ESERCIZIO

Sia $P(s) = \dfrac{1-s}{(s+1)(s+2)(s+3)}$

Determinare $\tilde{P}(z)$ e calcolare T in modo

che in $\tilde{P}$ non vi siano zeri evocati.

SOL

$$\tilde{P}(z) = [1 - z^{-1}]\, \mathcal{Z}\left[S_T\left[\mathcal{L}^{-1}\left[P(s)/s\right]\right]\right]$$

$$\frac{P(s)}{s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \lim_{s\to 0} s\frac{P(s)}{s} = P(0) = \frac{1}{6}$$

$$B = \lim_{s\to -1} (s+1)\frac{P(s)}{s} = \frac{2}{(-1)(1)\cdot 2} = -1$$

$$C = \lim_{s\to -2} (s+2)\frac{P(s)}{s} = \frac{3}{(-2)(-1)(1)} = 3/2$$

$$D = \lim_{s\to -3} (s+3)\frac{P(s)}{s} = \frac{4}{(-3)(-2)(-1)} = -2/3$$

$$\tilde{P}(z) = \frac{z-1}{z}\,\mathcal{Z}\left[S_T\left[A\,1(t) + B e^{-t} + C e^{-2t} + D e^{-3t}\right]\right]$$

$$= \frac{z-1}{z}\,\mathcal{Z}\left[A\,\delta_{-1}(k) + B\left(e^{-T}\right)^k + C\left(e^{-2T}\right)^k + D\left(e^{-3T}\right)^k\right]$$

$$= \frac{z-1}{z}\left[A\,\frac{z}{z-1} + B\,\frac{z}{z-e^{-T}} + C\,\frac{z}{z-e^{-2T}} + D\,\frac{z}{z-e^{-3T}}\right]$$

$$= A + \frac{z-1}{z-e^{-T}}\,B + C\,\frac{z-1}{z-e^{-2T}} + D\,\frac{z-1}{z-e^{-3T}}$$

$$= \frac{z^2\left[1-6p+9p^2-4p^3\right] + z\left[5p-10p^2+10p^4-5p^5\right] + 4p^3-9p^4+6p^5-p^6}{6\,(z-p)(z-p^2)(z-p^3)}$$

con $p := e^{-T}$

Per avere un solo zero (osserva assenza di zeri evocati)

devo imporre

$$1-6p+9p^2-4p^3 = 0$$

da cui $p = \frac{1}{4}$ oppure $p = 1$ (zero doppio)

L'unica sol accettabile è $p = e^{-T} = \frac{1}{4}$ da

cui $T = \ln 4$.