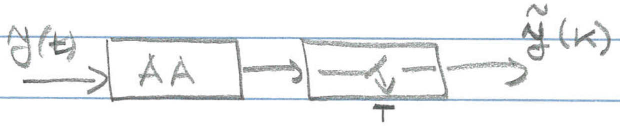
ESERCIZIO

Si considerino i segnali della classe

$$y(t) = A e^{-t/\tau} 1(t) \quad \text{con } \tau \in [1, \infty],\ A \in \mathbb{R}$$

Si consideri lo schema



(*)

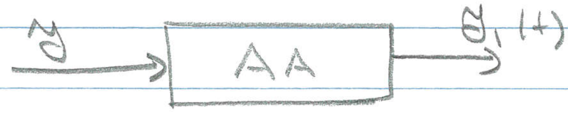
dove AA è un filtro Anti Aliasing ideale relativo al periodo di campionamento T.

Si calcoli T in modo che per qualunque segnale della classe considerata l'energia della parte di segnale eliminata dal filtro AA sia non superiore all'1% dell'energia complessiva.

SOL.

Si ha $Y(s) = \dfrac{A}{s + \frac{1}{\tau}} \Rightarrow Y(j\omega) = \dfrac{A}{j\omega + \frac{1}{\tau}}$

quindi se considero



$$Y_1(j\omega) = \begin{cases} \dfrac{A}{j\omega + \frac{1}{\tau}} & |\omega| \leq \dfrac{\Omega}{2} = \dfrac{\pi}{T} \\ 0 & |\omega| \geq \pi/T \end{cases}$$

$$E_Y = 2\int_0^{+\infty} \frac{A^2}{\omega^2 + \left(\frac{1}{\tau}\right)^2}\, d\omega$$

$$E_{Y_1} = 2\int_0^{\pi/T} \frac{A^2}{\omega^2 + \left(\frac{1}{\tau}\right)^2}\, d\omega$$

devo imporre

$$E_{Y_1} \geq 0.99\, E_Y$$

$$E_Y = 2\int_0^{+\infty} \frac{A^2\tau^2}{\omega^2\tau^2 + 1}\, \frac{\tau}{\tau}\, d\omega = 2A^2\tau \int_0^{+\infty} \frac{1}{\omega^2\tau^2+1}\, d\omega\tau$$

$$= \Big[2A^2\tau \, \mathrm{arctg}(\omega\tau)\Big]_0^{+\infty} = A^2\pi\tau$$

$$E_{Y_1} = 2\int_0^{\pi/T} \frac{A^2}{\omega^2 + \left(\frac{1}{\tau}\right)^2}\, d\omega = \left[\int_0^{\pi\frac{\tau}{T}} \frac{1}{x^2+1}\, dx\right] 2A^2\tau = 2A^2\tau\, \mathrm{arctg}\left(\frac{\pi\tau}{T}\right)$$

$$\frac{E_{y_1}}{E_y} = \frac{2}{\pi}\,\mathrm{arctg}\left(\frac{\pi\tau}{T}\right) \geq 0.99$$

$$\Rightarrow \mathrm{arctg}\left(\frac{\pi\tau}{T}\right) \geq 0.99\,\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi\tau}{T} \geq \mathrm{tg}\left(0.99\,\frac{\pi}{2}\right)$$

$$\Rightarrow T \leq \frac{\pi\tau}{\mathrm{tg}\left(0.99\,\frac{\pi}{2}\right)} \qquad \forall\ \tau \geq 1$$

$$\Rightarrow T \leq \frac{\pi\tau_{min}}{\mathrm{tg}\left(0.99\,\frac{\pi}{2}\right)} = \frac{\pi}{\mathrm{tg}\left(0.99\,\frac{\pi}{2}\right)} \simeq 0.05$$