H} , a set T of annotation types, and a types graph G_T , we introduce a **pathwise compatibility operator** $\xi(\mathcal{H}, T, G_T, dc_1, dc_2, dc_3) = C_{\xi}$ that $\forall a_1, a_2 \in A \mid \text{annotate}(a_1) = (dc_1, dc_2) \land$ annotate $(a_2) = (dc_2, dc_3)$ returns a compatibility score for the annotations connecting dc_1 and dc_2 with respect to the annotations connecting dc_2 and dc_3 .

Both forms of the compatibility operator make use of the types graph, which expresses the relationships among the different types of annotation, in order to determine the degree at which the type of two different annotations is compatible. Note that the annotation compatibility set C can be used to produce a ranking among the annotations connecting different DCs in order of severity of compatibility problems.

3. AUTOMATIC SUGGESTIONS OF RELA-TIONSHIPS AMONG DCS

The introduced model and operators can be exploited to create tools for helping the user of a digital archive to perform scientific research on its content. In particular, the annotation-based hypertext can be automatically analyzed to highlight possible inconsistencies among the annotations - e.g., two DCs are annotated with typed annotations that have different and possibly contrasting semantics – as well as to extract new information about possible relationships e.g., two DCs are not annotated but the surrounding set of edges suggest the possibility of a relationship among them. It has to be stressed that the automatic analysis of the graph can only provide the user with suggestions on possible new or different annotations. The final choice of which annotations are to be added or modified is made by the research user who, from his cultural and scientific background, can take the final decision on relationships among digital content. Moreover, the research on illuminated manuscripts is an ongoing work, for which temporary inconsistency and incompleteness are normal events. Yet, the automatic analysis may help the researcher by suggesting the creation of new annotations, because the task of accepting to author an automatic annotation is expected to be simpler than creating an annotation from scratch.

3.1 Suggestions of Possible Inconsistencies

As previously explained, the model allows for multiple annotations of the same pair of digital contents. This means that public annotations of different authors may be different, or that for a given author, public annotations may differ from private ones, or even that there can be different private annotations. These inconsistencies may be made on purpose, but may also be the result of an erroneous interaction with the system, or to changes in the view of the DC relationships over the years. In any case, the analysis of the graph may pinpoint particular relationships that need to be carefully checked by the user.

Definition 11. Given an annotation-based hypertext \mathcal{H} , a set T of annotation types, a types graph G_T , a subset of authors $AU_{\psi} \subseteq AU$ and a subset of scopes $S_{\psi} \subseteq S$, we introduce a **pair-wise inconsistency finder operator** $\psi(\mathcal{H}, T, G_T, AU_{\psi}, S_{\psi}) = C_{\psi}$ that firstly computes $\mathcal{H}^{\pi} = \pi(\mathcal{H}, AU_{\psi}, S_{\psi}, T)$ and secondly computes

$$C_{\psi} = \bigcup_{\substack{dc_1, dc_2 \in DC^{\pi} \mid \exists a \in A^{\pi}, \\ \text{annotate}(a) = (dc_1, dc_2)}} \xi \left(\mathcal{H}^{\pi}, T, G_T, dc_1, dc_2\right)$$

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$$C_{\psi} = \bigcup_{\substack{dc_1, dc_2, dc_3 \in DC^{\pi} \mid \\ \exists a_1, a_2 \in A^{\pi}, \\ \text{annotate}(a_1) = (dc_1, dc_2) \land \\ \text{annotate}(a_2) = (dc_2, dc_3)}} \xi \left(\mathcal{H}^{\pi}, T, G_T, dc_1, dc_2, dc_3\right)$$

Once that either the pair-wise or the path-wise operator is applied, from the set C_{ψ} it is possible to extract all the compatibility scores related to annotations made by authors AU_{ψ} (possibly belonging to the same group) with scopes S_{ψ} . It is then possible to apply a threshold function on the set of compatibility scores, in order to provide the user with all the annotations that may be inconsistent or even contradictory. The degree by which two annotations are inconsistent depends on the semantics that the users give to the annotation types and to the types graph. The approach is general enough to support different definitions of the compatibility score, which are based on the knowledge of the application domain. In the case of illuminated manuscripts, the compatibility scores are based on the particular kind of relationships that can express a hierarchical relationship or a relatedness (and non-hierarchical) one. Examples of inconsistencies are that the same two DCs could be annotated as in hierarchical and non-hierarchical relationship at the same time, or that a dc_1 has been set as an ancestor of dc_2 by one author and viceversa for a different author. Suggestions of inconsistencies can be exploited in different way: as placeholders for highlighting unclear relationships between DCs that a user is interested in investigating in detail; as an indication of the more debated relationships in the archive.

3.2 Suggestions of New Relationships

The analysis of the annotation-based graph can highlight that two DCs are not annotated, yet there is a path that connects them. Moreover, since DCs are made of different sets which are organized hierarchically, the annotation of two manuscripts may suggest a similar annotation between two details, and viceversa. Also in this case, the existence of similar relationships may only suggest the presence of new relationships, which must be validated by the research user. Yet it can be considered that the presence of suggestions would ease the user in creating the network of annotations of the digital archive. Of course, there is also the possibility that the automatic analysis of the graph will disclose new relationships, at least for non expert users.

Definition 12. Given an annotation-based hypertext \mathcal{H} , a set T of annotation types, a types graph G_T , a subset of authors $AU_{\psi} \subseteq AU$ and a subset of scopes $S_{\psi} \subseteq S$ we introduce a **relationship finder operator** $\rho(\mathcal{H}, T, G_T, AU_{\rho}, S_{\rho}) = C_{\psi}$ that functions as follow:

- 1. compute $\mathcal{H}^{\pi} = \pi(\mathcal{H}, AU_{\rho}, S_{\rho}, T)$
- 2. compute the transitive closure² $\mathcal{H}^{\pi+}$ of \mathcal{H}^{π}
- 3. $\forall dc_1, dc_2 \in \mathcal{H}^{\pi+} \mid \nexists a \in A^{\pi}, \text{annotate}(a) = (dc_1, dc_2),$ that is all of the DCs among which exists a path but are not directly connected, for each path $P = dc_1a_1 \dots dc_ma_hdc_n \dots a_kdc_2$ connecting dc_1 to dc_2 , compute $C_{\rho,P} = \bigcup_{dc_{i_1}, dc_{i_2}, dc_{i_3} \in P} \xi(\mathcal{H}, T, G_T, dc_{i_1}, dc_{i_2}, dc_{i_3})$
- 4. if exists a path P such that $\sum_{a_1,a_2 \in C_{\rho,P}} c(C_{\rho,P}, a_1, a_2) > T_{\rho}$ (alternatively $\prod_{a_1,a_2 \in C_{\rho,P}} c(C_{\rho,P}, a_1, a_2) > T_{\rho}$), with T_{ρ} given threshold, than it suggests the existence of a possible relationship between dc_1 and dc_2 .

The suggestion of new possible relationships may be particularly useful for annotations with a public scope, because each research group may study only a subset of all the illuminated manuscripts, and may not be aware of the relative importance of a manuscript for their particular research work. Moreover, for a research group exchanging information through shared annotations, the suggestion of additional relationships between the DCs that they are currently studying may help merging the individual results of each researcher. Finally, even for a single author, the possibility to annotate the archive by simply accepting the annotations proposed by the system is less time consuming than creating a new annotation.

As for the suggestion of inconsistencies, the degree by which two DCs are related depends on the semantics of the annotation types and of the types graph. For instance, two DCs may be related if they both have a hierarchical relationship with the same ancestor. Yet the strength of a potential relationship, which has to be validated by the user anyway, depends both on the distance between the two vertices and on the weight given to each annotation type.

4. CONCLUSIONS

We have discussed the problem of providing users with suggestions about annotations of digital content managed by a digital archive. To this end, annotations have been studied and formalized as an effective tool suitable for developing scientific research on illuminated manuscripts. We have proposed a formal model that introduces the notion of annotation-based hypertext and explores some of its properties, in order to automatically extract some information about the relationships among digital contents. This information is exploited to provide the user with suggestions about possible inconsistencies in the annotations that have been manually inserted, and to highlight possible new annotations that the user may choose to validate.

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²The transitive closure of a graph G = (V, E) is a graph $G^+ = (V, E^+)$ such that for all $v, w \in V$ there is an edge $(v, w) \in E^+$ if and only if there is a path from v to w in G that has at least one edge.