An Analysis of Cognitive Networks for Unslotted Time and Reactive Users

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***Abstract*—A novel framework for the analysis and optimization of cognitive wireless networks with unslotted time operations and reactive primary users is proposed. In the considered network setting, primary users' channel access is regulated by a carrier sense-based contention mechanism. As the sensing mechanism cannot distinguish between primary and secondary signals, secondary users' activity may interfere with primary users' channel contention, thus biasing the statistics of the stochastic process modeling primary users' transmissions. In fact, a primary user which wakes up during a transmission from a secondary user may sense a busy channel and enter backoff or generate a collision. The proposed framework considers these effects and optimizes the fraction of time a secondary user is allowed to transmit according to a constraint on the minimum throughput achieved by the primary users. Numerical results are presented which illustrate fundamental behaviors and tradeoffs in a network with one primary and one secondary user. Extension to more general scenarios is also discussed.**

***Keywords*-Cognitive radios, Renewal analysis, Wireless networks, Channel sensing, Reactive primary users**

I. Introduction

The coexistence in the same band of licensed (primary) and unlicensed (secondary) users has been widely investigated as an effective way to increase the efficiency of spectrum usage in wireless networks. While primary users are generally intended as *dumb* devices, secondary users implement smart and adaptive strategies in order to effectively exploit the available channel resource. This general framework is referred to as *cognitive radio* [1], [2].

As primary users are the legitimate owners of the bandwidth, secondary users must devise transmission strategies which maximize their own revenue while generating a bounded and limited degradation to primary users' performance [3]. The most popular approach to this challenging problem is the so called *white space* approach [4]–[8], which grants access to secondary users in time-frequency slots left unused by the primary users. The idea is to confine primary users' transmission in a channel resource orthogonal to that used by the primary users in the time and frequency domain in order to avoid packet collisions. Secondary users' transmission/idleness in a given band is determined based on the outcome of a binary hypothesis test on the existence of a primary user's signal.

In this context, most prior work assumes slotted time operations, *i.e.*, the existence of a time structure synchronizing channel access and packet transmission. Under this assumption, if secondary user channel sensing is idealized, that is, secondary users perfectly identify unused time slots, then secondary user activity is transparent to the primary users, which do not suffer any performance loss [9]–[11]. In practice, secondary user channel sensing is imperfect due to noise and fading. As a consequence, secondary user activity results in an increased failure rate of primary users' packets [6], [7], [12]. In fact, a secondary user which erroneously detects as idle a time-frequency slot accessed by a primary user, may transmit a packet, and then collide with the transmission of the primary user. Secondary users, then, need to account for sensing errors in their strategy in order to meet the constraints on the primary users' performance. Cooperation strategies can be used to balance the performance loss suffered by the primary users [8].

Recently, there has been increasing interest in cognitive networks with unslotted access [13]–[15], in which primary (and secondary) users asynchronously access the channel. The removal of the slotted structure significantly complicates the access strategy of the secondary users. In fact, a primary user may access the channel during a secondary user's transmission and generate a collision. Therefore, the transmission time of the secondary users, which in slotted time is generally equal to one slot, becomes an optimization variable in the unslotted scenario. [13] presents a learning-based algorithm for unslotted cognitive networks with multiple secondary users. [14] studies the joint design of secondary user sensing and transmission strategy, and formalizes the optimization problem as a partially observable Markov decision process. In [15], the authors investigate various sensing, backoff and transmission strategies for secondary user channel access in unslotted cognitive networks.

As mentioned before, primary users are generally intended to be oblivious devices, whose channel access and transmission strategy follows the same predetermined policy and protocol which regulates their operations in the absence of secondary users. However, although not as adaptive and smart as those implemented by the secondary users, the policy which controls the primary users' network may *react* to the activity of secondary users. In fact, the presence of secondary users may interact with those mechanisms designed to coordinate primary users channel access or to preserve the link quality of individual primary users. This framework, referred to as cognitive networks with *reactive* primary users, was introduced in [16]–[18] by Levorato *et al.* In [16]–[18] primary users implement an Automatic Retransmission reQuest protocol. Interference due to secondary users' transmissions increases the failure probability of primary users' transmission, thus biasing their retransmission process and the idle/busy pattern of the channel. In [19], Huang *et al.* study a reactive primary users scenario in which the access probability of primary users decreases if a collision occurs.

Prior work on unslotted cognitive networks [13]–[15] assume that the statistics of the stochastic process modeling primary users activity are independent of the operations of secondary users. In this paper, we propose an analytical framework which addresses an unslotted cognitive network with reactive primary users implementing a carrier sense-based contention mechanism. Carrier sensing is widely used to regulate users' access in practical wireless networks. Before transmitting, users perform a binary hypothesis test on the incoming power and classify the channel as busy or idle. If the channel is sensed idle, then the user transmits the packet, otherwise it enters a backoff period whose duration has a predefined distribution. This mechanism helps reduce the collision rate and avoid network congestion. We remark that, different from [17]–[19], we consider reactive primary users under the assumption of unslotted time operations. However, the contention mechanism cannot distinguish between primary and secondary signals. As a consequence, secondary users' transmissions may force a primary user to enter backoff, thus biasing the statistics of primary users' channel access. We present a framework which explicitly models this inter-dependence in a network with a single primary and a single secondary user and incorporates imperfect sensing. Primary user's activity is modeled as an alternate idle/busy renewal process, whose statistics depend on the transmission strategy of the secondary user. Even though this is a very specific case, the numerical results we obtain are able to shed light on fundamental behaviors and tradeoffs that arise in these types of systems. How this framework can be extended to the more general (and practically relevant) case of multiple primary and secondary users is also discussed.

The rest of this paper is organized as follows. In Section II, the network and problem considered in this paper are described and discussed. Section III presents the analytical computation of the fundamental statistics and metrics of the renewal reward process modeling the network. In Section IV, we provide a specific instantiation of the network and derive the closed-form expressions for this case. Section V presents and discusses numerical results showing some fundamental behaviors of the system under investigation. Section VI discusses the extension of the presented analysis to more complex networks. The paper ends with the conclusions in Section VII.

II. Network Model

We address hierarchical dynamic spectrum access [4], so that, even though the primary users are the legitimate owners of the channel resource, secondary users are also authorized to use the wireless medium, according to an *opportunistic spectrum access* rationale. This is permitted under the general rule that the performance of the primary users does not degrade too much due to the presence of the secondary users. The bound on the maximum degradation of the primary users' throughput is explicitly included in the optimization problem formalized in the following. For the sake of simplicity, we consider a single channel shared by all the users. The extension to multiple channels is straightforward, and beyond the scope of this paper.

As noted earlier, we include channel sensing and backoff in the model of primary users as a mechanism to regulate channel access. In particular, before transmitting, a primary user senses the channel to detect the presence of other users. If the channel is sensed as idle, it transmits the packet. If the channel is sensed as busy, the primary user enters a backoff period of random duration, after which it senses the channel again, and the process repeats indefinitely.

Each primary user generates an idle/busy alternating channel activity pattern. The j-th idle and busy periods are referred to as $I(j)$ and $B(j)$, $j{=}1,2,3,\ldots$.

The idle period $I(j)$ accounts for the interarrival time of the packets in the queue of the primary users and possible backoff time due to channel sensing. In particular, $I(j)$ is the sum of an interarrival time $U(j)$ and n backoff intervals $W_1(j),\ldots,W_n(j)$, with $n{\geq}0$, where $n{=}0$ means that the primary source immediately starts transmission after a packet arrives in its queue, *i.e.*, $I(j){=}U(j)$.

We include sensing errors in the model, and we define $\Gamma_{i\to b}$ and $\Gamma_{b\to i}$ as the probability that the primary source identifies as busy an idle channel and vice versa, respectively.

The secondary source exploits the idle periods $I(j)$, $j{=}1,2,3,\ldots$ to transmit its own packets. We assume a *persistent sensing* strategy for the secondary source, by which the latter continuously senses the channel during the transmission of the primary. This in order to identify the start of the idle period, corresponding to the first time instant in which the primary senses an idle channel. The secondary source then starts to transmit, and occupies the channel for a time τ. We will discuss in the next sections, how more refined and detailed sensing strategies modify the study presented in this paper. Secondary user channel sensing is assumed to be perfect in this paper.

Many papers in the related literature consider slotted time operations. Under such an assumption, the strategy of the secondary user is just to sense the channel until it becomes idle. In fact, when this condition is verified, the secondary user knows that the channel will be available for the entire duration of the slot. Therefore, the secondary user can just set its transmission time equal to the slot duration, which guarantees that any interference or collision with the primary is avoided. In this framework, the only cause of interference with the primary user is imperfect sensing.

In contrast, for unslotted networks, the primary user may start transmission at any time. If the secondary user senses the channel as idle, it does not know how long the channel will remain available. Thus, the strategy of the secondary users, beside channel sensing, must also include the specification of the transmission time duration τ.

If the interarrival interval U of the primary source is smaller than τ, then the secondary source is still transmitting when the primary source senses the channel and the latter enters backoff or collides depending on the outcome of the sensing. The same holds for the end of every backoff period.

We assume that the durations of the idle and busy periods are independent with respect to each other. As the statistics of $I(j)$ and $B(j)$ do not depend on j, we drop the subscript. The activity of the primary source, then, is modeled as an alternate renewal process, where the time between two consecutive renewal events (referred to as *renewal cycle* in the following) is divided into two parts, corresponding to the idle and busy period, respectively (Fig. 1.a). Therefore, the renewal event corresponds to the end of the transmission of the primary source.

The durations of the renewal cycles are independent, identically distributed and nonnegative random variables, denoted by $X{=}I{+}B$. We denote the cumulative density function (cdf) of the variables I and B by $F_I(i){=}P[I{\leq}i]$ and $F_B(b){=}P[B{\leq}b]$, where $P[\cdot]$ denotes the probability of an event. The cdf of the renewal cycle is thus:

$$F_X(x){=}P[X{\leq}x]{=}\int_0^x F_B(x{-}i)\mathrm{d}F_I(i), \qquad (1)$$

with $\mathrm{d}F(y){=}(\mathrm{d}F(y)/\mathrm{d}y)\mathrm{d}y$. The average duration is $E[X]{=}E[I]{+}E[B]$, where $E[\cdot]$ is the expectation operator.

Let us denote the throughput of the primary and secondary source as $\mathcal{T}_1(\tau)$ and $\mathcal{T}_2(\tau)$, respectively. Note that both are function of the transmission time of the secondary source τ. In fact, the success probability and the duration of the renewal cycle depend on τ, due to collisions and backoff.

According to reward renewal process theory, $\mathcal{T}_1(\tau)$ and $\mathcal{T}_2(\tau)$ can be expressed as

$$\mathcal{T}_1(\tau){=}\frac{R_1(\tau)}{E[X|\tau]} \quad \text{and} \quad \mathcal{T}_2(\tau){=}\frac{R_2(\tau)}{E[X|\tau]}, \qquad (2)$$

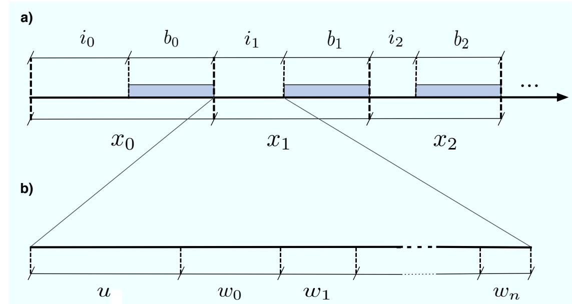

Figure 1. a) Alternate renewal process of the primary source, b) characterization of the idle time.

where $R_1(\tau)$ and $R_2(\tau)$ are the average per-cycle reward of the primary and secondary source, respectively, and τ is the deterministic duration of all secondary transmissions, and is therefore a fixed parameter.

The primary reward per cycle can be computed as

$$(1{-}\psi_1(\tau))\int_0^{+\infty}\phi_1(b, L_1(b))L_1(b)\mathrm{d}F_B(b), \qquad (3)$$

where $\psi_1(\tau)$ is the collision probability conditioned on τ, $L_1(b)$ is the length in bits of the transmitted packet (that may in general be a function of the busy time duration). $\phi_1(b, L_1(b))$ is the successful decoding probability conditioned on the transmission duration and the size of the transmitted packet (that determine the transmission rate). The reward of the secondary user admits an analogous expression.

The goal of the secondary user is to maximize its own throughput with a constraint on the maximum throughput loss suffered by the primary user, where the reference throughput of the latter is computed in the absence of the former.

The optimization problem can be formulated as

$$\begin{aligned} \overline{\tau}:\ & \arg\max_{\tau} \mathcal{T}_2(\tau) \\ \text{s.t. } & \mathcal{T}_1(0) - \mathcal{T}_1(\tau) < \mathcal{T}_1(0)\epsilon, \end{aligned} \qquad (4)$$

where ϵ is the maximum fraction of throughput loss, referred to as *maximum distortion* in the following.

It is important to observe that the bound on the maximum throughput loss of the primary user also implies a bound on the impact on the statistics of the renewal cycle of the transmission by the secondary user, that determines the average cycle duration and thus influences the overall throughput. However, the activity of the secondary user also affects the success probability, and thus the reward, of the primary user.

III. Duration of the Renewal Cycle and Collision Probability

In this section, we derive the duration of the renewal cycle conditioned on τ, *i.e.*, $E[X|\tau]$.

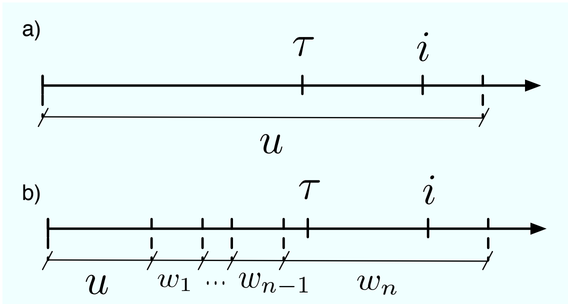

Figure 2. a) the interarrival time u is larger than i, b) the interarrival time u is smaller than i.

The average duration of the renewal cycle can be computed as

$$E[X|\tau]{=}E[I|\tau]{+}E[B|\tau]. \tag{5}$$

However, the duration of the transmission of the secondary user does not influence that of the primary one, and therefore

$$E[B|\tau]{=}E[B]{=}\int_0^{+\infty}(1{-}F_B(b))\mathrm{d}b. \tag{6}$$

The duration of the idle time, instead, depends on the value of τ, and thus we redefine the cdf of the idle period as $F_{I|\tau}(i)$, and the average of I can be found with an equation analogous to (6). We now need to compute $P[I{>}i|\tau]$ For the sake of simplicity, we assume $F_{W_k}(w_k){=}F_W(w)$, $k{=}1,2,3,\ldots$, *i.e.*, the backoff intervals are i.i.d. variables. The analysis presented in the following sections can be extended to consider the statistics of W_k that depend on k at the cost of more involved equations.

We first consider idealized sensing at the primary user, where $\Gamma_{b\to i}{=}\Gamma_{i\to b}{=}0$, and then develop the analysis for the case $\Gamma_{b\to i}{>}0$ and $\Gamma_{i\to b}{>}0$.

A. Idealized Sensing

In this case, the primary user always senses the channel correctly. Therefore, it can be observed that the duration I of the idle interval is larger than τ with probability one, *i.e.*, $P[I{>}\tau|\tau]{=}1$. In fact, the primary user does not start packet transmission if it senses a busy channel, and this event occurs with probability one while the secondary user is transmitting.

We thus focus on the case $I{>}\tau$. We can distinguish two cases, summarized in Fig. 2. If $U{=}u{>}\tau$, *i.e.*, the interarrival time of the primary user is larger than the duration of the transmission of the secondary user τ (see Fig. 2.a), then the primary user senses an idle channel at u and starts transmitting with probability one. Thus $I{>}i$ only if $U{>}i$, and this event has probability $1{-}F_U(i)$.

If $U{=}u{\leq}\tau$, then the primary user enters backoff with probability one at u (see Fig. 2.b). The primary source may *wake up* from the backoff again before τ, thus entering again backoff with probability one and so on. The duration of the idle time is thus equal to the first instant in which the primary user wakes up from backoff after τ.

We define the probability that k consecutive backoff intervals have a cumulative duration smaller than z as $F_W(z,k){=}P[\sum_{j=1}^{k} w_j{\leq}z]$. $F_W(z,k)$ can be recursively computed as

$$F_W(z,k){=}\int_0^z F_W(z{-}t)\mathrm{d}F_W(t,k{-}1), \tag{7}$$

with $F_W(z,1){=}F_W(z)$. If the cdf of the duration of each backoff interval depends on the index k, the convolution of the previous equation still holds where $F_W(z{-}t)$ is substituted with the cdf of the k–th backoff interval.

Assuming $U{=}u{\leq}\tau$

$$P[I{>}i|\tau,U{=}u{\leq}\ \tau\ ] = (1{-}F_W(i-u)) + \sum_{n=1}^{\infty}\int_0^{\tau-u}(1{-}F_W(i{-}u{-}z))\mathrm{d}F_W(z,n). \tag{8}$$

Thus, the previous probability is equal to the probability that the primary user wakes up from the first backoff after i, that is, the first backoff interval is larger than $i{-}u$ or there are n backoff intervals ending before τ, and more precisely at time $u{+}z$, with $z{<}\tau{-}u$ and for any $n{\geq}1$, and the last backoff is larger than $i{-}u{-}z$, that is, the primary user checks again the channel after i.

The probability $P[I{>}i|\tau]$ can be then computed as

$$P[I{>}i|\tau]{=}P[U{>}i]{+}\int_0^{\tau}P[I{>}i|\tau,U{=}u{\leq}\tau]\mathrm{d}F_U(u), \tag{9}$$

and $E[I|\tau]$ is then as in (6), and the overall average duration of the renewal cycle is the sum of the average duration of I and B.

Note that under the assumption of perfect sensing, both collision probabilities are equal to zero, that is, $\psi_1(\tau){=}\psi_2(\tau){=}0$.

B. Imperfect Sensing

If $\Gamma_{b\to i}{>}0$ and $\Gamma_{b\to i}{>}0$, the derivation of the distribution of the idle time is slightly more involved. In fact, we need to account for collisions that may be present when the primary user misses the detection of the secondary signal and additional backoff time due to the incorrect sensing of an idle channel, which is erroneously detected as busy so that a user refrains from transmitting unnecessarily.

We need to distinguish three cases:

- $I{\leq}\tau$,
- $I{>}\tau$ and $U{>}\tau$,
- $I{>}\tau$ and $U{\leq}\tau$,

which are illustrated in Fig. 3.a, Fig. 3.b and Fig. 3.c, respectively.

The objective is again the computation of the probability $P[I > i|\tau]$. In the first case, we may have either the arrival of the packet in the primary user's queue after a time $U = u{>}i$,

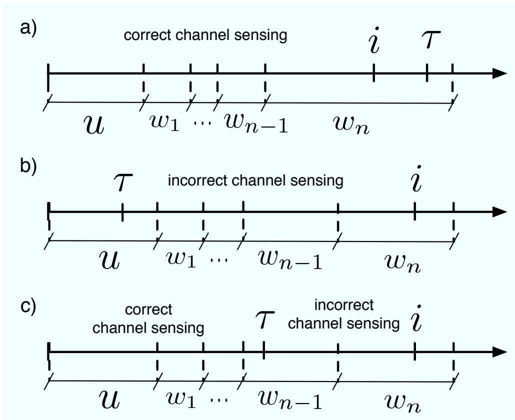

Figure 3. a) the transmission time τ is larger than i, b) the transmission time τ is smaller than i and the interarrival time u is larger than τ, c) the transmission time τ is smaller than i and the interarrival time u is smaller than τ. In the cases a and b, the figure refers to the nontrivial case in which $u \leq i$.

or a sequence of correct channel sensing by the primary user if $U = u \leq i$. In this latter case, there may have been an arbitrary number of backoff intervals that we need to account for.

According to the previous discussion, we write

$$P[I>i|\tau, i\leq\tau] = 1-F_U(i) + \int_0^i (1-\Gamma_{b\to i}) P[I>i|\tau, U{=}u\leq i\leq\tau]\mathrm{d}F_U(u), \tag{10}$$

where

$$P[I>i|\tau, U{=}u\leq i\leq\tau] = 1-F_W(i-u) + \sum_{n=1}^{\infty}\int_0^{i-u}(1-\Gamma_{b\to i})^n(1-F_W(i-u-z))\mathrm{d}F_W(z,n). \tag{11}$$

The second case, where the arrival of the packet in the primary user queue occurs after the secondary source has terminated its transmission, is rather similar. Nevertheless, if $U{=}u\leq i$, a sequence of incorrect channel sensing must take place upon the arrival of the packet and after the end of all the backoff intervals occurring in $(u, i]$. In fact, the primary user would otherwise start transmitting, thus ending the idle period before i.

In this case, we write:

$$P[I>i|\tau, U=u>\tau] = 1-F_U(i) + \int_\tau^i \Gamma_{i\to b} P[I>i|\tau, \tau<U{=}u\leq i]\mathrm{d}F_U(u), \tag{12}$$

with

$$P[I>i|\tau, \tau<U{=}u\leq i] = 1-F_W(i-u) + \sum_{n=1}^{\infty}\int_0^{i-u}\Gamma_{i\to b}^n F_W(i-u-z)\mathrm{d}F_W(z,n). \tag{13}$$

Note that in both the previous equations we used the probability that the primary user detects the presence of a signal when the channel is idle.

In the last case, we consider an idle period i larger than the transmission time of the secondary user τ, where a packet arrives in the queue of the primary user at time $u\leq\tau$. Thus, the primary user may remain idle as a consequence of a sequence of correct channel sensing in $(0,\tau]$ (during the transmission of the secondary user) and a sequence of wrong channel sensing in $(\tau, i]$ (when the channel is idle).

In particular, we have

$$P[I>i|\tau, i>\tau, u<\tau] = 1-F_U(i) + \int_0^\tau (1-\Gamma_{b\to i}) P[I>i|\tau, U{=}u<\tau\leq i]\mathrm{d}F_U(u), \tag{14}$$

where $P[I>i|\tau, U{=}u<\tau\leq i]$ is as in (15).

The first summand of the rather involved integral form of (15) simply corresponds to the probability that the first backoff ends after i. The second summand assumes that the first backoff ends after τ and before i. Thus, the idle period continues only if the primary user detects an interfering signal despite the channel being idle. As in the previously discussed cases, there may be an arbitrary number of backoff intervals, before the last one that causes the whole idle period to be greater than i. The last summand accounts for the case where the first backoff ends before τ. Thus, we may have a sequence of backoff intervals before τ, where

$$\begin{aligned} P\left[I>i|\tau, U{=}u<\tau\leq i\right] &= 1-F_W(i-u) + \\ &+ \int_0^{i-u}\Gamma_{i\to b}\left(1-F_W(i-z_1-u) + \sum_{n_1=1}^{\infty}\int_0^{i-u}\Gamma_{i\to b}^{n_1}(1-F_W(i-u-z_1-z_2))\mathrm{d}F_W(z_2,n_1)\right)\mathrm{d}F_W(z_1) + \\ &+ \sum_{n_2=1}^{\infty}(1-\Gamma_{b\to i})^{n_2}\int_0^{\tau-u}\int_0^{i-z_3-u}\left(1-F_W(i-z_2-z_3-u) + \sum_{n_1=1}^{\infty}\Gamma_{i\to b}^{n_1}\int_0^{i-z_2-z_3-u}(1-F_W(i-z_1-z_2-z_3-u))\mathrm{d}F_W(z_1,n_2)\right) \\ &\mathrm{d}\, F_W(z_2)\mathrm{d}F_W(z_3,n_2) \end{aligned} \tag{15}$$

$$\psi = \int_0^\tau\left[\Gamma_{b\to i} + (1-\Gamma_{b\to i})\left(\sum_{n=1}^{\infty}\Gamma_{b\to i}\int_0^{\tau-u}(1-\Gamma_{b\to i})^{n-1}\mathrm{d}F_W(z,n)\right)\right]\mathrm{d}F_U(u) \tag{16}$$

the primary user must sense the channel correctly. There may also be a sequence of backoff intervals lying within (τ, i), starting at the end of the last backoff started before τ, where the primary source remains idle only if it senses the idle channel as busy.

The collision probability is as in (16), where $\psi{=}\psi_1{=}\psi_2$. Thus, the collision probability is equal to the probability that, when the primary source first senses the channel, it does not detect the signal of the secondary user, plus the probability that the same occurs after all the possible backoff intervals ending before τ.

It is worth observing that the previously presented analysis can be extended to include more involved sensing strategies of the secondary user. For instance, similar equations result under the assumption that the secondary user senses the channel at instants divided by random idle periods. Moreover, also in this case it is possible to include a sensing error that generates an increased collision rate as well as wasteful idle time to both users.

IV. Case Study

In this section, we derive closed-form expressions for the previously presented formulae, under some specific assumptions. In particular, we assume that the interarrival and backoff intervals are exponentially distributed with parameters λ and ν, respectively. This assumption was used in all prior work on unslotted cognitive networks [13]–[15].

Thus, one can write

$$F_U(u){=}1{-}e^{-\lambda u}, \qquad F_W(w){=}1{-}e^{-\nu w}, \qquad (17)$$

$$\mathrm{d}F_U(u){=}\lambda e^{-\lambda u}\mathrm{d}u, \qquad \mathrm{d}F_W(w){=}\nu e^{-\nu w}\mathrm{d}w. \qquad (18)$$

It can be also shown that

$$\mathrm{d}F_W(w,n){=}\frac{\nu^n w^{n-1}}{(n{-}1)!}e^{-\nu w}\mathrm{d}w. \qquad (19)$$

The average duration of the interarrival and backoff intervals are $1/\lambda$ and $1/\nu$, respectively. Moreover, we assume that the error rate of the channel sensing is $\Gamma_{i\to b}{=}\Gamma_{b\to i}{=}\Gamma$.

For the sake of simplicity, we assume that the duration of the transmission period of the primary user is deterministic and equal to τ_B, and that the size of the packets sent by both the primary and the secondary user is fixed. Thus, the success probability of the primary user in the absence of collision can also be assumed to be fixed, and for convenience we set it to one. The inclusion in the framework of packet failures due to fading and noise only requires a slightly modified expression for the throughput of the primary user. In the following we refer to the throughput normalized over the bandwidth, and the rewards of the primary and secondary user can be then restated as $R_1(\tau){=}\rho_1(1{-}\psi(\tau))$ and $R_2(\tau){=}\rho_2(1{-}\psi(\tau))\tau$, respectively, where ρ_1 and ρ_2 are the transmission rates in b/s/Hz.

Most of the previous assumptions are stated just to simplify the notation and reduce the number of parameters of the system. The key assumption that enables the significant simplification of the equations describing the renewal cycle is the characterization of the various intervals as exponentially distributed intervals, thanks to the memoryless property of the exponential distribution.

A. Idealized Sensing

By plugging Eqs. (17)-(19) into the equations of Section III-A, it is possible to obtain a simple closed-form for the average cycle time and throughput. However, the same result can be obtained by simply reasoning on the memoryless property of the exponential distribution.

Recalling that $i{>}\tau$, consider first the probability $P[I{>}i|\tau, u{\le}\tau]$, that corresponds to the probability that the first sensing instant after τ occurs after i as well. Due to the memoryless property of the exponential distribution, we are allowed to forget about what happened before τ. Importantly, since $u{\le}\tau$, the first sensing instant occurs after an interval distributed according to an exponential distribution with parameter ν. Therefore, $P[I{>}i|\tau, u{\le}\tau]$ is the probability that the backoff interval lasts more than $i{-}\tau$ starting from τ, and then, $P[I{>}i|\tau, u{\le}\tau]{=}e^{-\nu(i-\tau)}$.

By substituting the expression of this probability in (9), and then computing the average we obtain

$$E[I|\tau]{=}\tau{+}\frac{1}{\nu}(1{-}e^{-\lambda\tau}){+}\frac{1}{\lambda}e^{-\lambda\tau}. \qquad (20)$$

In fact, the idle period of the cycle has a duration at least equal to τ, due to the idealized sensing assumption. The average additional time after τ depends on the distribution of the interval after which the primary user will sense the channel again. If $u{\le}\tau$, which happens with probability $1{-}e^{-\lambda\tau}$, the time before the primary user wakes up again is distributed according to an exponential distribution with parameter λ, otherwise the primary user is in backoff at time τ, and the remaining time is exponentially distributed with parameter ν. Since the average residual life of exponentially distributed intervals is equal to the average duration of the whole interval, it follows that in the two cases mentioned above, the average durations are $1/\lambda$ and $1/\nu$, respectively.

As said before, the collision probability in the idealized sensing is zero, and the expressions of the normalized throughput of the primary and secondary user can be trivially derived by the rewards stated at the beginning of the section.

The solution of the optimization problem (4) admits a quasi closed-form in this case, that is,

$$\overline{\tau}{=}\frac{1{-}\frac{\lambda}{\nu}{+}\epsilon{+}\Delta(-e^{1+\lambda/\nu-\epsilon-\tau_B(\nu-\lambda)-\log\nu})}{\lambda}, \qquad (21)$$

where $\Delta(y)$ is the Lambert function [20].

B. Imperfect Sensing

The imperfect sensing case is slightly more complicated, but also the assumption of exponentially distributed intervals grants a significant simplification of the equations.

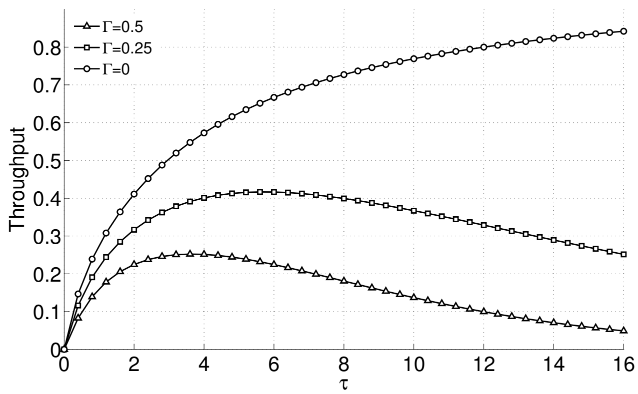

Figure 4. Throughput of the secondary user as a function of the duration τ of its transmission period.

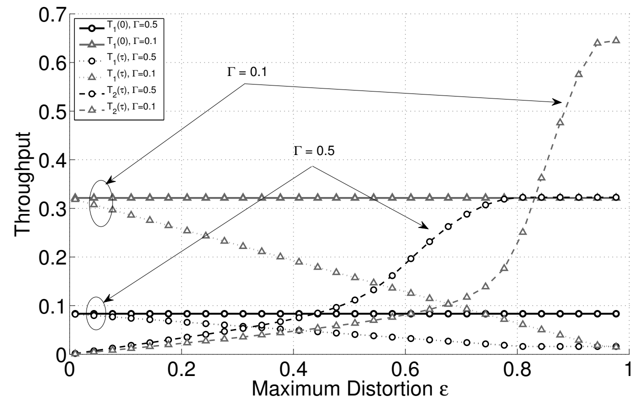

Figure 5. Throughput of the primary ($\mathcal{T}_1$, dotted) and secondary ($\mathcal{T}_2$, dashed) users as a function of the primary quality threshold ϵ.

Analogously to Section III-B, we consider three separate cases, namely $I{\le}\tau$; $I{>}\tau$ and $U{>}\tau$; $I{>}\tau$ and $U{\le}\tau$.

In the first case, through simple computations, we get

$$P[I{>}i|\tau, U{=}u{\le}i{\le}\tau]{=}e^{-(i-u)\Gamma\nu}, \tag{22}$$

that, when substituted in Eq. (10), yields

$$P[I{>}i|\tau, i{\le}\tau]{=}\frac{(e^{-i\Gamma}\nu{-}e^{-i\lambda})(1{-}\Gamma)\lambda}{\Gamma\nu+\lambda}. \tag{23}$$

The same probability in the second case admits a similar equation, that can be obtained by analogous calculations. In particular, we get

$$P[I>i|\tau, \tau{\le}U{=}u{\le}i] \tag{24}$$
$$=\frac{e^{-i(1-\Gamma)\nu-\tau\lambda-i\lambda}(e^{i(1-\Gamma)\nu+\tau\lambda}-e^{\tau(1-\Gamma\nu+i\lambda)})\Gamma\lambda}{\lambda{-}(1{-}\Gamma)\nu}.$$

The third case appears as a formidable integral expression. However, due to the memoryless property of the exponential distribution, it is possible to break the last summand into two separate integrals, one involving the period before τ, and the other involving the period after τ. We do not report here the long closed-form expressions for this last probability and the average duration of the idle period, as they do not add much to the discussion that follows.

The collision probability can be computed rather simply from Eq. (16). It can be found that the overall success probability of a packet, where $\phi_1{=}1$, is

$$(1{-}\psi){=}e^{-\lambda\tau}+\frac{(e^{\tau\Gamma\nu}{-}e^{\tau\lambda})\Gamma\lambda}{\Gamma\nu+\lambda}. \tag{25}$$

V. Numerical Results

In this section, we report some numerical evaluations obtained with the proposed analytical model in the case discussed before.

Unless other values are specified, we use the following parameters: the transmission time of the primary source is $\tau_B{=}1$ [s], the parameters of the interarrival and backoff distribution are $\lambda{=}1$ [1/s] and $\nu{=}0.5$ [1/s], respectively. The transmission rates are $\rho_1{=}\rho_2{=}1$ [b/s/Hz] and the maximum fraction of throughput loss of the primary source is $\epsilon{=}0.2$.

In Fig. 4 we show the throughput obtained by the secondary user as a function of τ. Increasing τ roughly corresponds to adopting a more aggressive transmission strategy, in the sense that once the secondary user gains access to the channel, it keeps the channel busy for a longer time. Thus, when the primary user performs perfect channel sensing, an increasing τ results in a higher throughput, since collisions are entirely avoided. This no longer holds if the sensing mechanism is imperfect and a more aggressive access strategy also leads to an increased number of collisions. Therefore, the curves for $\Gamma > 0$ have a maximum, which occurs when the performance gain due to a longer transmission time is completely balanced by the larger collision probability.

This implies, as will be highlighted also by other results, that there might be no point for the secondary user in increasing τ when the channel sensing mechanism of the primary is imperfect. Importantly, the secondary user may choose a τ smaller than the maximum transmission time forced by the constraint on the maximum throughput loss of the primary user.

Fig. 5 shows the throughput values of both primary and secondary users versus ϵ, which determines the maximum degradation of primary user's throughput. For ease of visualization, we also report (as horizontal solid lines in the figure) the throughput value of the primary user in the absence of the secondary user. This also corresponds to the value of the dotted curve (which denotes the performance of the primary user when the secondary is active) for $\epsilon = 0$. As is intuitive, when ϵ is increased, the throughput of the secondary user becomes higher, at the price of a decreased throughput for the primary user. However, from the figure, it is observed that the combined throughput of both users is generally higher than that of the primary user alone

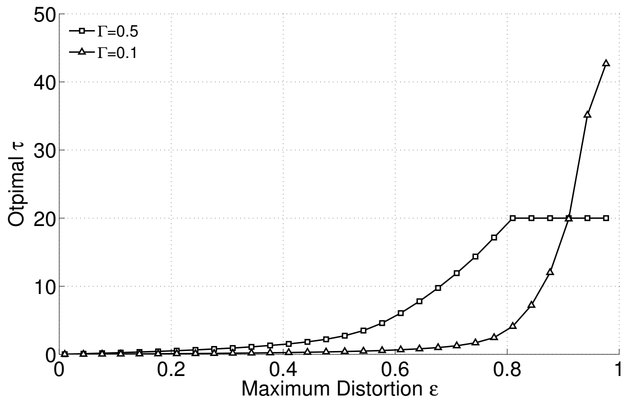

Figure 6. Optimal transmission duration τ of the secondary user as a function of the primary quality constraint ϵ.

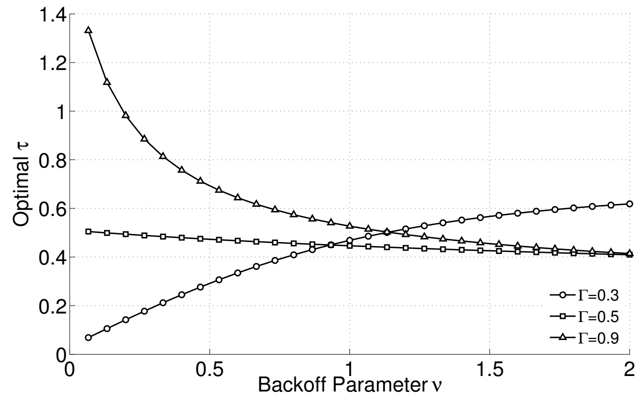

Figure 7. Optimal transmission duration τ of the secondary user as a function of the backoff parameter ν.

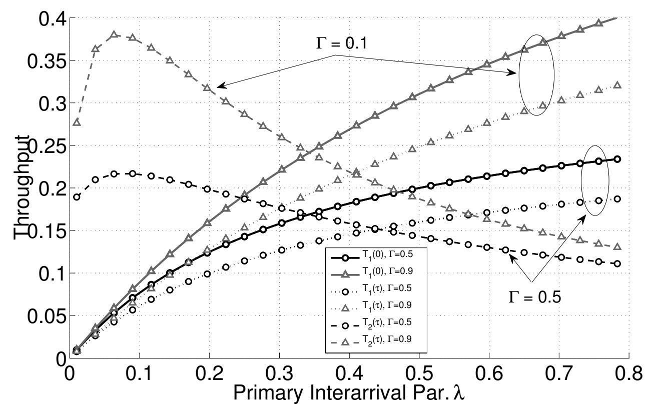

Figure 8. Throughput of the primary ($\mathcal{T}_1$, dotted) and secondary ($\mathcal{T}_2$, dashed) users as a function of the traffic intensity of the primary λ.

when the secondary is absent. This confirms the goodness of the cognitive network approach even in the proposed case. Finally, when ϵ becomes very large, both $\mathcal{T}_1$ and $\mathcal{T}_2$ saturate, and the larger Γ the lower the saturation value of ϵ. This means that, when ϵ is very large, the throughput of the secondary user is not pushed further, even though strong degradation of the primary performance is allowed. This happens because of the imperfect channel sensing performed by the primary user; when the secondary user can access the channel very often, a trade-off similar to the one mentioned above arises,*i.e.*, the additional throughput gained with a larger τ will be compensated by a higher collision rate, thus there is no way for the secondary to increase its throughput even further.

This conclusion is confirmed by Fig. 6 where the optimal value of τ is plotted directly as a function of ϵ. According to the previous discussion, the optimal transmission duration becomes longer as ϵ increases, since the secondary is allowed to access the channel more aggressively, but saturates when a certain limit value of ϵ is reached; the higher Γ, the lower the limit value of ϵ.

Fig. 7 investigates the dependence of the optimal τ on the backoff parameter ν of the primary user. Recall that backoff intervals are assumed to be exponential with parameter ν. Thus, the higher ν, the lower the average backoff interval. The figure shows two different trends according to the channel sensing performed by the primary being accurate or not. In the former case, when the backoff of the primary becomes shorter the secondary is better off by adopting a more aggressive transmission policy. A shorter backoff time implies indeed a more frequent sensing of primary user when the channel is found busy, thus resulting in a lower process distortion and therefore in a higher margin for longer transmissions of the secondary. When the sensing procedure is inaccurate instead, the parameter ν almost does not influence τ. However, we see a slightly decreasing trend as ν increases; this happens because when the backoff becomes shorter, multiple sensing operations by the primary during a transmission period of the secondary user are possible, each potentially leading to a collision. Thus, the secondary user may desire to reduce this risk and adopt a slightly less aggressive transmission policy.

We want to underscore this point. The previous discussion implies, as will be highlighted also by other results, that there might be no point for the secondary user in increasing τ when the channel sensing mechanism of the primary is imperfect.

Finally, Fig. 8 considers the effect of varying the traffic intensity of the primary, λ, and shows the throughput of both users as a function of λ. Similarly to Fig. 5, $\mathcal{T}_1$ and $\mathcal{T}_2$ are reported with dotted and dashed lines, respectively, while the solid line represents the performance of the primary user in the absence of the secondary user. Clearly, the throughput of the primary user is an increasing function of λ, in both cases with or without a secondary in the system. The performance when the secondary is present, and subject to an upper bound constraint to the process distortion caused on the primary, is simply scaled by the constant factor $1-\epsilon$. However, it is interesting to observe that the throughput of the secondary user $\mathcal{T}_2$ has an optimal point, whereas the throughput of the primary user decreases as λ is increased. The reason for this behavior is that when λ is very small, and therefore the throughput of the primary user is limited, it is very difficult to fulfill the constraint of a maximum process distortion on the primary user, since even a small degradation of its throughput is high in relative terms.

VI. Extension to Larger Networks

The analysis presented in this paper centers on the *distortion* of the primary user activity due to the secondary source's transmission. A deep understanding of the interdependence between primary and secondary users' activity is fundamental for the optimization of the secondary users' strategy in some network scenarios. In the considered setting, this effect involves both primary user and secondary user performance. On the one hand, the backoff mechanism implemented by the primary source amplifies the effect of collisions, inducing a larger degradation of the primary source performance. On the other hand, imperfect primary source channel sensing may result in collisions, thus hampering the secondary source's performance.

The presented framework enables the investigation of this mutual interaction in a simple network setting with one primary and one secondary source. By focusing on a simple basic system we shed light on the fundamental behavior of the system. However, the extension of the framework to larger networks with multiple primary and/or multiple secondary users is a further step in the direction of achieving practical adaptive control rationales determining the strategy of the secondary users in scenarios with complex interactions. In the following, we briefly discuss how the framework presented in this paper can be extended to larger networks.

The analysis of a network with multiple primary users requires a larger state space and a more complex renewal model. In general, the network can be modeled as a semi-Markov process [21], which associates an average reward and time duration to every transition from a state of the network to another. The average throughput is then obtained as the ratio between the sum over the state space of the reward and time associated with each state weighed with the steady state distribution.

For general distributions of the interarrival, backoff and transmission times, the computation of the average permanence time associated with each state is challenging, as it requires the construction of a complex state space of the embedded Markov chain in order to ensure that the process is semi-Markov. If the distribution of the arrival and backoff time is exponential, then the state space only needs to track the state of each individual primary source in terms of idleness, transmission and backoff, thanks to the memoryless property of the exponential distribution.

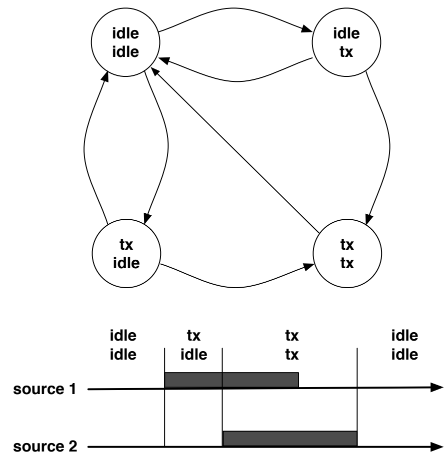

Figure 9. State space diagram and example of network state pattern for a network with 2 primary sources.

Fig. 9 shows the state space of the embedded chain of the semi-Markov process and an example of a sampling pattern for a network with two primary sources under the assumption of exponentially distributed interarrival and backoff times. From the state in which both sources are idle, the chain moves to a state in which one of the sources is transmitting. The average time and the probability of this transition only depend on the parameters of the arrival and backoff time distribution. The transition to the state in which both sources transmit has probability equal to zero because simultaneous arrivals/waking up instants have zero probability. If the transmitting source terminates the transmission while the other source remains idle, the chain moves back to the state in which both sources are idle. If the idle source wakes up from backoff or an arrival occurs, then it senses the channel, and if it does not detect the transmission from the other source, it starts transmitting. In this case, a collision occurs and the chain returns to the idle state. The average duration of the various transitions can be calculated based on the parameters of the exponential distributions modeling interarrival and backoff intervals. The state space can be extended to take into account different distributions of those intervals depending on the events that generated the idle state. The computation of the average reward and time associated with a cycle of the chain is entirely similar to that used for the renewal process addressed in this paper.

The coexistence of multiple secondary users in the network requires the design and analysis of specific coordination and channel assignment mechanisms, which may lead to even more significant modifications to the analysis. A detalied investigation of these more complex scenarios is left for future research.

VII. Conclusions

In this paper, we analyzed a cognitive network where the transmission of the primary and the secondary user is not constrained to a slotted structure. Under this assumption, the secondary user interferes with the primary user due not only to channel sensing errors, but also to the collision resulting from the start of the primary transmission during that of the secondary user. Prior work does not consider that primary users may implement a contention mechanism in order to coordinate among themselves. We present an analytical framework, based on renewal theory, which accounts for the channel sensing performed at the primary user. The secondary user thus interferes with the primary user by also distorting the statistics of its idle time due to backoff. Numerical results shed light on interesting behaviors and tradeoffs. For instance, the throughput of the secondary user is not a monotonically increasing function of its transmission time. In fact, long transmissions have a higher collision probability. As a consequence, the optimal transmission time of the secondary user may be shorter than the maximum transmission time imposed by the constraint on the throughput loss of the primary user.

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