Analytical Investigation with Markov Models of Selective Repeat Type II Hybrid ARQ

Leonardo Badia, Marco Levorato, Michele Zorzi
Dept. of Information Engineering (DEI), University of Padova, via Gradenigo 6/B, 35131 Padova, Italy.
{lbadia,mlevorato,zorzi}@ing.unife.it.

Abstract— **This paper presents an analytical model for the analysis of Hybrid ARQ techniques on Discrete Time Markov Channels by means of Markov chains. The first contribution is an original proposal to track the outcome of the packet transmissions in a generalized Hybrid ARQ transmission system, deriving an appropriate Markov chain. Also, an example is given of how to put this Markov chain in relationship with a Discrete-Time Markov Channel description. This framework can be used to evaluate from the theoretical point of view the performance of truncated Type II Hybrid ARQ techniques, deriving in general some useful insight on the behavior of such systems.**

Index Terms— **Queueing analysis, automatic repeat request, Markov processes, error analysis.**

I. Introduction

IN RECENT years, the investigation of Automatic Retransmission reQuest (ARQ) techniques over Markov channels has reappeared as a flourishing subject in the technical literature [1]–[3]. It is worth noting that, differently from the classic work studying ARQ systems [4], these contributions mainly focus on wireless links, at least implicitly, and therefore exploit the channel description with a Markov chain, as this has been proven to be an elegant and powerful method to derive analytical studies for these channels [5]. The effectiveness of Markov models is mainly due to the fact that, whereas in wireline links the average error probability (normally very low) often suffices to describe the error process, in wireless channels also higher order statistics are relevant (e.g., error correlation may in general impact in a non trivial manner the performance [1], [3]).

In this paper, we focus in particular on Selective Repeat (SR) Type II Hybrid ARQ (HARQ), trying not to depend on a specific implementation but rather keeping a general approach where several specific kinds of Type II HARQ can be framed. The analysis of plain SR ARQ techniques through Markov chains can be captured within this general framework [3]: Firstly, some basic assumptions are introduced in order to describe the transmission system. Among them, the most important is about the availability of a Markov Chain modeling the channel, hence called in the following *channel chain*, where different states correspond to different physical conditions of the channel and therefore different quality levels of the received packets. Through manipulation of the channel chain, a wider representation, called in the following *ARQ chain*, is derived, whose state comprises not only the channel but also the outcome of the previously transmitted packets.

When the ARQ transmission window comprises only one packet, which is a situation called *ideal ARQ* in the literature, the two chains coincide, the outcome of this single packet depending on the channel state only. This situation becomes very different when non-instantaneous feedback at the transmitter is considered, since packets are not always transmitted in increasing numerical order. Thus, even though the channel memory is concentrated in the current state due to the Markov chain representation, the outcome of previous packet transmissions is required. It is easy to prove [3] that it is necessary to keep track only of a number of packets which can be transmitted during a full round-trip time. In this way, the system chain can be derived and therefore throughput and delay performance can be obtained.

However, the above description holds for plain SR ARQ. Instead, Type II HARQ, which is our focus in this paper, requires a different analysis. We point out that Type II HARQ has also been investigated in some very recent contributions which might relate to the present paper, even though they focus more on specific versions of Type II HARQ [6] or analyze it in an implementation context [7]. Our contribution here is instead a general mathematical analysis (developed in a similar fashion to the classic SR ARQ but with completely different characteristics of the Markov model) of the Type II HARQ transmission system, which is still missing in the literature.

Thus, in this paper we bring up the following contributions: first, a multiple-level Markov channel model dedicated to HARQ studies with hard decision is proposed. Differently from the previously mentioned Discrete Time Markov channel used in classic studies of ARQ systems, a model for the HARQ packets must include a packet outcome larger than a binary one (i.e., correct vs. erroneous). We also obtain an entire analytical formulation of the problem by considering the multiple-level Markov chain as based on an underlying Discrete Time Markov channel. Finally, this model is used in order to investigate and quantitatively evaluate the performance of different implementations of Type II HARQ.

The remainder of this paper is organized as follows: in Section II we analytically describe the HARQ system by means of a Markov framework, deriving the ARQ chain from an N-State Discrete Time Markov Channel. The solution of the ARQ chain allows to evaluate different metrics, as shown in Section III. Finally, Section IV shows numerical evaluations and Section V concludes the paper.

II. Analytical Framework

HARQ techniques combine classic ARQ, since they involve retransmission of the erroneous packets, and Forward Error Correction (FEC), i.e., packets are protected from channel impairments by error-correcting codes, besides the only error-detecting codes which can be found in plain ARQ and allow

1-4244-0357-X/06/$20.00 ©2006 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2006 proceedings.

for parity check and subsequent acknowledgement (ACK) / not acknowledgement (NACK) of packets. For Type II HARQ, retransmissions always refer to the same information content, but the actual packet sent may be different. The transmitter replies to a NACK by sending *incremental redundancy packets*. These consist of redundancy bits not yet transmitted, which improve the error-correcting capability if attached to the previously received sequence.

For simplicity, we also assume that the size L of the packets is always the same, independent of their being either a first transmission or a retransmission. Indeed, allowing a different packet size for retransmissions would lead to a more cumbersome mathematical formulation, but would be conceptually straightforward. Instead, with this assumption we consider in the following a discrete (slotted) time, where a time slot equals the time required for transmitting one packet, and the round trip time equals a fixed number of slots m, in general greater than 1.

We analytically investigate the system under the assumption that packets are obtained through error-correcting codes, designed to re-transmit the same information content at most F times (the first transmission corresponds to re-transmission 0). This means that, for every $i = 0, 1, \dots, F$, we have a code C_i which is ideally seen as a $((i+1)L, k)$, where $k \leq L$ is the number of information bits in a single packet. Indeed, C_{i+1} codewords follows from the ones of C_i with the juxtaposition of L incremental redundancy bits.

We assume that it is possible to neglect the case of undetected errors, or misinterpretation of the codeword due to excessively high number of errors: In general, codes are properly designed exactly to keep these situations very unfrequent. Furthermore, we assume that the receiver's feedback is errorless, which might be seen as an abstraction of the case where it never happens that a NACK is transformed by errors into an ACK, but all feedback errors correspond to erasures, which are contrasted by using a time-out. In this way, the feedback errors can be neglected and simply moved to a higher error rate of the forward link. Finally, our work assumptions include that the receiver's buffer is unlimited and the sender always has a packet to transmit. These simplifications have been shown in the literature [8] to affect the analysis only simplifying it, but without changing the qualitative behavior.

For the Markov analysis purpose, we focus on a hard decision decoding process. In this case, we quantize the number of errors which can be present into a packet of length L in $K+1$ levels, i.e., a received packet can have an "error level" equal to 0, 1, $\dots$, K. However, since we deal also with juxtapositions of packets (at most $F+1$), on a generic sequence of J packets of length L, $1 \leq J \leq F+1$ we have $JK+1$ levels.

Finally, for every $i = 0, 1, 2, \dots, F$ we define θ_i (which satisfies $0 \leq \theta_i \leq K \cdot (i+1)$) as the error correction threshold of the codes C_i, i.e., a packet is correctly received after its first transmission if its error level is below θ_0 (if it is not, a retransmission will occur); at the ith retransmission, the error level, which is evaluated by summing the previous error level evaluated on a $(K \cdot i)$-step scale with the new one with K levels, must be below θ_i in order to have complete error-

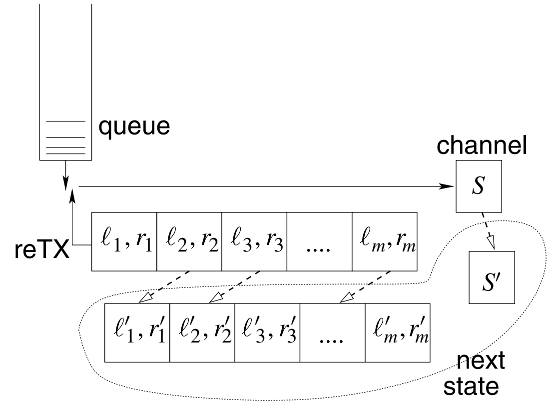

Fig. 1. The ARQ chain and its evolution

correction. The right definition of the θ_is depends on the actual way in which HARQ packets are coded and also on the error correction code used. This point will be further discussed in Section IV.

The above assumptions are suitable to describe the error process of the packets with an N-state Markov Chain, which is fully specified by an $N \times N$ matrix $\mathbf{T}$ whose generic element t_{ij}, $0 \leq i, j \leq N-1$, describes the probability of transition from state i to j. The N-state Markov Chain is used together with an integer function ξ which assigns an error level ξ_i in $0, 1, \dots, K$ to every state of the chain $i = 0, 1, \dots, N-1$.

This channel Markov chain describes the channel transition only, but, as already discussed, the selective repetition induces higher order memory. To derive the ARQ chain describing completely the state information, we follow an approach akin to the one used in [3], where we proved that one can use a Markov approach in which the status of last m transmissions plus the channel state is tracked. However, to realize this approach for the HARQ case, we have to consider more possibilities depending on the error level quantization and the maximum number of transmissions.

The derivation of the ARQ chain works as follows: we consider a state vector $\mathbf{v} = (\mathbf{a}, S)$, comprising two parts. S is the channel state, which takes a value in $0, 1, \dots, N-1$ and whose transitions are regulated by the $N \times N$ matrix $\mathbf{T}$. Also, each state S is associated with its error level ξ_S, taking values in $0, 1, \dots, K$. The term $\mathbf{a}$ is an m-sized array describing the outcome of the last m packet transmission. Every value of the array contains an ordered pair of integers, (ℓ_i, r_i), with $i = 1, \dots, m$. In particular, the right-most pair, i.e., (ℓ_m, r_m), refers to the current transmission, whereas (ℓ_1, r_1) describes the outcome of the oldest transmission considered, i.e., the transmission that occurred $m-1$ slots before. In general, (ℓ_i, r_i) refers to slot $t - m + i$, where t is the current time index. The element r_i, between 0 and F, corresponds to the number of retransmissions already performed for this specific packet. Values higher than F are not permitted since packets still in error after the Fth re-transmission are discarded. The first element ℓ_i of the pair is the error level associated with this specific packet, thus it is between 0 and $(r_i+1)K$ (at most, there are $(F+1)K+1$ possible values when $r_i = F$). To help understanding this process and the employed notation, look also at Fig. 1, where the system state at a given time and

1-4244-0357-X/06/$20.00 ©2006 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2006 proceedings.

its cyclical evolution are sketched. In the figure, it is shown that everyone of the past m time slots is assigned to the pair (ℓ_i, r_i) which is cyclically clocking as the time goes by, and the system state is also described by the channel state, which evolves following a Markov chain.

Note that a further simplification is possible, in order to describe the state with a lower number of possible values. In fact, since the impact of values of ℓ lower than θ_r is the same (they describe a packet which is anyway considered resolved once the ACK reaches the transmitter), we might identify with $(0, r_i)$ every entry in the array $\mathbf{a}$ of the form (ℓ_i, r_i), where $\ell_i \leq \theta_{r_i}$.

On a discrete time axis, the evolution of $\mathbf{v} = (\mathbf{a}, S)$ is fully described by a Discrete-Time Markov chain. We will call $\mathbf{G}$ its transition matrix. We omit the proof of this for brevity. However, it can be easily proven by showing that every transition depends on the current state only. For this Markov chain, balance equations can be written, to describe ACK/NACK of a packet at the first transmission, ACK/NACK of a packet after a retransmission and finally probability equal to 0 for the infeasible states.

Thus, if $\sigma(\mathbf{v})$ is the steady-state probability that the state vector is $\mathbf{v}$:

$$\begin{aligned}&\text{if } (\xi_S \leq \theta_0): \\ &\sigma((\ell_1, r_1), \ldots, (0,0), S) = \\ &= \sum_{c=0}^{N-1} t_{cS} \Bigg(\sum_{x=0}^{F} \sigma((0,x), (\ell_1, r_1), \ldots, (\ell_{m-1}, r_{m-1}), c) + \\ &\quad + \sum_{x=\theta_F+1}^{K(F+1)} \sigma((x,F), (\ell_1, r_1), \ldots, (\ell_{m-1}, r_{m-1}), c) \Bigg)\end{aligned} \tag{1}$$

$$\begin{aligned}&\text{if } (\xi_S > \theta_0): \\ &\sigma((\ell_1, r_1), \ldots, (\xi_S,0), S) = \\ &= \sum_{c=0}^{N-1} t_{cS} \Bigg(\sum_{x=0}^{F} \sigma((0,x), (\ell_1, r_1), \ldots, (\ell_{m-1}, r_{m-1}), c) + \\ &\quad + \sum_{x=\theta_F+1}^{K(F+1)} \sigma((x,F), (\ell_1, r_1), \ldots, (\ell_{m-1}, r_{m-1}), c) \Bigg)\end{aligned} \tag{2}$$

$$\begin{aligned}&\text{if } 0 < x \leq F \text{ and } \theta_x - \xi_S > \theta_{x-1}: \\ &\sigma((\ell_1, r_1), \ldots, (0,x), S) = \sum_{c=0}^{N-1} t_{cS} \cdot \\ &\quad \cdot \sum_{y=\theta_{x-1}+1}^{\theta_x - \xi_S} \sigma((y, x-1), (\ell_1, r_1), \ldots, (\ell_{m-1}, r_{m-1}), c)\end{aligned} \tag{3}$$

$$\begin{aligned}&\text{if } 0 < x \leq F, \quad \theta_x < y \leq xK + \xi_S: \\ &\sigma((\ell_1, r_1), \ldots, (y,x), S) = \\ &= \sum_{c=0}^{N-1} t_{cS} \cdot \sigma((y - \xi_S, x-1), (\ell_1, r_1), \ldots, (\ell_{m-1}, r_{m-1}), c)\end{aligned} \tag{4}$$

$$\begin{aligned}&\text{in any other case}: \\ &\sigma((\ell_1, r_1), \ldots, (\ell_m, r_m), S) = 0\end{aligned} \tag{5}$$

Note that these equations can be put into a matrix form, therefore obtaining the matrix $\mathbf{G}$.

III. Evaluation of SR ARQ metrics

We can now proceed to the evaluation of several metrics of interest for the SR ARQ analysis. The set of balance equations (1)–(5) can be used to derive the steady-state probabilities, solving $\sigma\mathbf{G} = \sigma$. In particular, this is obtained by imposing also the normalization condition, i.e.

$$\sum_{\mathbf{v}} \sigma(\mathbf{v}) = 1 \tag{6}$$

since the balance equations are homogeneous. After having derived the values of $\sigma(\mathbf{v})$ for all possible $\mathbf{v}$s, the following performance metrics can be evaluated: Average throughput T, Average number of transmissions per packet N_{tx}, probability of packet discarding P_{pd}.

The average throughput can be evaluated as the sum of the steady-state probabilities of the states in which a packet is correctly received and a new one is sent over the channel. This happens when $\ell_1 = 0$, so if we define the set $\mathcal{A}$ as collecting all states where a correct packet is received, i.e., $\mathcal{A} = \{\mathbf{v} \mid \mathbf{v} = ((0, r_1), (\ell_2, r_2), \ldots, (\ell_m, r_m), S)\}$, we have:

$$T = \sum_{\mathbf{v} \in \mathcal{A}} \sigma(\mathbf{v}) \tag{7}$$

An equivalent description of this metric may be obtained by considering a different position than the first, i.e., ℓ_i with $1 < i \leq m$ instead of ℓ_1, due to the fact that the shift of the locations from m through 1 is deterministic. However, we indicate ℓ_1 in the previous expression since it corresponds to the instant when the correct reception is known at the transmitter, so that a new packet is sent. Equivalently, the condition $\ell_m = 0$ would have the physical meaning of describing when the destination node correctly receives the packet.

Since r_1 describes the number of retransmissions of the packet whose feedback is currently arriving at the transmitter, we can evaluate the average number of transmissions as

$$N_{tx} = \sum_{\mathbf{v}} (r_1 + 1)\sigma(\mathbf{v}) = 1 + \sum_{\mathbf{v}} r_1 \sigma(\mathbf{v}). \tag{8}$$

Note that, as for the throughput, any r_i with $1 < i \leq m$ can be used instead of r_1.

Analogously to the calculation of the average number of packet transmissions, the condition of packet discarding is instead described by having (x, F) as the first pair of the vector $\mathbf{a}$, where $\theta_F < x \leq (F+1)K$. Defining $\mathcal{B}$ as the set of states where the received packet is going to be discarded, i.e., $\mathcal{B} = \{\mathbf{v} | \mathbf{v} = ((x,F), (\ell_2, r_2), \ldots, (\ell_m, r_m), S)$ with $\theta_F < x \leq (F+1)K\}$,

$$P_{pd} = \sum_{\mathbf{v} \in \mathcal{B}} \sigma(\mathbf{v}). \tag{9}$$

Again, a different choice of the position where to identify the pair (x, F) is also possible, since having this pair in another position of the vector $\mathbf{a}$ means that the corresponding packet is bound to be discarded, even though this is not happening in the current time instant.

IV. Numerical Results

The evaluation presented in the previous sections relies on the availability of an N-state Markov chain which describes the channel so that every state i is characterized by an error level ξ_i. Moreover, these levels have to be compared with the threshold θ_j, where j is the number of transmissions. For the numerical evaluations, we adopt a simple and practical approach, which derives an N-state channel chain from a simple two-state chain. This is just an example to directly validate the model presented before and show how this can be used to evaluate and compare different Type II Hybrid SR ARQ strategies with hard decision.

For the error correction we consider an (n, k) Reed Solomon (RS) linear erasure block code, with symbols from the Galois Field $\mathbb{Z}_{2^M}$, where k and n are the number of symbols of the uncoded and coded message, respectively. In order to encode a binary message $\mathcal{F}$ of kM bit, we first split $\mathcal{F}$ in k blocks of M bits, corresponding to k symbols of $\mathbb{Z}_{2^M}$. After that we apply the code to the k symbols, obtaining a coded message C of nM bit. The described code is equivalent to a binary linear code (nM, kM). The minimum distance of an (n, k) RS code is $d_{min} = n - k + 1$; thus, assuming that in a coded message of n symbols there are c erasures, i.e., symbols that are known to be in error, and e unknown errors, the message is successfully decoded if

$$2e + c \le d_{min} - 1 = n - k. \tag{10}$$

Our numerical evaluation is referred to a case of *known error positions* (KEP), which means that, similarly to [9], we tail a Cyclic Redundancy Check (CRC) to each block. Thus, the decoder knows the location of the symbol errors by detecting the bit errors contained in each block thanks to the CRC. Hence, the receiver knows which symbols it failed to detect, and $e = 0$. Thus, the correct reception of at least k symbols is sufficient for the message reconstruction.

Assuming that an information packet contains k symbols, for the HARQ system under analysis, we take $n = (F + 1)k$, recalling that F is the maximum number of retransmissions before packet discarding. In this way we may divide the overall codeword into $F+1$ parts, containing k symbols each, which are sent out in order at every transmission. With the notation used in Section II, L, K, k are all the same value, for simplicity. Thus, code C_0 corresponds to a (k, k) code, i.e., to information symbols only, C_1 is a $(2k, k)$ RS code and so on, and proper thresholds θ_i are defined as $\theta_i = ik$.

To model the channel with a Markov approach, we consider the errors at symbol level to be described by a two-state Markov process with transition matrix $\mathbf{P} = \{p_{ij}\}$, $i, j \in \{0, 1\}$, where state 0 means error-free channel and 1 on the other hand describes always erroneous condition. For this model, the steady-state error probability is $\varepsilon = p_{01}/(p_{10}+p_{01})$ and the average error burst length is $B = 1/p_{10}$. In this way, the $K+1$ error levels on a packet are obtained by considering its K symbols, each one of which could be correct or not according to the outcome of the two-state Markov chain.

This indeed determines $K + 1$ possible outcomes for what concerns the number of errors which are present in a single packet, that is level 0 is obtained when all symbols are correct (i.e., the two-state chain stays in state 0 for K subsequent instances), level 1 corresponds to all K possibilities where only one sub-packet is erroneous, and so on. However, to keep the Markov property of the model, we also need to memorize the outcome of the last symbol separately. In fact, all transitions to the next error level only depend on the outcome of the *last* symbol, since the two-state chain is Markov.

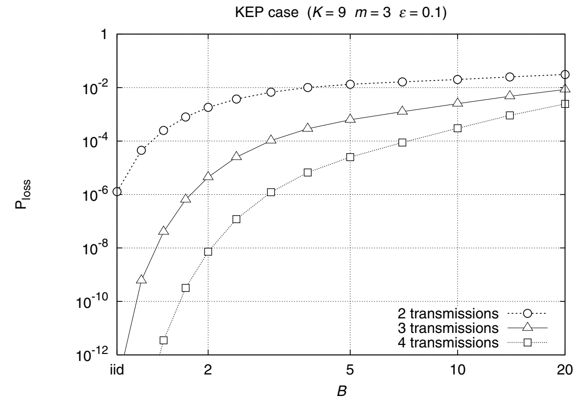

Fig. 2. SR Type II HARQ performance for RS codes in the KEP case: impact of channel burstiness on the probability of packet discarding

Thus, this approach is able to obtain a suitable $N \times N$ transition matrix $\mathbf{T}$ with $N = 2K$, since the N states describe all possibilities of error level ($K + 1$ values) and last symbol outcome (2 possibilities), but two cases never happen, since all correct (erroneous) symbols always imply that the last one is also correct (erroneous). We assume that the states are numbered so that 0 means that the error level of the packet is 0 (which implies last symbol is correct), $2K - 1$ means that the error level is K (which implies last symbol is erroneous). For every other intermediate case $0 < j < 2K - 1$, state j means that the error level is $\lceil j/2 \rceil$ and the last sub-packet is correct or erroneous according to $((j))_2$ being 0 or 1, where $((\cdot))_2$ denotes the modulo 2 operation.

Considering a transmission of K consecutive symbols, with time indices $1, 2, \ldots, K$, one can denote with $\varphi_{xy}(s, K)$, $x, y \in \{0, 1\}$ the probability that s symbols out of K are successful and the channel state is y for the Kth, given that the channel state was x at time 0 (i.e., for the last symbol transmitted before the sequence of K symbols starts), which is a well-known function that can be derived as shown in [10]. This allows to promptly compute the matrix $\mathbf{T}$ since t_{ij} is set equal to

$$t_{ij} = \varphi_{xy}(j, K) \tag{11}$$

where $x = ((i))_2$, $y = ((j))_2$.

In the following, we set the value of m to 3. To show the impact of the channel correlation [1] on the Type II HARQ performance, we mainly choose the value of B (average length of symbol error bursts) as the independent variable. We will speak in particular of *i.i.d.* (independent and identically distributed) case, referring to the choice of $B = 1/\varepsilon$ which causes the symbol errors to be i.i.d. The parameter ε has been taken equal to 0.1. This choice means that the steady state probability of the Markov channel chain being in the

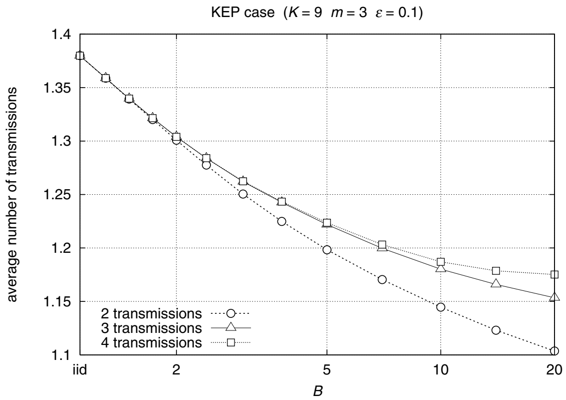

Fig. 3. SR Type II HARQ performance for RS codes in the KEP case: impact of channel burstiness on the average number of transmissions

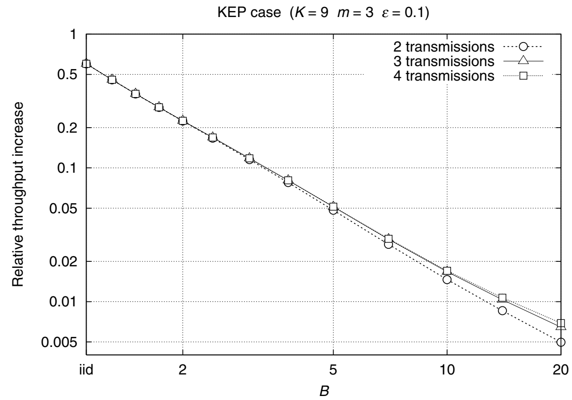

Fig. 4. SR Type II HARQ performance for RS codes in the KEP case: relative throughput increase from the single transmission case.

good state 0 is 38.7% in the i.i.d. case, but increases if the channel is bursty. For example, for $B = 20$ it will be 86.1%. Note that this value is the complement of the packet error probability if only one transmission is allowed. Therefore, the error probability for the single transmission case would be too high for any reasonable application; however, in this way we can show the capability of Type II HARQ of dealing with highly impaired channel conditions, obtaining lower error probabilities when retransmission is allowed.

Fig. 2 reports the packet loss probability. This figure highlights that even in the case of high error probability, a HARQ mechanism is able to significantly reduce the probability of discarding a packet if a sufficiently high number of retransmissions is allowed. For the case of weakly correlated channel, when 4 transmissions are allowed, the value of P_{loss} can be pushed down to less than 10^{-12}. [1] However, note that bursty channels have worse performance in this sense since their probability of not resolving a packet even with 4 transmissions may be still significant.

Fig. 3 shows the average number of packet transmissions, which generally presents a slight increase with F, especially when the channel is correlated. Indeed, the average number of packet transmissions is generally lower for correlated channels. This is because when the channel is correlated it is also likely that it stays in the good state for a longer time; thus, it is more likely that the packet is delivered upon its first transmission.

For what concerns throughput, we do not report absolute values since the curves are too difficult to distinguish. Rather, we plot in Fig. 4 the relative improvement obtained by comparing the throughput with the steady state probability of the Markov channel chain of being in state 0: recall that this corresponds to the throughput in the case of a single transmission. With our numerical choices we obtain the largest incremental improvement by allowing $F = 2$. The further increases in the throughput performance for $F = 3$ or $F = 4$ are very low. Note, in particular, that significant improvements are obtained only when the channel tends to the i.i.d. case, whereas the throughput variations for the correlated case are marginal. As a general conclusion, the channel correlation strongly affects the performance, which justifies the worthiness of our analysis.

[1] Actually, compatibly with the numerical precision of the computation tool, we found that P_{loss} is lower than 10^{-19} for the i.i.d. channel when $F = 4$.

V. Conclusions

We presented a Markov analysis for Selective Repeat Type II Hybrid ARQ techniques, which allows to study from a general perspective the behavior in terms of throughput, number of transmissions and delay. The presented analytical framework is entirely tunable and adaptable to different channel models; moreover, it can be promptly extended to consider also different assumptions for what concerns the transmission process or the employed coding. Exact results have been presented in order to evaluate Selective Repeat truncated Type II HARQ for the case of Reed Solomon linear erasure block codes with known error position. These results can be useful to gain precise understanding about the behavior of HARQ mechanisms.

References

[1] W. Luo, K. Balachandran, S. Nanda, and K. Chang, "Delay analysis of selective-repeat ARQ with applications to link adaptation in wireless packet data systems," *IEEE Trans. Wirel. Commun.*, vol. 4, no. 3, pp. 1017–1029, May 2005.

[2] T. Issariyakul, E. Hossain, and A. S. Alfa, "Markov-based analysis of end-to-end batch transmission in multi-rate multi-hop wireless networks," in *Proceedings IEEE ICC 2005*, vol. 5, 2005, pp. 3505–3509.

[3] M. Rossi, L. Badia, and M. Zorzi, "Exact statistics of ARQ packet delivery delay over Markov channels with finite round-trip delay," *IEEE Trans. Wirel. Commun.*, vol. 4, no. 4, pp. 1858–1868, July 2005.

[4] A. G. Konheim, "A queueing analysis of two ARQ protocols," *IEEE Trans. Commun.*, vol. 28, no. 7, pp. 1004–1014, July 1980.

[5] Q. Zhang and S. A. Kassam, "Finite-state Markov model for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, no. 11, pp. 1688–1692, Nov. 1999.

[6] J.-B. Seo, Y.-S. Choi, S.-Q. Lee, N.-H. Park, and H.-W. Lee, "Performance analysis of a Type-II Hybrid-ARQ in a TDMA system with correlated arrival over a non-stationary channel," in *Proc. ISWCS*, 2005, pp. 59–63.

[7] B. Bellalta i Jimenez and A. Graell i Amat, "Performance analysis of a Type II Hybrid ARQ protocol based on RCPC codes for the IEEE802.11a random access MAC protocol," in *Proc. IEEE VTC Spring*, vol. 4, 2005, pp. 2359–2363.

[8] L. Badia, M. Rossi, and M. Zorzi, "SR ARQ packet delay statistics on Markov channels in presence of variable arrival rate," *IEEE Trans. Wirel. Comm.*, vol. 5, no. 7, pp. 1639–1644, July 2006.

[9] L. Rizzo, "Effective erasure codes for reliable computer communication protocols," *ACM SIGCOMM Computer Communication Review*, vol. 27, no. 2, pp. 24–36, Apr. 1997.

[10] R. A. Howard, *Dynamic Probabilistic Systems*. New York: John Wiley & Sons, INC., 1971.