Statistical Analysis of Non Orthogonal Spectrum Sharing and Scheduling Strategies in Next Generation Mobile Networks

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***Abstract*—Spectrum sharing has been recently proposed as a promising paradigm to improve the efficiency of resource usage in next generation mobile networks. In particular, non orthogonal spectrum sharing allows the operators to re-use the available frequencies at the cost of higher interference at the receivers. In this paper, we mathematically analyze the performance of this technique and how it is statistically related to the channel coefficients. Moreover, we compare different kinds of schedulers that exploit various aspects of non orthogonal spectrum sharing. Finally, the resulting system performance is assessed, first through an exact statistical framework and then by simulating the schedulers in an LTE scenario with the open-source network simulator ns3.**

***Index Terms*—Next Generation Mobile Networks; MIMO; 4G mobile communication; interference suppression; scheduling algorithm.**

I. Introduction and Related Works

THE INCREASING demand for transmission capacity in NGMN (Next Generation Mobile Networks) is pushing the operators to develop new cooperation paradigms to provide high Quality of Experience (QoE) to the end users. The NGMN Alliance [1] identified the increase of spectral efficiency and the reuse of existing infrastructure as key requirements for the wireless networks of the future. In this paper, we focus on the downlink side of the Long Term Evolution (LTE) technology of the Universal Mobile Telecommunication System (UMTS) [2], where user multiplexing is based on Orthogonal Frequency Division Multiple Access (OFDMA). Also, LTE can use Multiple Input Multiple Output (MIMO) radio operation by leveraging multi-antenna technologies. These innovations lead to change the current exclusive resource allocation paradigm in favor of cost, spectrum, and energy-efficient voluntary physical resource sharing with an innovative use of the radio spectrum and network infrastructure. This scenario has been investigated by the EU-funded project SAPHYRE [3] with the aim to quantify the gain obtained by sharing resources in an inter-operator scenario. The aim of the project is not only to quantify if and how much the efficiency of resource usage can be improved, but also to identify new business models that can facilitate competition among the mobile networks and enhance the overall societal benefit. With a higher degree of competition on both spectrum and infrastructure, less regulation is needed, benefiting end users and society in general. We shall note that the European Commission (EC) has formulated a list of objectives for National Regulatory Authorities (NRA) to be taken into consideration [4]. Within the regulatory framework of electronic communications networks and services, these directives cover aspects such as competition on the market, efficiency of the spectrum usage and management, protection of customer benefits, limitation of radio interference , promotion of infrastructure investments.

There are several ways in which portions of the spectrum available to one or more operators can be used concurrently by all of them. One possible solution is to open the spectrum usage of a particular channel to all operators, still constraining the actual allocation of the channel to only one user of a specific operator at a time. This kind of sharing, referred to as *orthogonal*, can increase the multi-user diversity of the system, thereby improving the resource utilization efficiency [5]. In [6], this very approach is used to minimize the cell blocking probability by using the shared frequencies to enlarge the available bandwidth. Orthogonal sharing is relatively simple to implement, but provides performance gains only in asymmetric scenarios, i.e., whenever the amounts of traffic in the operators buffers are unbalanced. In fully loaded scenarios, the gain is given only by the increased multi-user diversity, and is marginal if the number of users is large [7].

A further improvement can be obtained by exploiting non-orthogonal spectrum sharing (NOSS) that allows multiple operators to use the shared spectrum resources at the same time. This configuration allocates multiple users to the same frequency simultaneously, thereby causing a degradation of the Signal-to-Interference-plus-Noise-Ratio (SINR) at the intended receivers. The interference has to be controlled through the use of multiple antennas at the BS and proper mitigation techniques, such as beamforming [8].

Moreover, the sharing paradigm can be extended to the infrastructure through the use by the operators of the same communication point (IS-NOSS) with further improvement in terms of capital and operational expense (CAPEX, OPEX)

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costs [9]. In both cases the performance can be improved with the combined use of scheduling algorithms that consider the users related to the different operators as one pool, so as to exploit their channel characteristics. As illustrated in Fig. 1, colocation and non-colocation of the base stations (BSs) pertaining to the operators result in two different arrangements of the channel coefficients. These configurations can be exploited by the scheduler for user selection in each resource block.

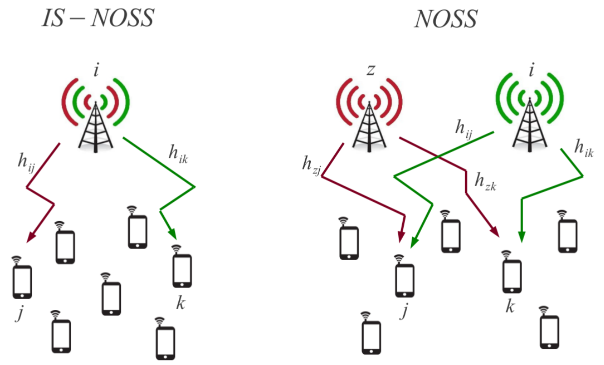

Fig. 1: IS-NOSS and NOSS scenarios

In this paper we make the following three contributions.

First, to characterize the degradation of the SINR in the NOSS scenarios we introduce a *novel performance indicator* called the Interference Suppression Ratio parameter (ISR) defined as the ratio between the SINR perceived in the NOSS case and the Signal to Noise Ratio (SNR) perceived when orthogonal scheduling is applied.

Moreover, we define *three different schedulers* that exploit in different ways the channel characteristics and the way the resources are shared, and we compare their performance through a statistical analysis of the ISR obtained.

Finally, we implement the same framework in the network simulator ns3 [10] to obtain an estimation of the spectral efficiency achievable by the proposed techniques in a more realistic LTE cellular network scenario and we compare the simulation results with the performance obtained by an optimal OSS scheduler and an optimal NOSS scheduler.

The paper is organized as follows. Section II gives some background on beamforming techniques and their implementation in NGMNs. Section III presents an analytical study of the NOSS systems and the related scheduler algorithms. In Section IV the performance of the algorithm is evaluated via simulation. Finally, conclusions are drawn in Section V.

Remark: Throughout the paper, we use boldface letters for vectors, we denote with $\|\cdot\|$ the norm of a vector and with $(\cdot)^H$ the conjugate transpose.

II. Background

We consider a Multi-Input Single-Output (MISO) system where the BSs use multiple antennas to transmit while the users are equipped with a single antenna to receive. Moreover, we assume a perfect knowledge at the BSs of the channel coefficients, represented as a column vector $\mathbf{h}_{ij}$, where indices i and j refer to the BS and the user, respectively. The use of multiple antennas in transmission permits to spatially steer the power to the receiver according to linear precoding vectors $\mathbf{w}_{ij}$ determined through the channel coefficients [11].

If a spectrum resource is used exclusively by one user, which is referred to as Orthogonal Spectrum Sharing (OSS), the linear precoding vector $\mathbf{w}_{ij}$ can be computed to maximize the useful signal level perceived by the user in the absence of interference through the Maximum-transmission Ratio Technique (MRT) [12]:

$$\mathbf{w}_{ij}^{(mrt)} = \frac{\mathbf{h}_{ij}}{\|\mathbf{h}_{ij}\|} \tag{1}$$

If a spectrum resource is instead shared non-orthogonally (either NOSS or IS-NOSS) between the operators, the MRT technique is inefficient due to the large amount of interference created by the BSs to the users. In this scenario, it is more convenient to exploit the cooperation between the two operators. The Zero Forcing (ZF) approach [13] permits to cancel the interference generated by the other link computing the unit-norm beamforming vector that is orthogonal to the channel of the second user, and which at the same time maximizes the product $\mid \mathbf{w}_{ij}^H \mathbf{h}_{ij} \mid$. Thus, we can determine the vector $\mathbf{w}_{ij}$ for ZF beamforming as

$$\mathbf{w}_{ij}^{(zf)}(k) = \frac{\mathbf{P}_{\mathbf{h}_{ik}} \mathbf{h}_{ij}}{\|\mathbf{P}_{\mathbf{h}_{ik}} \mathbf{h}_{ij}\|} \tag{2}$$

where $\mathbf{P}_{\mathbf{h}_{ik}}$ is defined as:

$$\mathbf{P}_{\mathbf{h}_{ik}} \triangleq \mathbf{I} - \mathbf{h}_{ik}(\mathbf{h}_{ik}^H \mathbf{h}_{ik})^{-1} \mathbf{h}_{ik}^H$$

and k is the index related to the other user that is sharing the same spectrum resource. For the sake of simplicity, in the following, we omit the dependence on k.

The SNR perceived by the user in the non spectrum sharing case ($SNR_j^{(nsh)}$) is then

$$SNR_j^{(nsh)} = \frac{p_{ij} |\mathbf{w}_{ij}^{(mrt)H} \mathbf{h}_{ij}|^2}{\sigma^2} \tag{3}$$

where p_{ij} is the power transmitted to user j and σ^2 is the noise power. In the NOSS case and in the IS-NOSS case the values of the SINR perceived by the user j ($SINR_j^{(noss)}$, $SINR_j^{(isnoss)}$) are, respectively,

$$SINR_j^{(noss)} = \frac{p_{ij} |\mathbf{w}_{ij}^{(zf)H} \mathbf{h}_{ij}|^2}{\sigma^2 + p_{zj} |\mathbf{w}_{zk}^{(zf)H} \mathbf{h}_{zj}|^2} \tag{4}$$

$$SINR_j^{(isnoss)} = \frac{p_{ij} |\mathbf{w}_{ij}^{(zf)H} \mathbf{h}_{ij}|^2}{\sigma^2 + p_{zj} |\mathbf{w}_{zk}^{(zf)H} \mathbf{h}_{ij}|^2} \tag{5}$$

where z is the index related to the other BS that is sharing the same spectrum resource. From (5) we can notice how the precoding matrix cancels the interference at the receiver but also reduces the power of the useful signal. This degradation is due to the non-perfect orthogonality among the channel matrices used for the construction of the beamforming matrices.

Since $p_{zj}|\mathbf{w}_{zk}^{(zf)H}\mathbf{h}_{zk}| \ll \sigma^2$, we can write

$$ISR_j = \frac{\frac{p_{ij}|\mathbf{w}_{ij}^{(zf)H}\mathbf{h}_{ij}|^2}{\sigma^2+p_{zj}|\mathbf{w}_{zk}^{(zf)H}\mathbf{h}_{zk}|^2}}{\frac{p_{ij}|\mathbf{w}_{ij}^{(mrt)H}\mathbf{h}_{ij}|^2}{\sigma^2}} \simeq \frac{|\mathbf{w}_{ij}^{(zf)H}\mathbf{h}_{ij}|^2}{|\mathbf{w}_{ij}^{(mrt)H}\mathbf{h}_{ij}|^2} = \frac{\left|\left(\frac{\mathbf{P}_{\mathbf{h}_{ik}}\mathbf{h}_{ij}}{\|\mathbf{P}_{\mathbf{h}_{ik}}\mathbf{h}_{ij}\|}\right)^H \mathbf{h}_{ij}\right|^2}{\left|\frac{\mathbf{h}_{ij}}{\|\mathbf{h}_{ij}\|}^H \mathbf{h}_{ij}\right|^2} = \frac{\left|\frac{\left(\left(\mathbf{I}-\mathbf{h}_{ik}\left(\mathbf{h}_{ik}^H\mathbf{h}_{ik}\right)^{-1}\mathbf{h}_{ik}^H\right)\mathbf{h}_{ij}\right)^H}{\|\left(\mathbf{I}-\mathbf{h}_{ik}\left(\mathbf{h}_{ik}^H\mathbf{h}_{ik}\right)^{-1}\mathbf{h}_{ik}^H\right)\mathbf{h}_{ij}\|}\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|^2} \quad (8)$$

Since $\mathbf{h}_{ik}^H\mathbf{h}_{ik} = \|\mathbf{h}_{ik}\|^2$ and applying the conjugate transpose operator we obtain that:

$$\begin{aligned}
ISR_j &= \frac{\left|\mathbf{h}_{ij}^H\left(\mathbf{I}-\frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H}{\|\mathbf{h}_{ik}\|^2}\right)\mathbf{h}_{ij}\right|^2}{\|\left(\mathbf{I}-\frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H}{\|\mathbf{h}_{ik}\|^2}\right)\mathbf{h}_{ij}\|^2} \cdot \frac{1}{\|\mathbf{h}_{ij}\|^2} \\
&= \frac{\left|\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - \mathbf{h}_{ij}^H\mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\|^2 - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} \cdot \frac{1}{\|\mathbf{h}_{ij}\|^2} = \frac{\left|-\mathbf{h}_{ij}^H\mathbf{h}_{ik}\right|^2}{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2} \cdot \frac{\left|\frac{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2}{-\mathbf{h}_{ij}^H\mathbf{h}_{ik}} + \mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}}{\|\mathbf{h}_{ik}\|}\|^2} \\
&= (1-\rho_{jk})^2 \cdot \frac{\left|\frac{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2}{-\mathbf{h}_{ij}^H\mathbf{h}_{ik}} + \mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}}{\|\mathbf{h}_{ik}\|}\|^2} = (1-\rho_{jk})^2 \cdot \left|\frac{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2}{\mathbf{h}_{ij}^H\mathbf{h}_{ik}} - \mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2 \cdot \frac{\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} \\
&= (1-\rho_{jk})^2 \cdot \frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - |\mathbf{h}_{ik}^H\mathbf{h}_{ij}|^2\right)}{|\mathbf{h}_{ij}^H\mathbf{h}_{ik}|^2} \cdot \frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - |\mathbf{h}_{ik}^H\mathbf{h}_{ij}|^2\right)'\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} \\
&= (1-\rho_{jk})^2 \cdot \left(\frac{1}{(1-\rho_{jk})^2} - 1\right) \cdot \frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - \left|\mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2\right)'\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} = 1-(1-\rho_{jk})^2
\end{aligned}$$

since:

$$\frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - \left|\mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2\right)'\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} = 1$$

III. Analytical Evaluation

To study the degradation of the SINR perceived by the user in the NOSS case in relation with the SNR obtained in the OSS scheme we define the parameter ISR of user j as:

$$ISR_j = \frac{SNR_j^{(sh)}}{SNR_j^{(nsh)}} \quad (6)$$

where $SNR_j^{(sh)}$ can be considered as $SNR_j^{(noss)}$, see (4), or $SNR_j^{(isnoss)}$, see (5).

We introduce another parameter, the Degree of Orthogonality (ρ) that describes the compatibility of the users which can be selected to share the same spectrum resource assigned by BS i related to their channel coefficients:

$$\rho_{jk} = 1 - \frac{|\mathbf{h}_{ik}^H\mathbf{h}_{ij}|}{\|\mathbf{h}_{ik}\|\|\mathbf{h}_{ij}\|} \quad (7)$$

where j and k are the indices related to the users.

Note that a small ρ_{jk} represents an inefficient coupling among the users while as $\rho_{jk} \to 1$ the losses due to the simultaneous usage of the frequency resource are reduced.

As demonstrated in (8), it is possible to express (6) as a function of the coefficients ρ_{jk}, which gives

$$ISR_j = 1-(1-\rho_{jk})^2 \quad (9)$$

Through (9) it is possible to obtain the statistical behavior of ISR from the probability distribution of ρ, that in turn depends on the choice of the scheduler. Therefore, we need to consider which scheduling policies can be adopted to select the users that share the allocation.

We call Max SNR scheduler (M-SNR) the scheduling policy where the allocation is based on the SNR perceived by the users in the case of no-sharing without considering the ρ parameter. In particular, for every spectrum resource the operators select from the overall pool of users those with the best SNR, exploiting the multi-user diversity derived from a larger number of users.

Assume a unit-variance Rayleigh fading, i.e., $\mathbf{h}_{ij}\sim CN(\mathbf{0},\mathbf{I})$, where $\mathbf{0}$ is the all-zero vector and $\mathbf{I}$ is the identity matrix; thus, the cumulative distribution function (cdf) of ρ is given by the regularized incomplete beta function $I_x(\alpha,\beta) = B_x(\alpha,\beta)/B(\alpha,\beta)$, where $B_x(\alpha,\beta)$ is the incomplete beta function and $B(\alpha,\beta)$ is the (complete) beta function [14]. The shape parameters α and β are obtained by simulation; we found that $\alpha = 1, \beta = 2$ are suitable values. This analysis can be extended to the case of a NOSS or IS-NOSS network for different scheduling policies.

We consider two other different schedulers in addition to the M-SNR scheduler: a max-ISR scheduler (M-ISR) and a priority scheduler (PS). The former uses, as the M-SNR scheduler, a single metric as the criterion for a greedy selection, and considers the users of both operators as belonging to a common pool; that is, it selects two users but not necessarily one for either operator. However, differently from M-SNR that aims at maximizing the SNR, M-ISR uses the ISR parameter instead. Thus, for every spectrum resource, the two users that mutually achieve the highest ISR are chosen without considering the SNRs.

In the PS case the two operators allocate their users separately, but their allocations are prioritized. More specifically, this policy accounts for the fact that either of them is the original licensed "owner" (O) of the resource, i.e., the operator that would exploit the spectrum resource in case of no sharing. The other operator (P) is just exploiting the same channel, but with lower priority. Basically, we aim at establishing a prioritization akin to the typical primary-secondary relationship of cognitive networks [5]. The rationale behind this motivation is that the licensed owner O should be able to assign the resource first, and the secondary operator P should act by avoiding disturbance to the primary. Thus, O assigns its user with the best no-sharing SNR, then P chooses the user achieving the best ρ, with the aim to preserve the utility of the selection performed by O. In a game theoretic context, this framework would be akin to that of a Stackelberg game [17].

In the IS-NOSS case, since the operators are sharing the same infrastructure, the channel coefficients and the ρ perceived by the operators are identical. Thus, the probability density function (pdf) for the values of ρ achieved by the different schedulers is given, as explained in the Appendix, by the pdf of the maximum of n standard beta variables. This leads to

$$f_{isnoss}(x) = n\left(\frac{B_x(\alpha,\beta)}{B(\alpha,\beta)}\right)^{n-1}\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \qquad (10)$$

where n is equal to the number of possible pairs in the network, i.e.:

- $\binom{N_i+N_z}{2}$ for M-ISR;
- N_i if the owner operator is i or N_z if the owner operator is z for PS.

where N_i and N_z are the number of users for the operators i and z.

In the NOSS case the BSs are not colocated and the ρ perceived by the operators are different. Then the objective of the scheduler is not to maximize a single value of ρ but rather the sum of the values perceived by the base stations when a given pair is selected. According to the results presented in [15], the pdf of the sum of the ρ values achieved for the two schedulers in the NOSS case is (see Appendix II)

$$f_{noss}(x) = n\left(\frac{B_x(\alpha,\beta,a,b)}{B(\alpha,\beta)}\right)^{n-1}\frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}} \qquad (11)$$

where n is equal to the number of possible pairs available in the network as per (10) and $\frac{B_x(\alpha,\beta,a,b)}{B(\alpha,\beta)}$ is a general beta variable with parameters $\alpha = \frac{7}{3}, \beta = \frac{14}{3}, a = 0, b = 2$. Also these numerical values are computed as per [15].

Figs. 2 and 3 depict the cdf of the ISR based on the ρ statistics described above when 5 users per operator are active in the network. In the NOSS case, the value of the sum of ρ that is computed statistically is divided using a uniform distribution among the users. As expected, since the ISR perceived by the selected users is the same, the IS-NOSS configuration obtains better results than the NOSS configuration. The markers are related to the simulation results and will be explained in Section IV.

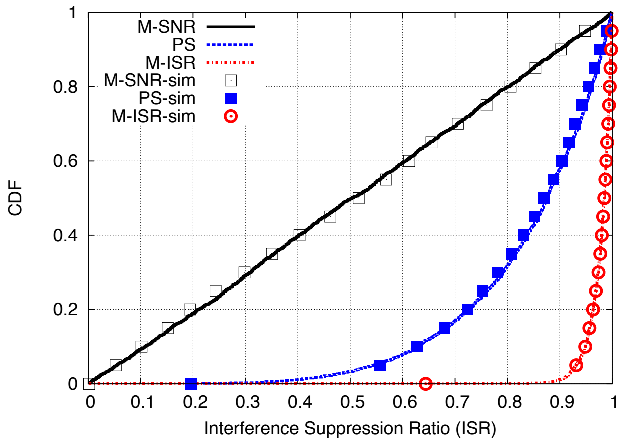

Fig. 2: ISR cdf and validation curves for the IS-NOSS case

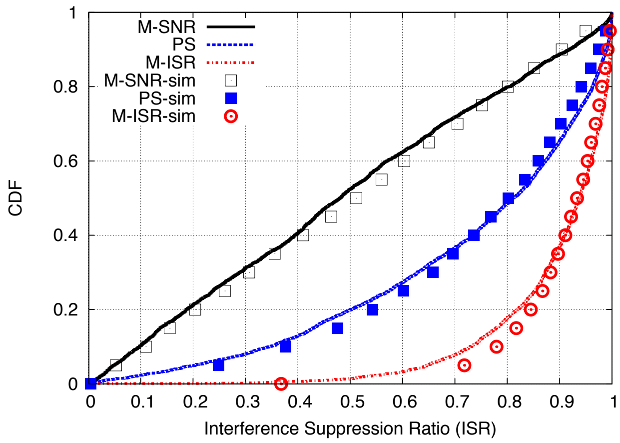

Fig. 3: ISR cdf and validation curves for the NOSS case

IV. Simulation Results

In the first part of this section we validate the analytical results obtained in Section III by simulating the proposed scheduling algorithms. In particular, we simulate a 2×1 MISO system with 5 users per operator and a unit-variance Rayleigh fading, i.e., $\mathbf{h}_{ij} \sim CN(\mathbf{0}, \mathbf{I})$. Thus, we compute the ρ for each possible pair of users and select one pair according to the different scheduling policies; finally, the ISR of the scheduled users is computed. Figs. 2 and 3 compare the cdfs obtained by simulation and by statistical analysis. In the IS-NOSS case the statistical curve shows a very good fit with the simulation results, while in the NOSS case the fit is slightly degraded (but still acceptable) due to the assumption of uniform distribution of the ρ among the users.

To evaluate the performance of the proposed scheduling algorithms in an LTE system, we used the network simulator 3 (ns3) [10]. This is an open-source simulation environment for networking research based on object oriented programming and spanning the entire protocol stack, from the physical layer up to the application. Its modular structure allows to develop new algorithms within the protocol stack of different communication standards. We extended an existing implementation for the evaluation of the OSS system performance [16] developed within the LTE module of ns-3. Starting from this framework, we developed the NOSS system and the proposed scheduling algorithms.

Within the simulator, we implemented the ISR statistical framework by generating users in the LTE cell and randomly assigning to each pair of them a degree of orthogonality ρ that is randomly generated according to the distributions given in (10) and (11). Through this approach, it is possible to evaluate the impact of the ISR parameter together with the SNR level perceived by the users on the downlink spectral efficiency. We compare the results also with: (i) an optimal OSS scheduler that for every RB chooses in the overall pool the user with the best SNR and (ii) the optimal NOSS scheduler that selects the pair of users that achieve the best spectral efficiency for every RB.

The scenario consists of two BSs, which may either be colocated (IS-NOSS scenario) or non-colocated (NOSS scenario), and are equipped with two antennas, plus a variable number of mobile users equipped with one antenna and randomly positioned (with uniform distribution) in a cell with a radius of 1.5 km. The downlink bandwidth available per operator of 5 MHz is divided into 25 RBs. Moreover, we assume a fully loaded scenario, i.e., the downlink traffic saturates each BS buffer, so all the RBs are used during each frame. Each operator has a total downlink power of 43 dBm that is equally divided among the used RB. The detailed system parameters are reported in Table I.

Parameter	**Value**
1-st sub-channel frequency	2110 MHz
Downlink Bandwidth per operator	5 MHz
Sub-Carrier Bandwidth	15 kHz
Doppler Frequency	60 Hz
Resource block bandwidth	180 kHz
Resource block carriers	12
Resource block OFDM symbols	7
BS downlink TX power	43 dBm
Noise spectral density	-174 dBm/Hz
Macrosopic Pathloss (distance R)	$128.1 + (37.6 \cdot \log(R))dB$
Shadow fading	log-normal, ϑ = 8 dB
Wall penetration loss	10 dB
Frame duration	10 ms
TTI (sub-frame duration)	1 ms
Target Bit Error Rate	5×10^{-5}
Cell coverage	1.5 km
BS distance (NOSS case)	50 m
Number of UEs per BS	1, 2, 5, 10, 20, 50

TABLE I: Main system parameters

Figs. 4 and 5 show the results obtained in the IS-NOSS case and in the NOSS case. We notice that in the user selection it is important to have both high SNR and high ISR for spectrum sharing to be efficient. In particular, using the M-ISR scheduler the performance is degraded due to the high probability of selecting two users with low SNR, while in the M-SNR case the higher probability to select users with low orthogonality causes a performance loss. By comparing these schedulers with the OSS scheduler, we notice that selecting the users without considering the orthogonality among their channel coefficients is inefficient. Only in the case of a high number of users (50 per operator) is the optimal OSS scheduler outperformed by the M-SNR scheduler, due to the more degrees of freedom in choosing the users. We emphasize also that, since the scenario is fully loaded, the power in the NOSS case is divided among all the RBs, while in the OSS case it is divided only among the RBs used by the operators. So, for a low number of users the OSS configuration is more efficient than the NOSS configurations, and this behavior is accentuated when the BSs are not colocated due to a worse coupling among the users scheduled in the same spectrum resource. Besides, this observation is confirmed by the results obtained when one user is available per operator, in particular, the users selection for the NOSS scheduler is mandatory while in the OSS case it is possible to exploit the multi-user diversity and a more efficient power allocation.

Differently, the PS scheduler results in a good trade-off be-

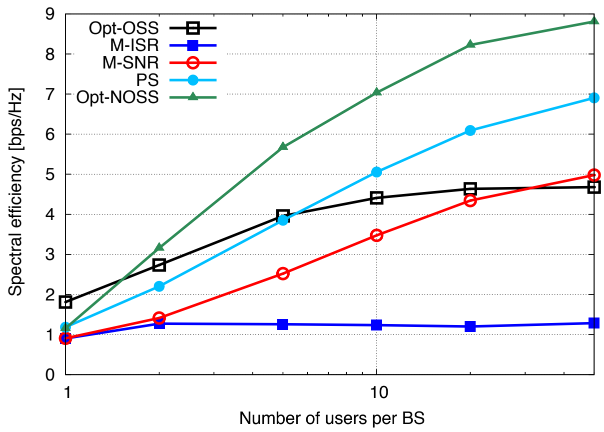

Fig. 4: Scheduler comparison for the IS-NOSS case

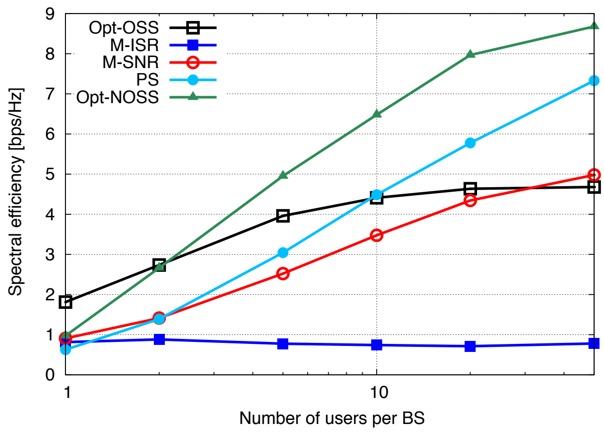

Fig. 5: Scheduler comparison for the NOSS case

tween the two metrics considered. Moreover, the performance obtained by the optimal NOSS algorithm is better than what obtained with the PS algorithm, but at the cost of a higher computational complexity. In particular, the complexity of the three allocation algorithms proposed in the choice of the two users for every resource is $O(N)$ while for the optimal NOSS algorithm it is $O\left(N^2\right)$, where N is the number of users in the cell. If we compare the IS-NOSS and the NOSS cases, we notice that the former achieves slightly better results; thus, sharing the infrastructure can give the operators a further improvement in terms of spectral efficiency for the scheduling algorithms that we investigated.

V. Conclusion and Future Work

We investigated NOSS techniques through a statistical analysis of the ISR and a simulation analysis of the spectral efficiency obtained with the use of several scheduling techniques in an LTE network. Non-orthogonal sharing appears to be a promising technique for the performance improvement in NGMN, and a joint user scheduling among the operators can give further improvements in terms of spectral efficiency.

From the results obtained, it also appears that an additional sharing concerning the infrastructure can, depending on the specific scheduling algorithm, further improve the overall performance of the network. As a possible extension of the present work, the same approach can be applied to other beamforming techniques, and also extended to scenarios with multiple cells.

Appendix

Consider a set of n independent and identically distributed random variables. The cdf of the maximum value of those variables is given by:

$$P\{\max(y_1, y_2, ..., y_n) \leq \alpha\} = [P\{y \leq \alpha\}]^n$$

In our case, the cumulative distribution is described by a regularized incomplete beta function $I_x(a,b) = B_x(a,b)/B(a,b)$, so the related pdf can be obtained as

$$\begin{aligned} \frac{\mathrm{d}\,[I(x))]^n}{\mathrm{d}x} &= \frac{\mathrm{d}\,[B_x(a,b)/B(a,b)]^n}{\mathrm{d}x} \\ &= n\left(\frac{B_x(a,b)}{B(a,b)}\right)^{n-1} \frac{\mathrm{d}B_x(a,b)/B(a,b)}{\mathrm{d}x} \\ &= n\left(\frac{B_x(a,b)}{B(a,b)}\right)^{n-1} \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)} \end{aligned}$$

A similar approach can be used for the case of a cdf that is a general beta variable, i.e.,

$$\frac{B_x(\alpha, \beta, a, b)}{B(\alpha, \beta)}.$$

By repeating the same procedure applied above, (11) can be derived.

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