Promoting Cooperation in Wireless Relay Networks through Stackelberg Dynamic Scheduling

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***Abstract*—This paper discusses a new perspective for the application of game theory to wireless relay networks, namely, how to employ it not only as an analytical evaluation instrument, but also in constructively deriving practical network management policies. We focus on the problem of medium sharing in wireless networks, which is often seen as a case where game theory just proves the inefficiency of distributed access, without proposing any remedy. Instead, we show how, by properly modeling the agents involved in such a scenario, and enabling simple but effective incentives towards cooperation for the users, we obtain a resource allocation scheme which is meaningful from both perspectives of game theory and network engineering. Such a result is achieved by introducing throughput redistribution as a way to transfer utilities, which enables cooperation among the users. Finally, a Stackelberg formulation is proposed, involving the network access point as a further player. Our approach is also able to take into account power consumption of the terminals, still without treating it as an insurmountable hurdle to cooperation, and at the same time to drive the network allocation towards an efficient cooperation level.**

***Index Terms*—Cooperative Relay, Dynamic Scheduling, Game Theory, Stackelberg**

I. Introduction

Coordination of wireless access is key to efficiently exploit the available channel resources. Many formalizations of this problem involve the use of game theory [1]. The earlier applications of game theory to wireless networks were limited to the formulation of wireless network problems as static games of complete information, that usually lead to the *tragedy of the commons* [2]. This problem models the inefficiencies occurring when independent agents share a common resource in a selfish manner. As an example, [3] shows that the IEEE 802.11 distributed MAC protocol can lead to an inefficient Nash Equilibrium (NE) and [4] shows that a situation similar to the traditional Prisoner's Dilemma arises in slotted Aloha MAC protocols. A closely related approach is taken in [5] that considers the routing problem for a multi-hop wireless network.

To improve the network performance, incentives for the users to cooperate may be provided. However, this requires to change the formulation of the game. In [6]–[9] cooperation is achieved introducing hierarchy in games, allowing some users to move before others. In [10]–[12] this is further advanced by considering repeated games, where cooperation is enforced by punishing deviating users in subsequent stages. In [13], cooperation stems from an agreement between players and the presence of an entity enforcing such an agreement is assumed. Cooperative games can be extended to *coalitional games* [14], where users are allowed to form coalitions; a coalition plays as a single entity against other (competing) coalitions, and the coalition members share the coalition benefit. The strategy of a player consists in its decision regarding joining a coalition.

Cooperative relaying is another important technique in a wireless network to improve connectivity and throughput in the network. The performance of relay channels has been widely studied in the literature [15]–[17]. However, relaying is possible in practice only if incentives are given to the individual users to overcome the disadvantages of their limited energy budgets. In this spirit, [18] promotes a fair packet forwarding mechanism balancing the relaying opportunities that each node gives to and receives from other nodes. Similarly, [19] introduces a virtual currency and mechanism for charging/rewarding service usage/provision. Both papers assume the application of a tamper-resistant module in each node to store the forwarding balance or the virtual currency credit. The virtual currency concept is also used in [20], while in [21] cooperation is reached by using a reputation mechanism. A distributed and scalable acceptance algorithm was proposed in [22], in order for the nodes of an ad hoc network to decide whether to accept or reject a relaying request. Finally, [23] considers an incentive mechanism where the nodes flexibly give transmission bandwidth in exchange for forwarding data.

In this paper we investigate cooperative relaying, not only improving the social welfare of the network, but also increasing the individual benefit of each single user, that is assumed to act selfishly and strategically. We first prove the potential gain of cooperation through a cross-layer scheme involving joint routing and medium access, which is analyzed by means of renewal process theory [24]. However, such a globally efficient allocation may not match the allocation equilibrium in a game theoretic sense. Thus, as a main contribution of this work, we propose an opportunistic relaying scheme involving a coordinator, that triggers cooperative behaviors increasing the access opportunities of users acting as relays. This kind of approach is framed as a Stackelberg game involving the coordinator as the leader and the users, whose strategic decision involves whether to act collaboratively, as followers.

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Differently from [18]–[21], that are based on the exchange on a network scale of abstract notions of worth (e.g., currency and reputation), our opportunistic relaying scheme represents a more tangible and immediate incentive mechanism. The repeated game formulation considered in [22] is efficient only if a user asking for a relay service can return the favor in future interactions. Our scheme can be applied in more general situations, even in strongly asymmetric scenarios where some users only ask for relay services and other users are only asked to act as relays. In fact, users acting as relays are immediately rewarded, independently of the future interactions with the other users. Our approach is closer in spirit to [23]. The main difference is that, instead of rewarding cooperative users in the frequency domain, giving them more bandwidth, we reward cooperative users in the time domain, increasing their access opportunities. Moreover, there are some different hypotheses that make the analysis of the two schemes very different, e.g., in this paper we assume that the users can select their modulation scheme which in turn determines the packet reception probability, while [23] adopts a more abstract formulation based on channel capacity.

The rest of this paper is organized as follows. We describe the scenario under investigation and the key assumptions in Section II. Then, Section III formalizes the analysis of cooperative versus non cooperative schemes by means of renewal process theory. Section IV utilizes game theory to provide network incentives towards cooperation and defines our proposed Stackelberg approach. Numerical results are provided in Section V. We discuss possible relaxations of some hypotheses in Section VI, and Section VII concludes the paper with some remarks.

II. Problem Statement

Consider a scenario as reported in Fig. 1, where a set $\mathcal{U} = \{1, 2, \dots, U\}$ of U nodes, hereafter called *users*, are distributed around a further node called *node 0*. This may represent an access point of a wireless local area network, or a base station of a cellular network. We focus on the uplink between each user and node 0; yet, we assume that node 0 is not only the end destination, but also a resource manager, as explained later.

We denote the signal to noise ratio (SNR) between user i and node 0 as γ_i and the SNR between users i and j as γ_{ij}. Users are labeled in decreasing order of SNR to node 0, i.e., $\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_U$. We consider time invariant channels and fixed transmission powers P_{pkt}, so that the γ_i and γ_{ij} terms are constant over time. We also assume perfect channel state knowledge.

A Time Division Multiple Access (TDMA) scheme is adopted, with a fixed slot duration T_{pkt}. Node 0 controls the time shares of the users by selecting, in each slot n, a specific user that is allowed to transmit. The probability that user i is selected in slot n is $P_i^{(n)}$. The selected user transmits a single packet over the entire slot, comprising a number of bits that depends on its modulation scheme M_i. M_i is chosen over a finite set $\mathcal{M}$ according to the channel quality and in turn determines the probabilities q_i and q_{ij} that the packet is

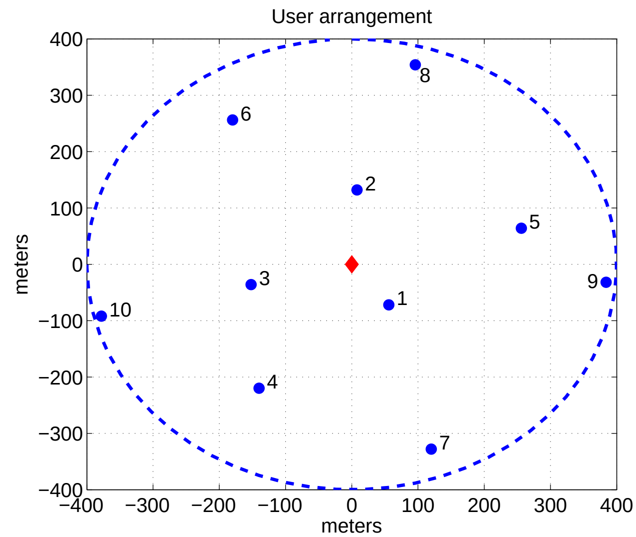

Fig. 1. The considered scenario: an access point surrounded by user nodes

correctly received by 0 and j. We denote with $E_{pkt} = P_{pkt} T_{pkt}$ the energy consumed by a user for a single packet transmission.

Automatic Repeat reQuest (ARQ) is used as the mechanism to achieve reliable communication [25]. If the packet transmitted by user i is not correctly received by node 0, the packet is retransmitted the next time user i is scheduled, until the packet is received or the maximum number of retransmissions is reached. For the sake of simplicity, in this paper we consider at most one retransmission per packet, although the extension to multiple retransmissions would be conceptually straightforward. Users are assumed to be backlogged, i.e., they always have packets to transmit. In the following, we will start by considering that retransmissions of a packet are only performed by the node that has originated that packet, i.e., the node that performed the first transmission attempt. We will refer to this situation as the *no cooperation* case and denote its corresponding quantities with a superscript N. $P_i^{(n)}$ can be set as a constant/static value for all n, which makes the selection process independent and identically distributed (iid). The scheduling policy can be described by a vector $\mathbf{P} = (P_1, P_2, \dots, P_U)$, where $\sum_{i=1}^{U} P_i = 1$, so that $P_i^{(n)} = P_i$ for all n; for example, a fair sharing is represented by $\mathbf{P} = (1/U, 1/U, \dots, 1/U)$.

We will also consider two evolutions of this scheme, where retransmissions of faulty packets may not be carried out by the same node performing the first attempt. This is enabled by assuming that during the transmission phase of a generic node i the other nodes listen to the channel and store i's packet if they have correctly received it. Thus, they can retransmit it if needed. If more than one user can retransmit the packet, node 0 selects the one with the best channel.

In the first scheme, called *forced cooperation* (denoted by superscript F), we assume that the users have no say in deciding whether or not to cooperate, but must follow node 0's directions when instructed to do so, hence the name. Since

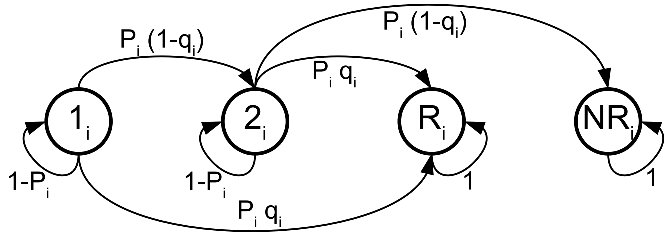

Fig. 2. Non-cooperative transmission process of a packet of user i

cooperation does not come from a free decision, there is no need for rewarding the collaborative users with a higher access probability. Thus, similarly to the no cooperation case, the access probabilities $P_i^{(n)}$ stay the same for every n. However, their physical meaning changes: they represent the event that the packet originated from i is transmitted during slot n; if it is the first transmission attempt, it will be performed by i, while this is not necessarily true for a retransmission.

Finally, we will consider a further cooperative case, called *voluntary cooperation* (denoted by superscript V), where the users freely decide whether or not they want to cooperate in the retransmission process of other users. In this case, node 0 rewards them with a higher access probability, decreasing by the same amount the access probability of the users being helped. Thus, $P_i^{(n)}$ changes over time. Suppose node i cooperates with node j in slot n, retransmitting a packet originated from node j. We define K_{ij} as the number of scheduling instants, after slot n, where the scheduling policy is changed, and $\Delta P_{ij}^{(s)} > 0$ as the variation of the scheduling policy, with respect to the reference policy $\mathbf{P} = (P_1, P_2, \ldots, P_U)$, in slot s, i.e.,

$$P_j^{(s)} = P_j - \Delta P_{ij}^{(s)} \quad ; \quad P_i^{(s)} = P_i + \Delta P_{ij}^{(s)} \tag{1}$$

$s = n+1, \ldots, n+K_{ij}$. To compare the three cases, we define the bit rate of user i in slot n as

$$BR_i^{(n)} = \begin{cases} \dfrac{N_i}{T_{pkt}} & i\text{'s packet correctly received by } 0 \text{ in slot } n \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where N_i is the number of bits in user i's packet, which depends on the chosen modulation scheme M_i. Finally, we define the asymptotic bit rate of user i as

$$BR_i = \lim_{T\to\infty} \frac{1}{T} \sum_{n=0}^{T-1} BR_i^{(n)} \tag{3}$$

III. Renewal Theory Analysis

In the no cooperation scheme, the transmission process of a generic packet originated from user i can be represented by the Markov Chain of Fig. 2. The successful reception probabilities q_i and q_{ij} depend on the modulation scheme M_i and the SNR values γ_i and γ_{ij}. In the following we will omit all these dependencies in favor of a clearer notation.

The initial state of the Markov Chain is 1_i, which means that the next time user i is scheduled it will transmit the packet for the first time. Analogously, state 2_i implies that by scheduling user i the packet will be transmitted for the second time. State 2_i is entered if the first attempt failed. The term P_i that influences the transition probabilities results from the scheduling process. The absorbing states R_i and NR_i represent the events that user i's packet is eventually received or not, respectively, by node 0. When either of the absorbing states is entered, the transmission process of another packet of node i is considered, restarting again from state 1_i.

The time intervals of the packet transmission processes are positive, independent, identically distributed random variables. These variables define a renewal process which can be studied exploiting renewal theory results [24]. The asymptotic metrics of the network can be obtained studying the (statistical) average behavior of the Markov process. In particular, the asymptotic throughput of each user is equal to the average number of received bits divided by the average time to be absorbed in the Markov chain associated to that user.

We denote with $P_{R_i}^N$ the probability to be absorbed in state R_i and with v_i^N the average number of time slots to be absorbed starting from state 1_i. Therefore,

$$\begin{aligned} P_{R_i}^N &= q_i + (1-q_i)\, q_i = q_i\,(2-q_i) \\ v_i^N &= \frac{1}{P_i} + \frac{1}{P_i}\,(1-q_i) = \frac{2-q_i}{P_i} \end{aligned} \tag{4}$$

Thus, i's asymptotic bit rate for the no cooperation case is

$$BR_i^N = \frac{P_{R_i}^N\, N_i}{v_i^N\, T_{pkt}} = P_i q_i \frac{N_i}{T_{pkt}} \tag{5}$$

The best modulation scheme for user i is simply obtained maximizing its throughput

$$M_i^N = \arg\max_{M_i \in \mathcal{M}} q_i N_i \tag{6}$$

Recall that both N_i and q_i depend on M_i. Finally, the asymptotic bit rate of the network for the no cooperation scenario is

$$BR^N = \sum_{i=1}^{U} BR_i^N = \frac{1}{T_{pkt}} \sum_{i=1}^{U} P_i q_i N_i \tag{7}$$

where the modulation scheme for each user is selected according to (6).

In the forced cooperation scheme, the packet transmission process of user i follows the Markov Chain in Fig. 3. Differently from the no cooperation case, the retransmission of i's packet is performed by the best user k among those that have received the packet during i's first attempt, $k < i$, otherwise the retransmission is performed by i itself. In the retransmission, k will use the same modulation order used by i, M_i. In fact, although the optimal modulation M_k for k may be higher, i's packet dimension cannot be increased. [1] We define q_k^i as the correct reception probability of a packet transmitted by k using the same modulation scheme of i. Since $k < i$, we have $q_k^i \geq q_i$.

[1] Actually, node k can even improve its amount of transmitted data by stuffing i's packet with its own data up to $N_k - N_i$ bits. In the present paper, we neglect this further advantage which, however, would be immediate to include.

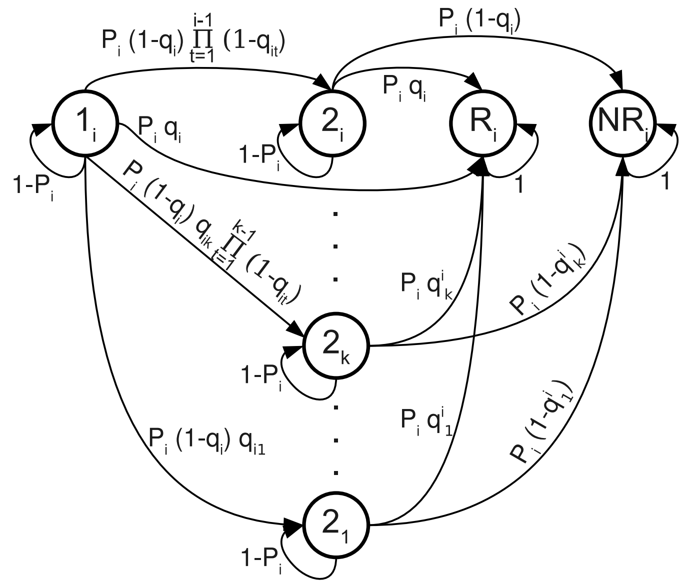

Fig. 3. Transmission process of a packet of user i in the forced cooperation scheme

The probability $P^F_{R_i}$ to be absorbed in R_i and the mean number of steps v^F_i to absorption are

$$\begin{aligned} P^F_{R_i} &= q_i + (1-q_i)\sum_{k=1}^{i} q_{ik}\prod_{t=1}^{k-1}(1-q_{it})\,q^i_k = \\ &= q_i\,(2-q_i) + \sum_{k=1}^{i-1}(1-q_i)\,q_{ik}\prod_{t=1}^{k-1}(1-q_{it})\,(q^i_k - q_i) \\ v^F_i &= \frac{2-q_i}{P_i} \end{aligned} \tag{8}$$

where we took $\prod_{t=1}^{0}(1-q_{it}) = 1$ and $q_{ii} = 1$. In particular, $(1-q_i)\,q_{ik}\prod_{t=1}^{k-1}(1-q_{it})\,(q^i_k - q_i)$ is the probability that 0 has not correctly received i's packet in the first attempt while k has received it but no user better than k has received it, multiplied by the difference between the probabilities that the packet is correctly retransmitted by k and i. It represents the contribution of k to the probability that i's packet is eventually received by 0.

Considering for the moment that user i is adopting the same modulation scheme M^N_i as in the no cooperation case, we have obtained $P^F_{R_i} \geq P^N_{R_i}$ and $v^F_i = v^N_i$. The latter is a consequence of considering a single retransmission (for the multiple retransmission case, $v^F_i \leq v^N_i$ in general).

Similar to (5), the asymptotic bit rate of user i in the cooperative scenario is

$$BR^F_i = P_i\left[q_i + \frac{1-q_i}{2-q_i}\sum_{k=1}^{i-1} q_{ik}\prod_{t=1}^{k-1}(1-q_{it})\,(q^i_k - q_i)\right]\frac{N_i}{T_{pkt}} \tag{9}$$

and the best modulation scheme M^F_i for the cooperative case is

$$M^F_i = \underset{M_i\in\mathcal{M}}{\arg\max}\left[q_i + \frac{1-q_i}{2-q_i}\sum_{k=1}^{i-1} q_{ik}\prod_{t=1}^{k-1}(1-q_{it})\,(q^i_k - q_i)\right]N_i \tag{10}$$

Finally, for the aggregate throughput we obtain

$$BR^F = \frac{1}{T_{pkt}}\sum_{i=1}^{U} P_i\left[q_i + \frac{1-q_i}{2-q_i}\sum_{k=1}^{i-1} q_{ik}\prod_{t=1}^{k-1}(1-q_{it})\,(q^i_k - q_i)\right]N_i \tag{11}$$

where the modulation scheme for each user is selected according to (10). Comparing this result with the no cooperation case, if in both cases users are adopting the modulation schemes according to (6), we obtain $BR^F \geq BR^N$. This relation is further enforced if we calculate BR^F considering the best modulation schemes for the forced cooperation case, according to (10).

IV. Game Theoretic Model

To study the *voluntary cooperation* scheme, we need to introduce a game theoretic framework modeling interactions among users and their decision to cooperate / not to cooperate. We assume that users are selfish, i.e., they make decisions to maximize their individual advantage.

In the voluntary cooperation scheme, each user is free to choose whom to cooperate with, as well as its own modulation scheme. For the time being, we consider that the strategy of user i consists only in choosing the set of users it cooperates with, which we denote as $\mathcal{C}_i \subseteq \mathcal{U}$. Since each user can cooperate only with users having a worse channel, $\mathcal{C}_i$ is actually a subset of $\{i+1, \ldots, U\}$. The choice of the modulation scheme can be added in a later step as a superposition to the choice of $\mathcal{C}_i$, and anyway it does not represent a strong interaction factor among users. Also, denote with $\mathcal{A}_i$ the set of users that cooperate with i, i.e., $\mathcal{A}_i = \{j \in \mathcal{U} : i \in \mathcal{C}_j\}$.

We represent the preference of each user i through a utility function $\Psi_i(B_i, E_i)$ which depends on the number of transmitted bits B_i and on the energy spent E_i per unit time. Actually, for the analysis of the game we use the *incremental utility* $\psi_i(\Delta B_i, \Delta E_i)$ representing the increase in Ψ_i with respect to the no cooperation case[2], i.e.,

$$\psi_i(\Delta B_i, \Delta E_i) = \Psi_i(B^N_i + \Delta B_i, E^N_i + \Delta E_i) - \Psi_i(B^N_i, E^N_i) \tag{12}$$

By definition, $\psi_i(0,0) = 0$. Also, it is reasonable to assume that ψ_i is a continuous and increasing (respectively, decreasing) function of the variation of transmitted bits (respectively, energy consumption) per unit time, ΔB_i (respectively, ΔE_i).

Note that ΔB_i and ΔE_i can be split into the contributions due to the individual interactions with other users: $\Delta B_i = \sum_{j\in\mathcal{U}\setminus\{i\}}\Delta B_{ij}$ and $\Delta E_i = \sum_{j\in\mathcal{U}\setminus\{i\}}\Delta E_{ij}$, where ΔB_{ij} and

[2] The game's outcomes are invariant to this choice. In fact, they depend only on the ranking of the preference of each user, which is preserved if a (user-dependent) constant is subtracted from the utility of each user.

ΔE_{ij} are the variations, per unit time, of transmitted bits and energy expenditure of i due to the interaction with j. Now, we assume that the incremental utility $\psi_i(\Delta B_i, \Delta E_i)$ can be additively split as a sum of local contributions $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij})$, each due to the interaction between i and j, with ψ_{ij} having the same characteristics of ψ_i (continuity and monotonicity). Then we can write:

$$\psi_i(\Delta B_i, \Delta E_i) = \sum_{j \in \mathcal{U}\setminus\{i\}} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) = \\ = \sum_{j \in \mathcal{A}_i} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) + \sum_{j \in \mathcal{C}_i} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) \quad (13)$$

where we exploited the fact that if $j \notin \mathcal{A}_i \cup \mathcal{C}_i$, i.e., j has no interaction with i, then $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) = 0$.[3]

In (13), ψ_i is re-arranged in two sum terms. The former involves the users of set $\mathcal{A}_i$ offering their cooperation to i; therefore, in the corresponding terms, ΔB_{ij} and ΔE_{ij} are positive (as we will see in Section IV-A) and negative, respectively. This means that user i will always benefit from cooperation by another user j with a better channel; however, the strategic choice whether to cooperate or not is left to user j. The latter term includes instead the variation of ψ_i due to i offering cooperation to other nodes belonging to set $\mathcal{C}_i$, which is where the decision of i comes into play.

The term ψ_{ij} can therefore be regarded as the specific utility of user i in a simple 2-player game between i and j, $i < j$, where the only user who can make a non-trivial decision is i. It will cooperate with j if and only if $\psi_{ij} \geq 0$ (it is not restrictive to assume cooperation in the equality case). Note that i's strategy has no influence on the utilities of lower index users and, therefore, on their decision process. Hence, i's decision to cooperate or not with j, with $i < j$, can be made by maximizing just the partial utility ψ_{ij}. In this way, the original U-player game is decoupled into $\binom{U}{2}$ 2-player games whose outcomes can be easily predicted.

In particular, without any incentive mechanism, the option of relaying packets for another node would never be advantageous. In fact, in this case $\Delta B_{ij} = 0$ and $\Delta E_{ij} > 0$, hence, ψ_{ij} is negative. Thus, no node would ever relay a packet. This is why we also include node 0 that can provide incentives for cooperation, through a reshaping of the transmission probabilities. In this way, users can now get a positive utility when they act as relays, since they may have higher energy consumption but also higher throughput.

A. Stackelberg Formulation

In light of the above discussion, we consider node 0 as an active player in the game, which, to promote cooperation in the network, can change the scheduling policies of users, with respect to the reference scheduling policy $\mathbf{P} = (P_1, P_2, \ldots, P_U)$, according to (1). We want that, after this intervention by node 0, the users exploiting a collaborative relay still have a throughput improvement, i.e., if $j \in \mathcal{C}_i$ then $\Delta B_{ji} \geq 0$; note that they always have an energy saving, i.e., $\Delta E_{ji} < 0$, since i performs a retransmission in j's stead. Moreover, as cooperation rewards are granted by node 0, the transmission probability of i can be increased according to (1) only if node 0 correctly received the packet retransmitted by i. In order to reach both objectives, we impose the following change in the allocation conditioned on the event that the packet retransmitted by i is correctly received by node 0

$$\sum_{s=1}^{K_{ij}} \Delta P_{ij}^{(n+s)} q_j^N N_j^N = w_{ij} \frac{q_i^j N_j - q_j^N N_j^N}{q_i^j} \quad (14)$$

where $w_{ij} \in [0, 1]$ is the *cooperation weight* of i with respect to j. The left hand side represents the average decrease of the number of bits transmitted by j during the following K_{ij} slots, given that $P_j^{(n+s)} = P_j - \Delta P_{ij}^{(n+s)}$, $s = 1, \ldots, K_{ij}$. Therefore, the average (non conditioned) decrease of the number of bits is obtained multiplying it by the probability that the packet retransmitted by i is correctly received by node 0, and we have imposed it equal to $w_{ij}\left(q_i^j N_j - q_j^N N_j^N\right)$. Since $w_{ij} \in [0, 1]$, the average increase in the number of bits transmitted by j during slot n, $q_i^j N_j - q_j^N N_j^N$, is higher than the average decrease of the number of bits transmitted by j during the subsequent K_{ij} slots, hence, $\Delta B_{ji} \geq 0$ as we wanted.

The cooperation weight w_{ij} is a tunable parameter describing how valuable it is to reward cooperation by i towards j. If w_{ij} is equal to 1, during the $K_{ij}+1$ time slots from n to $n+K_{ij}$ user j transmits an average number of bits equal to what it would have transmitted during the same interval in the no cooperation case. The lower w_{ij}, the higher the throughput of user j, but at the same time the lower the incentives given to user i, until $w_{ij} = 0$, where no incentives are given to user i.

The cooperation weight w_{ij}, $\forall i, j : i < j$, represents the strategy of node 0, i.e., the strength of incentives given to cooperating users. We suppose that w_{ij} are fixed by node 0 at the beginning of the communication and are transmitted to all users. In this way, any user knows in advance the gain it obtains by cooperating with each other user and can select its best strategy. This type of interaction between node 0 and other users can be cast in the framework of the Stackelberg games [1], where node 0 plays first and the users act afterwards. The player moving first can predict the behavior of other players and optimize its own strategy.

We can rewrite (14) as

$$\sum_{s=1}^{K_{ij}} \Delta P_{ij}^{(n+s)} = \frac{w_{ij}}{q_i^j}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right) \quad (15)$$

under the constraint $\Delta P_{ij}^{(n+s)} \leq P_j$, $s = 1, \ldots, K_{ij}$.

There are infinitely many solutions $\left\{K_{ij}, \Delta P_{ij}^{(n+s)}, s = 1, \ldots, K_{ij}\right\}$ that satisfy the above equation. However, cooperating users should be rewarded as early as possible, so as to enable faster convergence to the

[3] A linear $\psi_i(\cdot,\cdot)$ will satisfy (13). In particular, if $\psi_i(\cdot,\cdot)$ is linear then $\psi_i(\cdot,\cdot) = \psi_{ij}(\cdot,\cdot), \forall i, j$. Moreover, the converse is also true: if $\psi_i(\cdot,\cdot)$ satisfies (13) and $\psi_i(\cdot,\cdot) = \psi_{ij}(\cdot,\cdot), \forall i, j$, then $\psi_i(\cdot,\cdot)$ is a linear function.

asymptotic throughput. Thus K_{ij} is set as the lowest integer such that

$$K_{ij}P_j \geq \frac{w_{ij}}{q_i^j}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right) \qquad (16)$$

which results in the following scheduling policy variation:

$$\Delta P_{ij}^{(s)} = P_j \quad ; \quad s = n+1, \ldots, n+K_{ij}-1$$
$$\Delta P_{ij}^{(n+K_{ij})} = \frac{w_{ij}}{q_i^j}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right) - (K_{ij}-1)P_j \qquad (17)$$

B. User strategies

Now, we study the interaction between users considering generic cooperation weights w_{ij} and introducing the selection of the modulation scheme M_i.

In the voluntary cooperation scheme, the packet transmission process of user i follows the Markov Chain in Fig. 4, which is conceptually similar to Fig. 3 with the difference that only users belonging to $\mathcal{A}_i$ cooperate with i and the scheduling is dynamic according to (1). The access probability of user i at the beginning of a slot depends on the users i has cooperated with and on the users that have relayed i's packets in the preceding slots. In order to derive the exact metrics associated to the voluntary cooperation scheme, the Markov chain of Fig. 4 should be expanded to take into account that i might cooperate with other users when it is not scheduled. The transition associated to the probability $1 - P_i^{(n)}$ should be divided into a number of transitions equal to the cardinality of $\mathcal{C}_i$ plus 1, representing the events that i is not scheduled and it does not act as a relay or it acts as a relay for one of the users belonging to $\mathcal{C}_i$. These transitions would end in as many chains, all of them similar to the lower chain of Fig. 4, with the only difference that the access probabilities of user i are different. To obtain simple analytical expressions of the asymptotic metrics of the voluntary cooperation scheme, instead of exactly tracing the temporal variation of the scheduling probability we consider an approximate approach that takes into consideration just the average value $\overline{P}_i$ of the scheduling probability of a generic user i. This allows us to obtain the following results

$$P_{R_i}^V = q_i(2-q_i) + \sum_{k\in\mathcal{A}_i}(1-q_i)\,q_{ik}\left[\prod_{t\in\mathcal{A}_i,t<k}(1-q_{it})\right](q_k^i - q_i)$$
$$v_i^V = (2-q_i)/\overline{P}_i$$
$$BR_i^V = \overline{P}_i\left[q_i + \frac{1-q_i}{2-q_i}\sum_{k\in\mathcal{A}_i} q_{ik}\prod_{t\in\mathcal{A}_i,t<k}(1-q_{it})\,(q_k^i - q_i)\right]\frac{N_i}{T_{pkt}}$$
$$BR^V = \sum_{i=1}^{U} BR_i^V = \frac{1}{T_{pkt}}\sum_{i=1}^{U}\overline{P}_i\Bigg[q_i + $$
$$+ \frac{1-q_i}{2-q_i}\sum_{k\in\mathcal{A}_i} q_{ik}\prod_{t\in\mathcal{A}_i,t<k}(1-q_{it})\,(q_k^i - q_i)\Bigg]N_i \quad (18)$$

As per (1)

$$P_i^{(n)} = P_i + \sum_{j\in\mathcal{C}_i}\Delta P_{ij}^{(n)} - \sum_{k\in\mathcal{A}_i}\Delta P_{ki}^{(n)} \qquad (19)$$

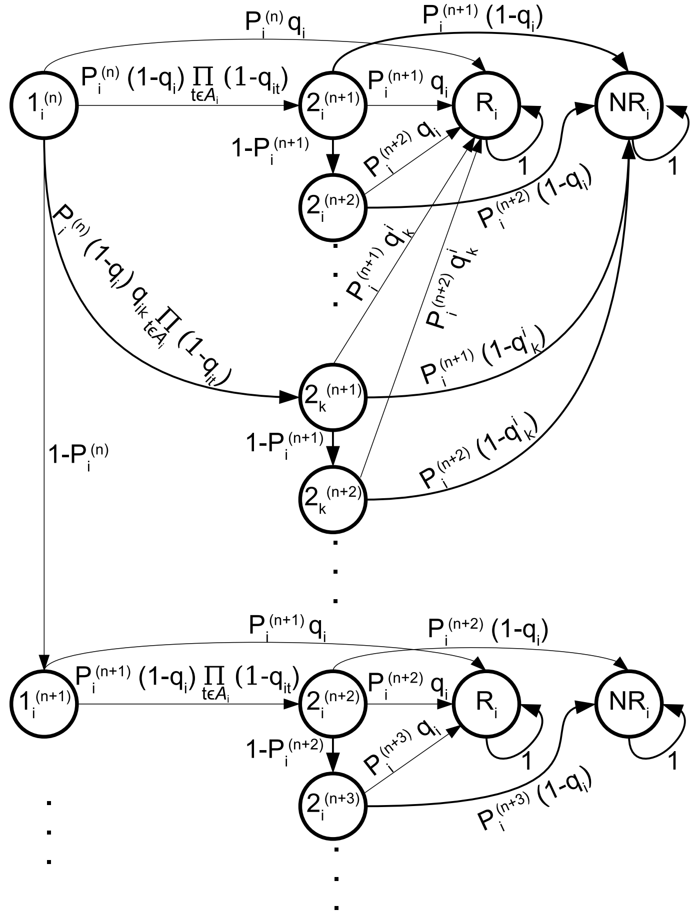

Fig. 4. Transmission process of a packet of user i in the voluntary cooperation scheme

where $\Delta P_{ij}^{(n)}, \Delta P_{ki}^{(n)} \geq 0$ are according to (17). $\Delta P_{ij}^{(n)} > 0$ if and only if i cooperated with j during one of the preceding K_{ij} slots. $\Delta P_{ki}^{(n)} > 0$ if and only if k cooperated with i during one of the preceding K_{ki} slots. As per (17), ΔP_{ki} depends on q_k^i and N_i that in turn depend on the modulation scheme M_i. This must be taken into account when optimizing M_i. In particular, since the access opportunity of user i is decreased after being helped, the *net* average increase of i's transmitted bits due to the cooperation of user k is scaled by a factor $(1-w_{ik})$. We define

$$D_i = q_i + \frac{1-q_i}{2-q_i}\sum_{k\in\mathcal{A}_i}(1-w_{ik})\,q_{ik}\left[\prod_{t\in\mathcal{A}_i,t<k}(1-q_{it})\right](q_k^i - q_i) \qquad (20)$$

Then, the optimal modulation scheme of user i can be computed as

$$M_i^V = \arg\max_{M_i\in\mathcal{M}} D_i N_i \qquad (21)$$

where both D_i and N_i depend on M_i. If i cooperates with j, the average variation $\Delta B_{ij} > 0$ and $\Delta E_{ij} > 0$ of i's transmitted bits and energy consumption per unit time are equal to

$$\begin{aligned}\Delta B_{ij}=&q_i^j\sum_{s=1}^{K_{ij}}\Delta P_{ij}^{(n+s)}D_i\frac{N_i}{K_{ij}T_{pkt}}=\\&=w_{ij}\left(\frac{q_i^jN_j}{q_j^NN_j^N}-1\right)D_i\frac{N_i}{K_{ij}T_{pkt}}\\\Delta E_{ij}=&\left[1+q_i^j\sum_{s=1}^{K_{ij}}\Delta P_{ij}^{(n+s)}\right]\frac{E_{pkt}}{(K_{ij}+1)\,T_{pkt}}=\\&=\left[1+w_{ij}\left(\frac{q_i^jN_j}{q_j^NN_j^N}-1\right)\right]\frac{E_{pkt}}{(K_{ij}+1)\,T_{pkt}}\end{aligned}\tag{22}$$

where M_i^V is chosen according to (21). Thus, the evaluation of the partial utility $\psi_{ij}(\Delta B_{ij},\Delta E_{ij})$ depends (through D_i) on $\mathcal{A}_i$, i.e., the cooperation choices adopted towards i by users with lower indices.

Before presenting our first result, we define the Nash Equilibrium (NE). A strategy profile is a NE if no user can increase its own utility by unilaterally changing its own strategy. The NE is an important solution concept in game theory. However, in general, its existence and uniqueness are not guaranteed.

Theorem 1. *Assuming that users cooperate in case their utility is flat with respect to this choice, the sub-game between users admits one and only one NE, $\mathcal{C}^*=\{\mathcal{C}_1^*,\dots\mathcal{C}_U^*\}$.*

Proof: The proof follows a constructive and iterative procedure. Let us consider user 1, which can not be helped by any other node: $\mathcal{A}_1=\emptyset$, $P_{R_1}^{\mathcal{A}_1}=P_{R_1}^N$ and $D_1=P_{R_1}^N/\left(2-q_1\right)$. Since the probability error function q_1 varies with continuity, the set of allocation policies that optimizes (21) is a singleton, therefore user 1 can uniquely select its best modulation scheme M_1^V. Then user 1 can compute the optimal set of users to cooperate with, i.e., its best strategy $\mathcal{C}_1^*$, depending on the modulation selected by each user. This can be done by calculating ΔB_{1j} and ΔE_{1j} according to (22) and evaluating ψ_{1j}, $\forall j\neq 1$, $\forall M_j\in\mathcal{M}$.

This procedure can be repeated for any other user. For a generic user i and for each modulation scheme M_i, if we know the strategies of users $1,2,\dots,i-1$, we can uniquely calculate $\mathcal{A}_i$, M_i^V, $P_{R_1}^{\mathcal{A}_i}$, ΔB_{ij}, and ΔE_{ij}, $\forall j>i$, $\forall M_j\in\mathcal{M}$; from these, we obtain ψ_{ij}, depending on the modulation selected by the users with worse channels. In the end, we obtain the best modulation scheme for all users and the unique NE strategy profile $\mathcal{C}^*$. ■

We say that a strategy profile is Pareto Efficient (PE) if it is not possible, by changing the strategy profile, to increase the utility of at least one user without decreasing that of some other user.

Corollary 2. *The Nash Equilibrium is Pareto Efficient.*

Proof: The utility of user 1 is the highest possible since it is not affected by other users' strategies and it selects its own strategy to maximize its own utility. In the same way, the utility of user 2 is the highest possible given the strategy of user 1. Moreover, if we change the strategy of user 1 we make user 1 worse off, except for the case in which user 1's utility is flat in its choice to cooperate with user 2. However, in this case we have assumed that 1 chooses to cooperate with 2, hence, if 1 changes its strategy, the utility of 2 can not increase. This procedure can be repeated for any other user. ■

C. Access point strategy

Theorem 1 states that the sub-game between the users has only one possible outcome. Moreover, the constructive proof provides an algorithm to calculate this outcome. The access point can predict, for each strategy $\mathbf{w}=(w_{ij})_{ij}\in[0,1]^{\binom{U}{2}}$, the strategies of all users. Therefore, it can choose its best strategy $\mathbf{w}^*=(w_{ij}^*)_{ij}$ to drive the network performance toward a desired outcome.

Assume that the network performance is quantified by a utility function $u_0:[0,1]^{\binom{U}{2}}\to\Re$, whose argument is the strategy selected by node 0. It can be thought as the composition of two functions f and g, i.e., $u_0=g\circ f$, such that $f:[0,1]^{\binom{U}{2}}\to\Re^U$ gives the utility of the users as a function of 0's strategy and $g:\Re^U\to\Re$ gives the utility of 0 as a function of all users' utilities. It is reasonable to assume that g is a continuous function.

Take w_{ij}^{th} as the value such that $\psi_{ij}(\Delta B_{ij},\Delta E_i)=0$, which can be derived from (22). It is the minimum w_{ij} such that i cooperates with j. The only interesting case is when w_{ij}^{th} exists and $w_{ij}^{th}\in[0,1]$, otherwise it is not possible to trigger i's cooperation with respect to j without decreasing the throughput of j. Since ψ_{ij} are continuous, then f is continuous in $[0,1]$ except in w_{ij}^{th}. Indeed, user i changes its cooperation behavior towards j at w_{ij}^{th}. However, from a practical point of view, if $w_{ij}\in[w_{ij}^{th},1]$ the utility of both users i and j increases. In fact, user j achieves at least the same throughput, while decreasing its energy consumption, whereas the increase in throughput of i compensates the additional energy spent to cooperate with j. That is, promoting cooperation under this scheme is always beneficial for both users involved. For this reason, it is reasonable to assume that u_0 is upper semi-continuous.

Theorem 3. *If u_0 is upper semi-continuous then there exists at least one Stackelberg Equilibrium (SE). Moreover, all SEs are equivalent from a network performance point of view.*

Proof: The utility u_0 can be maximized since the sub-game NE exists and is unique. The strategy space of node 0 is closed and bounded, and $u_0(\cdot)$ is upper semi-continuous. An SE can be found by combining the best strategy $\mathbf{w}^*$ of node 0 and the NE strategy profile of the sub-game among the users when the strategy of node 0 is $\mathbf{w}^*$. There may be more than one optimal $\mathbf{w}^*$, but they all achieve the same maximum utility of node 0. ■

Finally, for result comparison, we consider the following access point strategy

$$w_{ij}^*=\begin{cases}w_{ij}^{th} & \text{if } 0\leq w_{ij}^{th}\leq 1\\ 0 & \text{otherwise.}\end{cases}\tag{23}$$

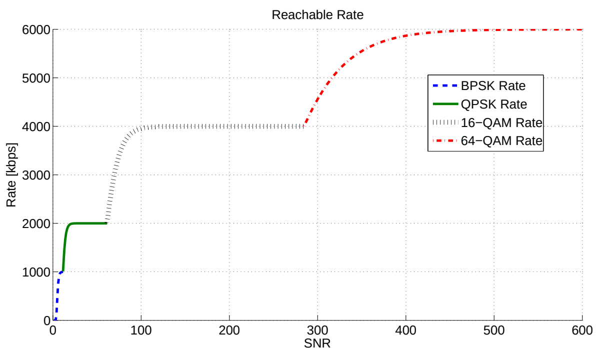

Fig. 5. Reachable rate varying the modulation depending on the SNR

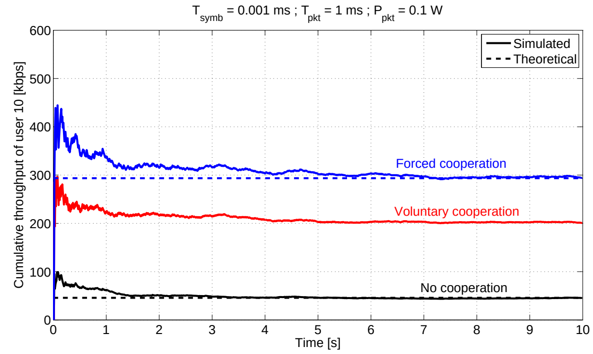

Fig. 6. Cumulative throughput of user 10

We chose this strategy to promote cooperation, i.e., increase network performance while keeping a high level of fairness (fairness metrics will be defined in the following section).

Note that $w_{ij}^{th} \notin [0,1]$ means that it is impossible, with the considered scheme, to provide an incentive for user i to cooperate with j. In this case the system functionality is independent of w_{ij}^*, and we have arbitrarily chosen $w_{ij}^* = 0$.

V. Performance Evaluation

Prior to comparing the 3 cooperation schemes, we introduce some performance metrics.

For any vector of U real numbers, $\mathbf{x} = (x_1, \ldots, x_U)$, we define a fairness metric $J(\mathbf{x})$ over $\mathbf{x}$, called Jain index [26], as

$$J(\mathbf{x}) = \frac{\left(\sum_{i=1}^{U} x_i\right)^2}{U \sum_{i=1}^{U} x_i^2} \tag{24}$$

We will evaluate this index for the vectors of throughput ($\mathbf{BR} = (BR_1, \ldots BR_U)$) and utility values ($\mathbf{\Psi} = (\Psi_1, \ldots \Psi_U)$). We use superscripts N, F, and V to relate these metrics to the no cooperation, forced cooperation and voluntary cooperation schemes, respectively.

A scenario with U users uniformly placed within a 400 meters radius from an access point has been simulated in Matlab. We consider a time slot $T_{pkt} = 1\ ms$ and a symbol period of $T_{sym} = 1\ \mu s$, that is, each packet is made of 1000 symbols. The number of bits per packet for a generic user depends on the number of bits per symbol, i.e., on the modulation scheme selected by that user. We consider $\mathcal{M} = \{BPSK,\ QPSK,\ 16-QAM,\ 64-QAM\}$, that correspond to the rates represented in Fig. 5.

Each user transmits with a fixed power of $P_{pkt} = 100$ mW. The time invariant channel attenuation coefficient is given by the superposition of two effects: a power law decay with exponent equal to 3 and a Rayleigh distributed coefficient. The signal to noise ratio obtained at a reference distance of 10 m considering a unit-power Rayleigh coefficient is 10. We consider the initial allocation policy $\mathbf{P} = (1/U, 1/U, \ldots, 1/U)$.

We take $\Psi_i(B_i, E_i) = B_i - c_i E_i$, i.e., $\psi_i(\Delta B_i, \Delta E_i) = \Delta B_i - c_i \Delta E_i$, which satisfies (13) with $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) = \Delta B_{ij} - c_i \Delta E_{ij}$, $\forall i, j$, where $c_i > 0$ is a measure on how important the throughput is for user i with respect to its power expenditure. We consider $c_i = \dfrac{q_i N_i}{2E_{pkt}}$ where q_i and N_i are calculated with a modulation scheme according to (6), i.e., c_i is equal to half i's energy efficiency (rate divided by power consumption) in the non cooperative case. In this way, users having a low non cooperative rate are more inclined to cooperate with other users, consuming their energy to obtain a higher throughput, with respect to users having already a high non cooperative rate. We obtain

$$w_{ij}^{th} = \frac{c_i E_{pkt}}{\left(\dfrac{q_i^j N_j}{q_j^N N_j^N} - 1\right)(D_i N_i - c_i E_{pkt})} \tag{25}$$

We first present some results for a specific topology with $U = 10$, which is actually the one in Fig. 1. Fig. 6 shows the evolution of throughput over time for user 10 (the one with lowest SNR), for the 3 different schemes. The dashed lines represent the average throughput of no cooperation and forced cooperation schemes according to (5) and (11). The cumulative throughput asymptotically converges to these average values. This convergence is quite fast, as the curves are already stable after few iterations and become practically indistinguishable from the asymptotic value within 10 seconds.

Fig. 7 compares the asymptotic throughput reached by each user. Roughly speaking, this specific topology includes some users (with indices 1-3) that are able to reach a maximal throughput of 600 kb/s already under the no cooperation scheme, by using the highest modulation (64-QAM) without ever incurring in packet retransmission. Conversely, users 7-10 have very poor channel conditions (lower modulation scheme, and possibly frequent retransmissions), and users 4-6 are in an intermediate condition. Interestingly, in the forced cooperation scheme the users with the highest indices obtain the greatest benefit. They know that users 1, 2 and 3 are forced to act as relays. Thus, since they have a good channel towards at least one of these relays, they select the highest modulation and their packets are transmitted in two hops exploiting the relays, allowing them to reach a bit-rate of about 300 kb/s. On

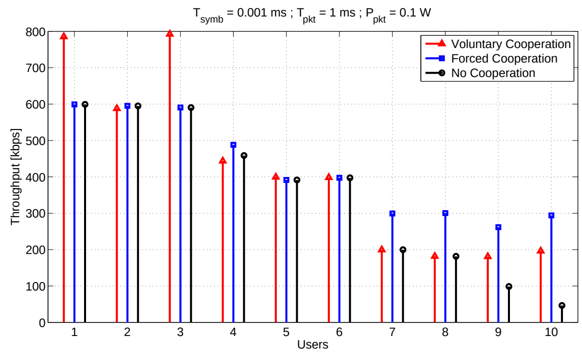

Fig. 7. Asymptotic throughput of each user

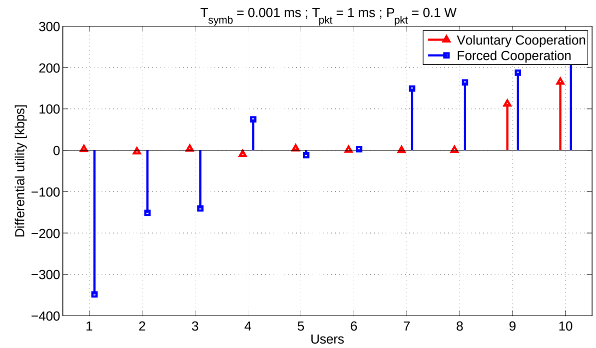

Fig. 8. Incremental utility ψ of each user

the contrary, the cooperating users do not obtain any improvement. Instead, the voluntary cooperation scheme increases the throughput of cooperating users as well. Especially, users 7 and 8 are not helped since none of the users with good quality finds a worthwhile incremental advantage in doing so.

Fig. 8 represents the incremental utility ψ of each user and emphasizes even more the differences between the forced and voluntary cooperation schemes. For the forced cooperation case, the utility of high index users considerably increases, though at the expense of low index users which have no reward in their cooperating behavior. When cooperation is forced by node 0, users 7-10 significantly increase their own throughput and at the same time cut in half the transmission power because retransmissions are performed by users 1-3, which in turn only suffer higher power expenditures. The voluntary cooperation scheme improves this situation, since no user worsens its incremental utility ψ. The highest index users improve their utility, even though by a smaller extent than with forced cooperation, and no user is worse off than before. Indeed, this happens because cooperation is offered even in the marginal case where the incremental utility is equal to 0; however, setting a higher requirement for cooperation would yield similar results, i.e., a utility value which is higher for some users, lower for none. In this sense, the voluntary cooperation scheme *Pareto dominates* the no cooperation scheme [1]. Moreover, the figure suggests that the voluntary cooperation scheme achieves a more fair distribution of the utility function among the users. Finally, Fig. 8 validates the analysis carried out in Section IV. In fact, even though the incentives to cooperative users are calculated using the approximate equations (18), the *real* throughput gain for cooperative users is just enough to compensate the *real* additional energy consumption to relay the packets of the other users, as we wanted.

To obtain general results, not constrained over a particular network topologies and channel realization, we ran a simulation campaign over many network topologies drawn at random with a variable number of users, and averaged the results. Fig. 9 represents the average throughput increase of the whole network thanks to cooperation, for both forced and voluntary cooperation schemes. The values are normalized to the total throughput ob-

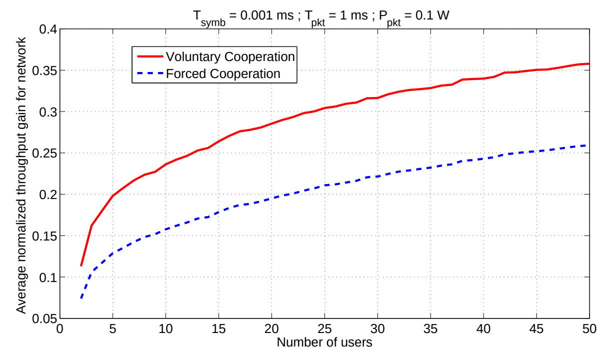

Fig. 9. Average throughput gain normalized to the no cooperation scenario

tained in the no cooperation scenario. Both forced and voluntary cooperation schemes obtain a significant gain; for 50 users, they improve the total throughput by more than 25% and 35%, respectively. Remarkably, voluntary cooperation performs better than forced cooperation; this is due to the better redistribution of additional resources gained through cooperation, which in the forced cooperation scheme are given just to the users with bad channel quality, while in the voluntary cooperation scheme are distributed more evenly. It is also worth noting that the cooperation gain increases in the number of users, which is due to multi-user diversity, i.e., with more users it is just more likely to find a suitable relay. However, the voluntary cooperation scheme does better in this sense, i.e., it increases more rapidly in the number of users, in fact it is more likely to find a suitable relay which is also willing to cooperatively participate in the retransmissions.

Fig. 10 shows the Jain index related to the throughput vector, i.e., $J(\mathbf{BR})$. Clearly, the no cooperation case just reports what is the average situation for what concerns fairness in the considered scenario if no cooperation is applied. Apparently, the forced cooperation scheme achieves the best value of fairness for throughput. In fact, users with lower throughput are helped by collaborative relays which have no other choice, therefore throughput gaps are smoothed out. After an initial decrease, the Jain index becomes even larger as the number of users increases. In fact, the higher the number of users, the higher the probability of finding a suitable relay (not necessarily a

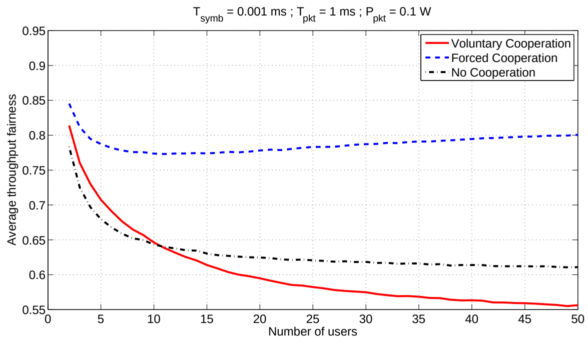

Fig. 10. Average throughput fairness

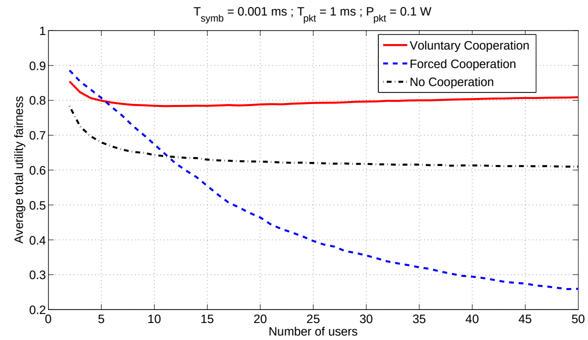

Fig. 11. Average utility fairness

willing one, since cooperation is forced). The fairness decreases quite rapidly for the voluntary cooperation scheme. This is due to the fact that users with good channel conditions, which already have a higher throughput than others, are rewarded by the access point if they cooperate, which means that they further increase their throughput. This pulls fairness even below the no cooperation case. However, it is worth noting that, although fairness is decreased, throughput is never decreased for anybody. Moreover, evaluating fairness over throughput just gives a very partial picture. Even though users with good channel increase their throughput, they also have to pay this gain in terms of power consumption, since they retransmit packets on behalf of bad users (which in turn can save energy); even their reward in terms of increased scheduling probabilities also implies more transmissions and therefore higher energy consumption.

Fig. 11 shows the Jain index related to the utility vector, i.e., $J(\mathbf{\Psi})$. The situation is inverted with respect to the preceding case. As the number of users increases, the fairness rapidly decreases for the forced cooperation scheme. This is due to the fact that a small subset of users, i.e., those having a very good channel quality and able to act as relays for a large area, are more and more forced to cooperatively relay packets. This pulls their utility much more below the utility of users that are exploiting them as relays, decreasing the total fairness of the network to values even below the no cooperation case. On the other hand, in the voluntary cooperation scheme users acting as relays do not experience a decrease in their utility while helped users can increase their own utility, which results in smoother utility gaps. Note that, if the utility fairness is considered as the social welfare metric, Fig. 11 gives a representation of the Price of Anarchy, defined as the ratio between the overall system welfare in the worst Nash equilibrium and in the best Pareto efficient case. In fact, the highest value of the utility fairness is 1, obtained when the users' utilities are equal, while the worst Nash equilibrium coincides with the unique equilibrium of the game under consideration.

To sum up, the comparison between the three schemes shows that voluntary cooperation is able to significantly improve the network performance over the case without cooperation. In all the comparisons, the forced cooperation scheme is to be regarded as a theoretical upper bound, as it implies a centralized scheduling determined a priori with full system knowledge, to which all the users adhere. Conversely, the voluntary cooperation scheme may be applied dynamically (based on transmission outcomes) and in a distributed manner, since each user decides freely whether to cooperate or not. The goal of the coordinator is just to set the system in an NE, for which the exchange of information required is rather limited and the convergence is pretty fast. Note also that the forced cooperation scheme does not operate in a stable point, i.e., at a NE. Thus, with the same system conditions of rational decision and distributed action the forced cooperation scheme will become identical to the no cooperation scheme. On the contrary, the voluntary cooperation scheme is robust toward strategic and self-interested users. Moreover, the performance of the voluntary cooperation scheme can be regarded as an improvement not only over the basic case without cooperation, but even over the forced cooperation scheme, especially since it achieves a higher total throughput and a more fair overall utility distribution.

VI. Discussions and Future Works

The results obtained in this paper have been derived considering a simplified model of a wireless communication network. In this section we discuss possible relaxations of some hypotheses we have made.

First, we consider the time invariant channels and the perfect channel state knowledge hypotheses, that allow to calculate the performance of each user and of the system by means of an analysis based on renewal process theory. If channels are time varying, the asymptotic performance is not longer equivalent to the statistical mean. However, for slowly varying channels, there is enough time for the physical quantities under investigation (i.e., throughput and energy consumption) to approach the statistical means, as Fig. 6 confirms. Hence, our formulation can be applied to the slowly varying channels scenario as well, by considering adaptive estimates. This work can also be extended to highly varying channels and imperfect channel state knowledge, assuming that the entities involved aim at maximizing the statistical mean of their performance, which might not coincide with their asymptotic performance. In this case, the statistics of the channel evolution and of the channel estimates are needed.

As frequently considered in many game theoretic studies, we assumed that *every* user is self-interested and strategic. In a network there might be some users that act individually or cooperatively independently of their personal advantage. Our framework and results can be easily extended assuming that a mix of no cooperation and forced cooperation nodes are present in the network of voluntary cooperation nodes. The former might receive the cooperation of the other users, but never offer their cooperation. Thus, the indices of such nodes do not belong to set $\mathcal{A}_i$ and do not appear in the summation and multiplication of Eq. (18). The latter always offer their cooperation, hence, there is no need to give them incentives by increasing their access opportunities, i.e., their cooperation weights can be set to 0. Thus, the indices of such nodes belong to set $\mathcal{A}_i$ and appear in the summation and multiplication of Eq. (18). It is straightforward to demonstrate that Theorems 1 and 3 and Corollary 2 are still valid, excluding the no cooperation and forced cooperation nodes from the sub-game (they do not play a game since their actions are fixed).

Another aspect which may be worth looking at is the evaluation of the overhead introduced by the forced and voluntary cooperation schemes with respect to the no cooperation scheme. This point is key to translate the theoretical framework proposed in this paper into an effective and realistic MAC protocol. However, it can be shown through simple computations that such an additional overhead is minimal and can be neglected. We do not consider the overhead for the estimation and communication of the channel states (which is needed in every scheme) and the computation of the cooperation weights (which is needed for the voluntary cooperation scheme), as these operations are performed sparsely since channels are slowly varying. Instead, we investigate the overhead to schedule different users, to identify eligible relays and to select one of them.

For the no cooperation scheme, at the beginning of each time slot, we assume that node 0 broadcasts a short packet indicating the user scheduled in that slot. Such a user, after a short time interval[4], sends the data packet. Finally, after another short time interval, node 0 sends an ACK to the user if it has received the packet correctly.

We modify such a simple MAC protocol to support the forced cooperation and voluntary cooperation schemes. In this case, during the scheduling phase, node 0 has to indicate not only the packet to transmit, but also who has to perform such a transmission, in case a relay service is required. Moreover, the user that transmits the packet adds, at the end of the packet data, a series of bits, one for each node, to communicate to node 0 the users for which it is available to act as a relay. This MAC protocol is not suitable if there are some users that are scheduled rarely, as in this case node 0 might not be updated about the relay opportunities offered by such users. In this case another option should be considered to inform node 0 about relay opportunities, e.g., a short contention window can be added after the ACK.

The additional overhead introduced in the considered MAC protocol can be easily quantified. Consider a time slot $T_{pkt} = 1$ ms, a symbol period of $T_{sym} = 1$ μs and a network of 50 users. Hence, the additional number of bits needed in the scheduling packet is equal to 6 while the additional number of bits needed in the data packet is equal to 50. Assuming, in the worst case, a $BPSK$ modulation, the additional overhead is equal to 56 μs over 1 ms, i.e., about 5%, that is very low compared to the throughput gain of the forced cooperation and voluntary cooperation scheme that are equal to 25% and 35% in such a scenario (see Fig. 9).

Finally, in this paper we have not considered the cost incurred by every node to listen and store the transmission of all the other nodes. Even though the power spent in reception is typically lower than the transmission power, such an effect might become predominant for a high number of users. Moreover, in the worst case each user might have to store up to $U-1$ additional packets, requiring a large buffer. These problems might be counteracted considering a simplified version of the proposed schemes, where we limit, for each user, the number of users to ask for a relay service and to cooperate with. Such a simplified version is motivated by the high gain that the voluntary cooperation scheme is able to obtain for a high number of users, as shown in Fig. 9. Such a potential gain might not be completely exploited if we limit the relay opportunities, but at the same time the scheme becomes more practical as the number of users increases. We will take into consideration the study of such a scheme in our future work.

VII. Conclusions

We tackled the problem of promoting cooperative relaying in a wireless network with coordinated time-division access, by giving the following contributions. First, we outlined mathematical models, based on Markov chains and renewal theory, to quantify the achievable throughput. Moreover, we modeled the cooperation option of the single users through game theory and we proposed an incentive scheme for voluntary cooperation that gives transmission resources to cooperating users when they retransmit a packet on behalf of other users. We modeled this access scheme as a 2-stage Stackelberg game, where a network unit plays the role of access coordinator. We presented a constructive approach to determine the NE of the sub-game, proven to be unique. We also proved the existence of a Stackelberg equilibrium, which results in the best incentive strategy that the coordinator can adopt.

Finally, we numerically compared the three schemes of no cooperation, forced cooperation, and voluntary cooperation. A careful analysis of these results justifies the voluntary cooperation scheme as a valid solution to increase the network performance in a viable manner from an implementation standpoint.

Such an approach may lead to framing the problem under study here in the more general context of coalitional games with

[4]In the $802.11\,g/n/ac$ standards the SIFS (short inter-frame space), defined as the sum of the RX/TX turnaround time, MAC processing delay and total receive delay from the antenna, is equal to $16\,\mu s$

transferable utility. As a matter of fact, the entire application of rewarding collaborative relay intervention may be seen as a way of transferring (although partially) the individual throughput terms among the users. The results of the present paper for cooperation at the physical/access/network layer appear to be more successful than similar evaluations made from a higher layer (network/transport) perspective [3], [5], [12], where the results often imply that cooperation is hard to achieve. Perhaps a meaningful solution, to be addressed in future research, could be identified in cross-layer approaches, where the benefits of a collaborative physical/access layer compensate the shortcomings of inefficient collaborative routing. The present paper may be a good starting point in this sense.

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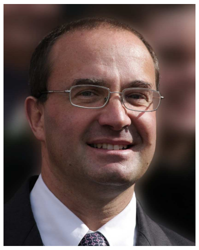

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