A Zero-Sum Jamming Game with Incomplete Position Information in Wireless Scenarios

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(Invited Paper)

***Abstract*—We consider a jamming problem, modeled as a zero-sum game with incomplete information played by a wireless network and a malicious jammer that wants to disrupt communication, in a wireless scenario with distance-dependent loss of the wireless medium. Multiple access is achieved by means of parallel channels, characterized by different propagation parameters. The jammer is unaware of the exact positions of the network nodes, but knows the prior distribution of where they can be located. We investigate the dependence of the equilibria of this game on the actual position of the jammer itself. We conclude that there are regions where the actual gameplay of the jammer at the Nash Equilibrium only consists of pure strategies, and therefore the wireless network can consider itself to be insensitive to the presence of the jammer. This does not mean that the jammer has no effect whatsoever, but if it is known to be physically located in such regions, its damage to the network can be quantified in advance with certainty; on the other hand, there is also no way of counteracting this jammer, and if its presence is not known, the network is not able to distinguish it from background interference.**

***Index Terms*—Wireless communication; Jamming; Game theory; Zero-sum games; Incomplete information.**

I. Introduction

JAMMING problems are a classic application of game theory to wireless networks. The situation where a legitimate transmitting network is contrasted by a malicious attacker that acts on the purpose of disrupting communication can be faced in several contexts; wireless ad hoc networking for military and civil purposes can be considered, where the jammer tries to disable the communication capabilities of the network [1], or the investigation can be extended to the physical layer, where the jammer is also able to corrupt the messages exchanged by the legitimate transmitters [2]. Also, cognitive spectrum access [3] can be investigated as another scenario of application, where the jammers can exploit vulnerabilities of reconfigurable cognitive radios.

From the modeling standpoint, a jamming problem has a quite direct appeal in that, if properly formulated, it involves just two agents and therefore can be naturally framed as a two-player game between a legitimate network owner/user, hereafter called the transmitter (T), and the adversary that wants to cause disturbance or disruption in the communication, referred to as the jammer (J).

In this paper, we focus on a simple setup where T can use multiple wireless channels for communicating and can select among them based on their quality, described, e.g., through some capacity expressions that ultimately depend on the signal-to-interference-plus-noise ratio (SINR). Thus, if the ambient noise is fixed, the data capacity of a channel only depends on the signal attenuation on it, and of course on the actions of J. On the other hand, we assume that J is only capable of "brute-force" attacks at the physical layer, which aim at raising the interference level on a given channel [4]. Also the interference that J can cause on a channel depends on the attenuation that its transmission gets on it.

We adopt a standard formulation of the problem as a zero-sum game [2], [4], where T is the maximizer and J is the minimizer. The value of the game will simply be the sum capacity that T is able to achieve on the channels used for transmission. Our goal is to investigate the role of mutual positioning between T and J, since the attenuation can be related to the path loss experienced on the channel, which in the end is related to the distance. For the sake of simplicity, we will consider a simple distance-based path loss model, but it is immediate to generalize the results to any situation where the channel quality is position-dependent.

Also, we consider a Bayesian approach as done for jamming in [5] (and also in [6], [7] for other networking problems), where T has different types depending on the positions of its nodes. This reflects the imperfect knowledge that J may have on the network structure. Our goal is to investigate the role of the position of J, and how this can impact on the resulting equilibrium. For this reason, J is *not* considered to be a player with type, i.e., its position is known to all the players. Actually, with this assumption we want to investigate whether the presence (or the knowledge thereof) of a jammer in a given position is a relevant element for the network and it changes its gameplay.

It turns out that, depending on the propagation scenario, there may be several situations where the effect of J is limited. These of course include cases where J is so far from the network that it cannot cause any damage; in this sense, our investigation includes physical layer security considerations [8], but it is not limited to them. In reality, the cause of J's irrelevance is more game theoretic than physical. Depending on the structure of the game, if the resulting zero-sum formulation has a single Nash Equilibrium (NE) in pure strategies for J (that is to say, there is a single behavior that a rational jammer can adopt), its presence can be ignored, not because it does not cause any harm, but rather there is no effective countermeasure that can be taken against it.

One can also argue that such a jammer would not even be detected in the first place [4]. Indeed, if J's equilibrium move is to play along a certain pure strategy, T cannot tell whether J is really present in the network or there simply is a high interference level. Because of this ignorance, T has no margin of loss, and no margin for improvement either. Conversely, there are positions of J where the resulting NE dictates that J plays a mixed strategy, which means that T should counteract more than one possible jamming action. We label these positions as *critical*, and we identify them by looking at where the maximin of the game is different from the minimax and therefore, roughly put, there is something at stake for the communication network. As a general conclusion, the choice of the proper countermeasures against jamming attacks should take into account the position of the jammers and possibly scan the interested area with different purposes, depending on whether a potential position of a jammer is critical or not.

The rest of this paper is organized as follows. In Section II we describe the propagation scenario, with simplified fundamentals of wireless communications. In Section III we give the game theoretic model for the problem, and in Section IV we compute the NEs. Finally, we present some numerical results in Section V and we draw the conclusions in Section VI.

II. Wireless Scenario

Radio waves propagate in different ways depending on the frequency used, and more in general on the physical scenario of communication [9]. However, the most important characteristic of interest in our analysis is that the perceived signal strength of a radio signal depends on the mutual positions of transmitter and receiver. In many simplified models, the larger the distance, the weaker the signal. This is also the rationale that we adopt in our analysis, which allows for a simple description by means of few parameters.

It is worthwhile noting, however, that radio propagation is much more complex than what discussed in the following; it involves a thorough description of the frequency channel and the geometry of the area surrounding the transmitter and the receiver to account for factors such as: the presence or the absence of a direct path (line-of-sight) between the terminals; the effect of reflections, refractions, scattering, and related phenomena, from other objects present in the area; the characterization of the noise on said channel; the existence of other transmitters located close-by that can cause interference; and many other relevant issues. Also, these characteristics of the wireless channel are inherently time-varying [10]. Rapid fluctuations of the signal strength are possible, especially in mobile environments. Thus, the proper characterization of the wireless channel is that of a stochastic process and what we give in the following is to be meant as its statistical description derived from different realizations (which can be just samples at different time instants if the process is ergodic).

All these issues are outside the scope of the present paper, where, for simplicity reasons, we consider a stationary channel with a straightforward distance-dependent attenuation through a power law, whose exponent is the only parameter that summarizes the scenario. More refined models are certainly possible but do not qualitatively change the conclusions that we draw later.

Thus, a signal transmitted with power P_{T} is received with a power attenuation $\mathtt{a}(d_{\mathrm{T}})$ depending only on the distance d_{T} between the transmitter and the receiver, and therefore called the *path loss* at a distance d_{T}. We assume that $\mathtt{a}(d_{\mathrm{T}})$ follows a power law with positive exponent α, usually $\alpha \in [2,4]$, i.e.,

$$\mathtt{a}(d_{\mathrm{T}}) = K_0 \left(\frac{d_{\mathrm{T}}}{d_0} \right)^{\alpha} \tag{1}$$

where d_0 is a reference distance (in particular, we can consider $d_0 = 1$ m, so as to drop it from the equation) and K_0 is the attenuation at d_0. The received power will be $P_{\mathrm{T}}/\mathtt{a}(d_{\mathrm{T}})$.

We can assume a background noise with power spectral density N_0 to be present on the channel, whose bandwidth is equal to B. It is common to take the noise term as an additive white Gaussian noise (AWGN). The noise power will thus be $N_0 B$. The meaning of this term can also be extended to include interference effects from other transmitters. Moreover, note that the background noise does not necessarily have to be white (and thus N_0 be a constant), nor Gaussian [11]; however, basic communication theory descriptions often regard the noise in terms of equivalent thermal noise characterizations, e.g., through noise temperature. Or alternatively, the term $N_0 B$ can be replaced with a more complicated expression where we compute the integral of the power spectral density over the channel band [9].

In short, we use $N_0 B$ to include both noise and also any unintentional interference. We can therefore compute the SINR at the receiver when the transmitter is at a distance d_{T}, denoted as $\Gamma(d_{\mathrm{T}})$, in the absence of jamming, as

$$\Gamma(d_{\mathrm{T}}) = \frac{P_{\mathrm{T}}}{\mathtt{a}(d_{\mathrm{T}})\, N_0 B} \, . \tag{2}$$

Instead, if a jammer is present on the same channel, located at a distance d_{J} from the receiver and using a jamming power P_{J}, we must consider it when computing the SINR; therefore we denote it as $\Gamma(d_{\mathrm{T}}, d_{\mathrm{J}})$, and put it equal to

$$\Gamma(d_{\mathrm{T}}, d_{\mathrm{J}}) = \frac{P_{\mathrm{T}}}{\mathtt{a}(d_{\mathrm{T}}) \big[N_0 B + P_{\mathrm{J}}/\mathtt{a}(d_{\mathrm{J}}) \big]} \, . \tag{3}$$

We can use (1) to model the attenuation of both the transmitter and the jammer. However, to take into account that the propagation models of the two may be different, we consider two different parameters in the equation, i.e., we denote with α (as before) the path loss exponent of the transmitter, and conversely we use β for the jammer (for simplicity, we use the same K_0, though). From (3), we obtain

$$\Gamma(d_{\mathrm{T}}, d_{\mathrm{J}}) = \frac{P_{\mathrm{T}}}{{d_{\mathrm{T}}}^{\alpha} \big[K_0 N_0 B + P_{\mathrm{J}}/{d_{\mathrm{J}}}^{\beta} \big]} \tag{4}$$

If noise, bandwidth, and transmission powers (both the useful and the jamming terms) are constants, the impact of the distance can be summarized as

$$\Gamma(d_{\mathrm{T}}, d_{\mathrm{J}}) = \big[{d_{\mathrm{T}}}^{\alpha} (K_1 + K_2\, {d_{\mathrm{J}}}^{-\beta}) \big]^{-1} , \tag{5}$$

with K_1 and K_2 being suitable constants. An infinitely far jammer has no effect on the SINR, as $\Gamma(d_{\mathrm{T}}) = \Gamma(d_{\mathrm{T}}, \infty)$.

A suitable performance metric that may describe what a transmitter would like to maximize (and a jammer to minimize) is the channel capacity C. For example, we can use the formula of the Shannon capacity for an AWGN channel, which results in defining C as

$$C = B \log_2 \left[1 + \Gamma(d_\mathrm{T}, d_\mathrm{J})\right] , \tag{6}$$

which depends on the SINR (and thus, also on the presence of a jammer).

If the transmitter operates over F multiple channels spanned by index $i = 1, 2, \dots, F$, and each of them has bandwidth B_i, we take χ_i to represent an indicator function that equals 1 whether the jammer is disturbing with power P_J the transmission on the ith channel, and 0 otherwise. The sum capacity available in the system in the presence of a jammer is therefore

$$C_\mathrm{tot} = \sum_{i=1}^{F} \Big\{ \chi_i B_i \log_2 \left[1 + \Gamma(d_\mathrm{T}, d_\mathrm{J})\right] + (1-\chi_i) B_i \log_2 \left[1 + \Gamma(d_\mathrm{T})\right] \Big\} \tag{7}$$

Remarkably, the Shannon capacity is a good choice in terms of utility in the micro-economic sense [12], since it is a concave function of the SINR. In the following section, we will use it to characterize the payoff received by T in a zero-sum game.

III. Game Theory Model

We consider a wireless network with 2 transmitters, denoted as T_1 and T_2, sending data to a receiver (sink node), whose distances from the receiver are d_1 and d_2, respectively. The available spectrum is divided into 2 channels (c_1 and c_2) of equal bandwidth B. For the sake of simplicity, all propagation aspects of the channels will be represented through the path loss exponent of (1). In particular, we assume that c_1 and c_2 are characterized by parameters α_1 and α_2, respectively, when the useful transmitters operate on them. The entire network is represented by the single player T in the game, which thus comprehends T_1, T_2, and the receiver, as they are assumed to act towards the same goal, i.e., maximizing the total network capacity. To coordinate multiple access of T_1 and T_2, the network adopts a frequency-division multiple access (FDMA) scheme, i.e., either of the channels c_1 and c_2 is assigned to T_1 and the other to T_2, without overlap. Nodes T_1 and T_2 use the same transmission power P_T.

However, the network activity is menaced by a malicious jammer J whose aim is to minimize the overall capacity of the network. The jammer is placed at distance d_J from the receiver, and can operate over only either c_1 or c_2, with a fixed power P_J, as per (3). Also, the path loss exponent perceived by J on channels c_1 and c_2 is equal to β_1 and β_2, respectively.

Our goal is to model the interaction between T and J as a zero-sum game with incomplete information. Thus, we consider a finite set $\mathcal{D}$ that has cardinality D and contains pairs of distances, i.e., $\mathcal{D} = \big((d_1^{(1)}, d_2^{(1)}), \dots, (d_1^{(D)}, d_2^{(D)})\big)$, where $d_i^{(k)}$ is the ith transmitter's distance in the kth pair. Each of the elements of $\mathcal{D}$ is considered to be a *type* of player T, denoted as $\theta_k = (d_1^{(k)}, d_2^{(k)})$. T can be of type θ_k with probability p_k, where the terms p_k are values between 0 and 1 for which $\sum_k p_k = 1$. For example, if a finite set of distances $\mathcal{Z} = \{z_j\}_{j=1,\dots,L}$ is available for both transmitters, and the transmitters can even have the same distance, then $\mathcal{D} = \mathcal{Z} \times \mathcal{Z}$ and $D = L^2$. For the analysis of this paper we consider that d_J can only have one value (a possible extension of the present investigation is to allow also uncertainty for d_J). The distance d_J is known by both players. Conversely, type θ_k is only known to player T, but the probability distribution $\mathbf{p} = (p_1, p_2, \dots, p_D)$ is common knowledge in the game. This characterization with types follows that of Bayesian Games (BGs) [5].

In game theory terms, our BG is initiated by a virtual player "Nature" that selects the type for T according to distribution $\mathbf{p}$ [12]. Once the type θ_k of T is given, T can perform either of these two actions: (A_1) assign c_1 to T_1 (and, consequently, c_2 to T_2); (A_2) the exact opposite, i.e., assign c_2 to T_1. Since the channels are fully described by the propagation parameter α_i, if A_1 is chosen, then the index i of α_i is the same of T_i; for A_2, they do not match. Hence, T has $N = 2^D$ pure strategies, since a strategy is defined as a D-tuple of actions, one per each type that T can be, and each action has two alternatives to choose from. We write the ith pure strategy of T as X_i, then the kth element of the associated D-tuple, referred to as $X_i(k)$, defines whether T plays action A_1 or A_2 when its type is θ_k. J can instead perform either of the two actions: (Y_1) attack channel c_1 with attenuation parameter β_1, or (Y_2) attack c_2 with attenuation parameter β_2. Therefore, J has just two pure strategies, coinciding with its actions Y_1 and Y_2.

We denote the sets of all possible pure strategies of the players as $\mathcal{X} = \{X_i\}_{i=1,\dots,N}$ and $\mathcal{Y} = \{Y_1, Y_2\}$. We define a mixed strategy $\boldsymbol{\xi}$ for T as an N-tuple $(\xi_1, \dots, \xi_N)$ that belongs to the space of probability distributions over $\mathcal{X}$, $\Delta\mathcal{X}$, i.e., $\xi_i \geq 0$ for all $i = 1, \dots, N$ and $\sum_{i=1 \dots N} \xi_i = 1$. So, ξ_i denotes the probability that T plays X_i. Similarly, a mixed strategy $\boldsymbol{\eta}$ for J belongs to $\Delta\mathcal{Y}$ and is the pair of probabilities η_1 and $\eta_2 = 1 - \eta_1$ that J plays Y_1 and Y_2, respectively.

Given the prior probabilities $\mathbf{p} = (p_1, \dots, p_D)$ for the type of T, we can put this BG in normal form using a $N \times 2$ matrix $\mathbf{M} = \{m_{ij}\}$, where entry m_{ij} with $i = 1, ..., N$ and $j = 1, 2$, represents the expected payoff for T when players T and J play their ith and jth pure strategy, respectively, computed as

$$m_{ij} \quad = \quad \sum_{k=1}^{D} p_k C_\mathrm{tot}(X_i(k), Y_j) \tag{8}$$

where $C_\mathrm{tot}(X_i(k), Y_j)$ is the sum capacity of the network according to (7) when T is of type θ_k, and thus performs action $X_i(k)$ with probability p_k, while J chooses action Y_j.

Also note that since the game is zero-sum, we do not need to represent J's payoff, which will be $-m_{ij}$. Finally, the expected payoff of a joint mixed strategy $(\boldsymbol{\xi}, \boldsymbol{\eta})$ can be computed by averaging the entries of $\mathbf{M}$ with weights equal to the probabilities of $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$.

IV. Nash Equilibria computation

The NEs of the zero-sum game, given the prior distribution $\mathbf{p}$ for the type of T and the resulting payoff matrix $\mathbf{M}$, can be found via von Neumann's Minimax Theorem [13]. Since the game is a classic zero-sum game, it can be found to have at least one NE in mixed strategies, and actually all NEs yield the same payoffs. The mixed strategies played at NE will be *maximinimizer* strategies for both players, i.e., they assure to the players the highest payoff they can get in the worst-case scenario of the strategy played by the opponent.

We denote one of these equivalent NEs as $(\boldsymbol{\xi}^*, \boldsymbol{\eta}^*)$, where $\boldsymbol{\xi}^*$ is the mixed strategy played by T and $\boldsymbol{\eta}^*$ is that played by J. There will be a unique quantity v, called the *value* of the game, such that each NE yields a payoff equal to v to T (and correspondingly a payoff equal to $-v$ to J), for which

$$\begin{aligned} v &= \max_{\Delta\mathcal{X}} \min_{1\le j\le 2} \sum_{i=1}^{N} \xi_i^* m_{ij} \\ &= \min_{\Delta\mathcal{Y}} \max_{1\le i\le N} (m_{i1}\eta_1^* + m_{i2}\eta_2^*) \,. \end{aligned} \tag{9}$$

To solve numerically, we search for w_1 and w_2, with

$$w_1 = \min_{1\le j\le 2} \sum_{i=1}^{N} \xi_i^* m_{ij} \tag{10}$$

$$w_2 = \max_{1\le i\le N} (m_{i1}\eta_1^* + m_{i2}\eta_2^*) \,, \tag{11}$$

so that w_1 is maximized and w_2 is minimized. The Minimax Theorem guarantees that the pair $(\boldsymbol{\xi}^*, \boldsymbol{\eta}^*)$ is a NE and the value is given by $w_1 = w_2$. To solve the maximization of (10) and the minimization of (11) we reduce them to two linear programs with proper slack variables for w_1 and w_2 [12]. Thus, the problems become

$$\begin{aligned} &\max \ w_1 \\ &\text{s.t.} \ w_1 \le \sum_{i=1}^{N} \xi_i m_{i1} \,, \quad w_1 \le \sum_{i=1}^{N} \xi_i m_{i2} \\ &\xi_i \ge 0 \quad \forall i = 1, \ldots, N \,, \quad \sum_{i=1}^{N} \xi_i = 1 \end{aligned} \tag{12}$$

and analogously

$$\begin{aligned} \min \ & w_2 \\ \text{s.t.} \ & w_2 \ge \eta_1 m_{11} + \eta_2 m_{12} \\ & \vdots \\ & w_2 \ge \eta_1 m_{N1} + \eta_2 m_{N2} \\ & \eta_1 \ge 0, \ \eta_2 \ge 0, \ \eta_1 + \eta_2 = 1 \end{aligned} \tag{13}$$

Both problems can be solved by means of optimization techniques; we used Dantzig's simplex algorithm [14]. We evaluate and compare the solution for different choices of the set $\mathcal{D}$, the prior $\mathbf{p}$, the distance of the jammer d_{J}, and the propagation coefficients α_i and β_i.

We also compute the `maximin` and the `minimax` in pure strategies, which can be done by considering the same problems as above but imposing $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ to have all the elements equal to 0 but one, which is equal to 1. In this case, w_1 gives the `maximin` and w_2 the `minimax`. As a matter of fact, these values can be immediately found by scanning the matrix $\mathbf{M}$ and taking minima and maxima over rows and columns.

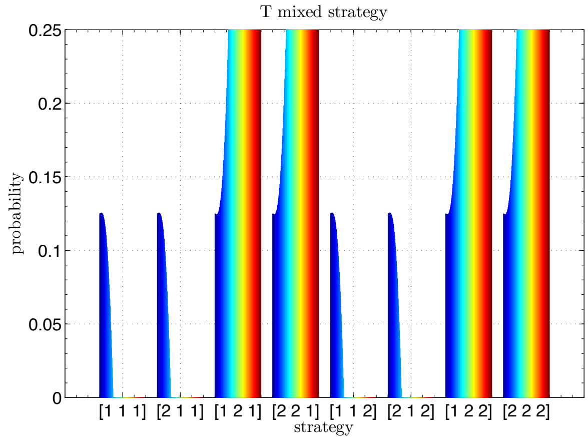

Fig. 1. NE strategy of T as a function of $d_J \in [0.01, 15]$ m (α_1=α_2=β_1=2, β_2=2.5, prior distribution for T's type: [1/3 1/3 1/3]).

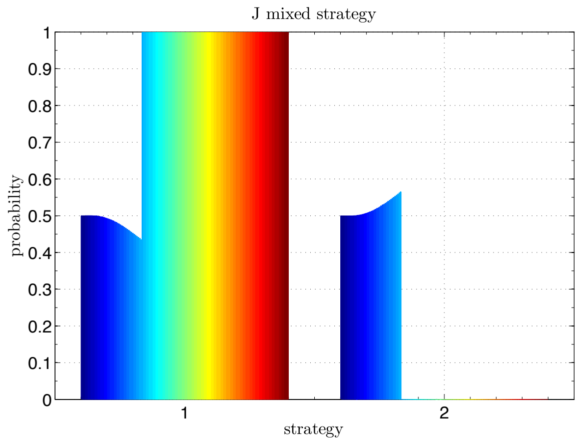

Fig. 2. NE strategy of J as a function of $d_J \in [0.01, 15]$ m (α_1=α_2=β_1=2, β_2=2.5, prior distribution for T's type: [1/3 1/3 1/3]).

V. Numerical Results

We evaluate a scenario where the propagation parameters α_1, α_2, β_1, β_2 take different values in the range $2 \div 3$. We take $N_0 B$, which represents the noise power plus the external unintentional interference, as equal to -120 dBm. We set the transmission power terms of the transmitter and the jammer to be the same, i.e., $P_{\mathrm{T}} = P_{\mathrm{J}} = 0$ dBm.

We set 3 types for T in $\mathcal{D} = \{(5,5), (5,10), (10,10)\}$ m. As a consequence, each of the 8 pure strategies of T is a triple of binary values (for brevity, we just write 1 or 2 instead of A_1 or A_2). For example, strategy $[1\ 2\ 1]$ denotes that T assigns c_1 (i.e., the channel with attenuation parameter α_1) to transmitter T_1 when it is of type 1 or 3, while it assigns c_2 (i.e., the one with attenuation parameter α_2) to T_1 if it is of type 2; in other words, this strategy means that channel c_1 is assigned to T_1 when the transmitters are at the same distance, and to T_2 if T_1 is closer to the sink. Finally, we place the jammer at a distance d_{J} ranging from 0.01 to 15 m.

In this scenario, we evaluate the NE of the BG, and its dependence on d_{J}. First, we considered the case of a uniform distribution for the types of T, i.e. $p_k = \frac{1}{3}$, $k = 1, 2, 3$, with attenuation parameters $\alpha_1 = \alpha_2 = 2$ for T, and $\beta_1 = 2$, $\beta_2 = 2.5$ for J. Figs. 1, 2, and 3 refer to this setting.

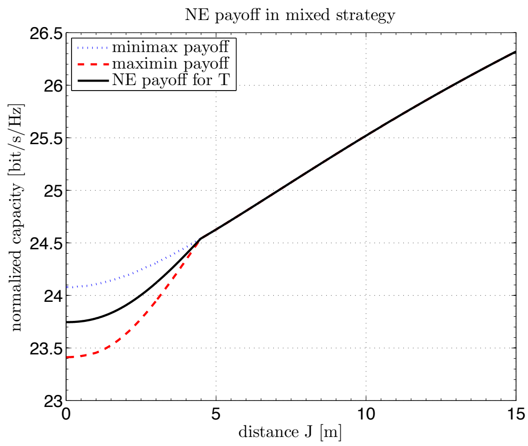

Fig. 3. NE payoff in mixed strategies as a function of $d_J \in [0.01, 15]$ m (α_1=α_2=β_1=2, β_2=2.5, prior distribution for T's type: [1/3 1/3 1/3]).

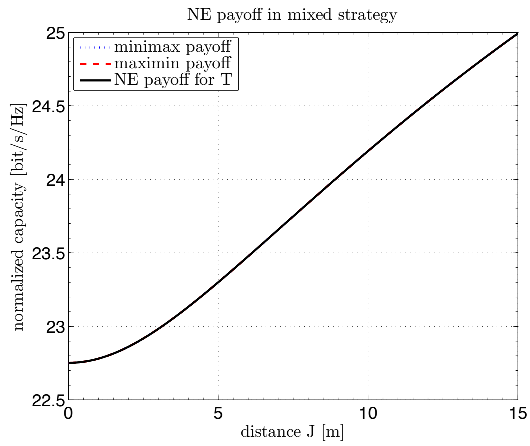

Fig. 4. NE payoff in mixed strategies as a function of $d_J \in [0.01, 15]$ m (α_1=β_1=β_2=2, α_2=2.5, prior distribution for T's type: [1/3 1/3 1/3]).

Figs. 1 and 2 show T's mixed strategy and J's mixed strategy, respectively, depending on d_{J}. In particular, Fig. 2 reveals that there are values of d_{J} for which the strategy of J at equilibrium is pure. For these cases, the corresponding strategy of T is a mixed strategy with probability $1/4$ split over a support represented by all pure strategies [$\cdot$ 2 $\cdot$], i.e., those for which action number 2 is played by T when its type is θ_2. In other words, T surely plays A_2 when the nodes T_1 and T_2 are placed at different distances, otherwise it randomly chooses between its alternatives; indeed, when the strategy of J is pure, and the nodes are placed at identical distances, T is indifferent on which channel to assign to the nodes.

This result is also visible from figures such as Fig. 3, in which we plot the NE payoff for T (quantified as the capacity per unit of bandwidth: this is also what displayed in all similar plots afterwards), together with the `maximin` and `minimax` payoff in pure strategies. In the very region where J's equilibrium move is a pure strategy, the three curves coincide. In these cases, the transmitter strategy can be regarded as *insensitive* to the presence of a jammer. From Fig. 3, the game has some degree of uncertainty in its outcome only when d_{J} is less than 4.484 m, which means that the jammer has an unpredictable behavior more or less only when closer to the receiver than the closest position available to the transmitters.

Fig. 4 shows the comparison between NE's payoff, `maximin`'s payoff and `minimax`'s payoff for a configuration slightly different from the previous one. We considered the same a priori probability but we set $\alpha_1 = \beta_1 = \beta_2 = 2$ and $\alpha_2 = 2.5$. In this case, the NE always imply a pure strategy for the jammer, regardless of the distance of J from the destination, since the `maximin` and `minimax`, and also the NE payoff, coincide for every d_{J}, and thus T's strategy is always insensitive to J's presence. This result is also correct since, in the presence of an unbalanced situation for the transmitter but not for the jammer (a channel has lower α_i and therefore has better quality, while the two β_i are identical), intuitively the jammer should always cause disturbance on the better channel.

Fig. 5 shows the comparison between NE's payoff, `maximin`'s payoff and `minimax`'s payoff for $\alpha_1 = \beta_2 = 2$ and $\alpha_2 = \beta_1 = 2.5$. In this case, the NE strategy of the jammer

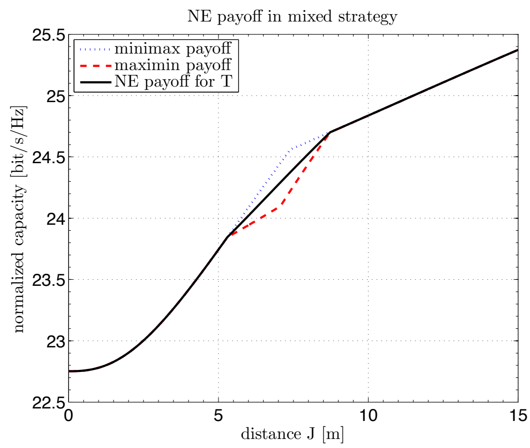

Fig. 5. NE payoff in mixed strategy as a function of $d_J \in [0.01, 15]$ m (α_1=β_2=2, α_2=β_1=2.5, prior distribution for T's type: [1/3 1/3 1/3]).

does not degenerate into a pure strategy for a whole region of intermediate distances, 5.298 m $< d_{\mathrm{J}} < 8.730$ m, which is more or less comprised between the available positions for the two transmitters.

Figs. 6, 7, and 8 show the results obtained considering the same values for the parameters used in Figs. 3, 4, and 5, respectively, for a non-uniform distribution of T types, where p_2=2/3, and p_1=p_3=1/6. This leads to an expansion of the *critical regions* in which the jammer plays a mixed strategy. In particular, in Fig. 6 the border of the critical region is pushed to 5.830 m, i.e., beyond the smallest value in $\mathcal{D}$, i.e., 5 m, and a similar and even more relevant enlargement of the region happens in Fig. 8, where J plays a mixed strategy in the entire interval $0 < d_{\mathrm{J}} < 10.012$ m. Also, in Fig. 7, differently from the similarly shaped Fig. 4, the equilibrium strategy of the jammer is not a pure strategy for a very wide range of positions, namely, for $d_{\mathrm{J}} < 11.740$ m, even though the differences are minor.

VI. Conclusions

We investigated a game theoretic setup of the jamming problem with variable distances of the players involved. In

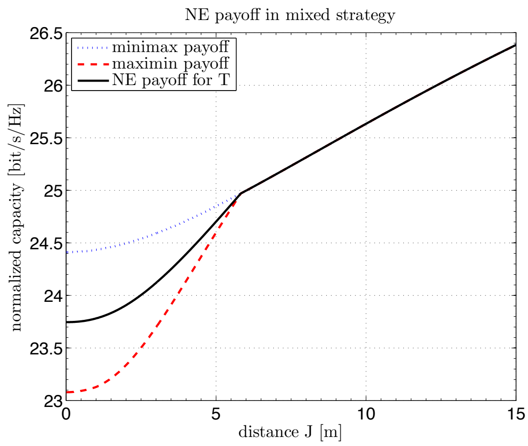

Fig. 6. NE payoff in mixed strategy as a function of $d_J \in [0.01, 15]$ m (α_1=α_2=β_1=2, β_2=2.5, prior distribution for T's type: [1/6 2/3 1/6]).

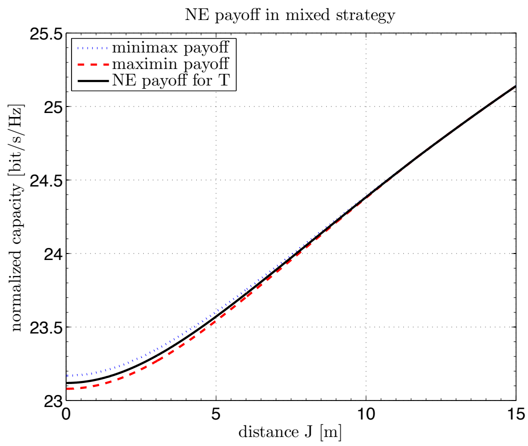

Fig. 7. NE payoff in mixed strategy as a function of $d_J \in [0.01, 15]$ m (α_1=β_1=β_2=2, α_2=2.5, prior distribution for T's type: [1/6 2/3 1/6]).

particular, we formulated a BG where we framed the positions of the useful transmitters as the type of the maximizer player, whereas the jammer, i.e., the minimizer player, can only occupy a fixed position, which is characterized by a distance d_{J} from the intended receiver. We investigated the dependence of the NEs and the resulting payoff on d_{J}; the main result is that, depending on the propagation parameters, there are intervals (in certain cases quite wide) for d_{J} where the NE solution implies that the jammer adopts a pure strategy, which means, a given channel is jammed with probability 1. As a result, the network manager should especially control those situations where this *does not happen*, i.e., the jammer occupies a position according to which at the equilibrium its strategy mixes multiple jamming actions. Such scenarios have been classified as *critical* positions for the jammer. This also leads to the conclusion that any security enforcement should especially check those areas for jammers.

However, it would make sense to consider a deeper interaction between the network and the jammer, where the game formulated here is used as a basis. In particular, it can be thought of relaxing the assumption of knowing the jammer's

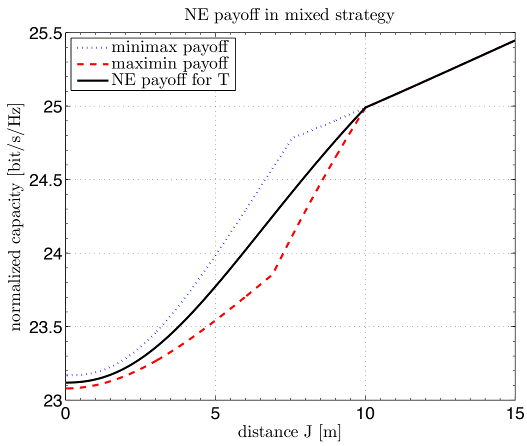

Fig. 8. NE payoff in mixed strategy as a function of $d_J \in [0.01, 15]$ m (α_1=β_2=2, α_2=β_1=2.5, a priori probability [1/6 2/3 1/6]).

position with certainty, and let this to be the *type* of the jammer. Our preliminary investigations indicate that the game becomes even more interesting. Especially, a more complicate interaction takes place and in the end the region where a jammer exhibits critical influence may change and even shrink further. Future analysis will be required to extend the analysis and identify challenges in an expanded setup with more advanced game theory instruments.

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