Bayesian Game Analysis of a Queueing System with Multiple Candidate Servers

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***Abstract*—We combine queueing theory and game theory to evaluate the performance of a queueing system with multiple strategic candidate servers. The intent is to model a transmission system where packets can be sent via multiple options, each incurring a cost and controlled by a distributed management. Our purpose is to analyze the effects of the presence or the lack of both cooperation and communication between servers. The mathematical characterization of the uncertainty about the characteristics of the transmission alternatives available is captured through a Bayesian game formulation. In this setup, we compute both the Price of Anarchy, quantifying the inherent inefficiency arising from selfish management of each server, and the Price of Stability, which is the loss due to distributed system management, under different conditions of signaling exchange among the servers.**

***Index Terms*—Queueing analysis, telecommunication networks, game theory, Bayesian games, Price of Anarchy.**

I. Introduction

THE NEED for scalability of modern communication networks has led to the practical establishment of several distributed management algorithms. Keeping the network intelligence spread throughout the network is both a low-cost management solution and also the proper way to involve the increased computational power of communication devices.

To mathematically characterize this aspect, in this paper we consider a joint application of queueing theory [1] and game theory [2]. In more detail, we consider a queueing system with multiple candidate servers, whose clients can be thought of as packets arriving for transmission. Each server represents an option available at the device, which is managed by its own controller, and has its own specific success probability for transmission. We model this situation as a game, where the servers are the players and they can decide to be *inactive* or *active*; staying active has a fixed local cost for the server, while a successful transmission is beneficial for the whole system. Clearly, in a system with centralized supervision, it would be best to only keep active the best server (i.e., the one with highest success probability). In our scenario, this is hindered by two facts: (i) servers act selfishly, as in any game theory setup; (ii) servers do not even know who the "best" server is, since they do not know each other's characteristics.

While the former is a classic ingredient of game theory capturing the lack of cooperation, the latter is due to the lack of *communication* between the agents, which is represented by modeling the problem as a Bayesian Game [3]. In this setup, we compute the *social welfare* of our system seen as the sum of the individual expected gains. As a measure of inefficiency we compute (i) the Price of Anarchy (PoA), which indicates the loss due to the lack of cooperation and measures how the efficiency of a system degrades due to selfish behavior of its agents; and (ii) the Price of Stability (PoS), i.e. the inefficiency due to the lack of communication compared with respect to the optimal outcome.

Up to our knowledge, this work is the first to combine these two mathematical approaches in this way. Various works in the literature study different characterizations of a queueing model using game theory; however, they consider the point of view of the customers, instead of the servers as we do. Indeed, some research works include whether to avoid or follow the crowd [4], and the proper period of time in which to arrive to a queue in order to minimize waiting and tardiness costs [5]. Other papers study the network as a whole and its strategic structure as a complex system made of customers and servers [6], [7]. However, our contribution is novel in that it considers multiple candidate servers of the queue as the players.

We remark that our analysis has strong consequences in modeling a transmission system with multiple alternatives controlled by distributed agents, a rationale that can be applied throughout the entire protocol stack. For example, at the network layer it can be exploited for a distributed routing selection in multi-hop environments [8]. At the datalink layer, multiple access techniques can be coordinated similarly [9]. Finally, at the physical layer, this can be applied for devices powered by multiple energy sources [10].

In the present contribution, we use a game theoretic approach to describe the strategic behavior of the servers; in particular, we refer to a game in which each player is characterized by a *type*, which is unknown to the others. We considered different levels of cooperation. Our goal is to compare the effects of cooperation and communication between the servers. For this reason, we consider different scenarios and we compute the PoA, which is a measure of how much the efficiency of a system is reduced due to the selfish behavior of its members [3]. In our case the PoA is evaluated as the ratio between the gain obtained in the best possible scenario, in which everything is known and there is cooperation, and the gain related to the worst scenario, in which just a single server works and the others are off. Moreover, we compute also the efficiency loss from a system in which communication between servers is allowed to a system in which there is no communication. For this evaluation, we consider the PoS, whose definition relates the optimal solution with the results obtained considering the best equilibrium. Not only do we find that allowing signaling among the servers improves the system efficiency, but we are also able to assess how much.

The rest of this paper is organized as follows. Section II gives some preliminaries on game theory. In Section III, we describe the system model and the game theory application, we compute the NEs in Section IV. Section V presents some numerical results and Section VI draws the conclusions.

II. Preliminaries

Queueing system theory was developed to predict behaviors of systems subject to randomly arising demands. This was also the spirit of the early contributions by Erlang in 1909 [11], the works by Pollaczek and Khinchine in the 1930s [1] and subsequent studies such as [12] and [13]. Nowadays, queueing theory finds many applications in management of communication networks or air traffic, planning of manufacturing systems, computer program scheduling, and facility dimensioning.

The idea to use a mathematical theory to make predictions on the behavior of multiple agents is also shared by game theory, which is now becoming more and more commonly applied to telecommunication problems [14], [15]. Differently from queueing theory, which still adopts a system view, game theory is even more extreme in considering that individual agents, called *players*, interact according to their particular interests that may be different for every one of them [2].

In game theory, players act towards the maximization of their personal payoff, whose evaluation takes into account the actions played by all players. This represents the quantification of the goodness (utility) coming from the eventual outcome that depends on the joint selection of actions. A static game of complete information is expressed by a triple $\mathcal{G} = (\mathcal{N}, \mathcal{S}, \mathcal{U})$, where $\mathcal{N}$ is the set of players, $\mathcal{S}$ is the set of all strategies allowed to each player, and $\mathcal{U}$ includes the payoffs, which are functions of the choice of actions by all players. In case of a static game with complete information, a strategy corresponds to playing an action, but this may become more complex under more advanced setups. We remark that, for the problem at hand, we compute payoffs as the sum of a positive profit due to the transmission service regardless of which server transmits, and a negative cost that depends on the server being active.

An equilibrium is a joint strategy profile where all individual payoffs of players are locally maximized. In particular, starting from the simplest case of a static game of complete information, a Nash Equilibrium (NE) [3] is defined as strategy profile where all players do not have an incentive to unilaterally change their courses of action. This means that each player is playing a *best response* to its *belief* about the strategy chosen by the opponents. Within a game theoretic context, players can either follow pure or mixed strategies. The former define the move a player will make for any situation it could face, the latter represents an assignment of a probability distribution to the set of pure strategies.

For the case under exam, we consider a Bayesian Game, in which players can be of different *types*, and each type has a different utility function. This is actually a way to capture that the players may behave in different ways. Also, we assume that each player is aware of its own type only. About the other players' types, each player solely knows the probability distribution over types, which is common knowledge among the players [2]. Thus, Bayesian games are games of incomplete information. In this setup, a strategy identifies a complete plan of actions that covers every contingency of the game, also including types. In game theory, players assume beliefs about their opponents, to which they react; the equilibrium condition holds whenever these beliefs are consistent and correct. The crucial point in a Bayesian game is to require players to form correct beliefs about both the actions and also the types of their opponents.

In the sequel of the paper, we will exploit these concepts to capture the inherent uncertainty of distributed systems, which is very relevant to communication systems. We will show how this theoretical background enables a descriptively powerful evaluation of the system and paves the road to a useful characterization for many distributed network problems. In [8], nodes forward packets randomly and in an uncoordinated behavior. It may be interesting, for example, to evaluate how much efficiency is lost with respect to the case of coordinated management. In [9], multiple access techniques are integrated; this is a strong research topic also in view of upcoming 5G cellular networks. In this case, the access selection may be done with or without coordination among the controllers allowing a comparison between the different performance evaluations obtained. Finally, in [10] the authors consider the problem of optimizing the transmission strategy of two devices with energy harvesting capability that share a wireless channel, in order to maximize the long-term average importance of the transmitted data. It is assumed that a central controller is kept informed on the energy level and packet importance of both nodes. Actually, it may be useful to compare the case of a distributed behavior (either collaborative or competitive) of the nodes and compare the resulting performance with respect to the central controller case. Furthermore, in the same spirit also the coexistence of multiple uncoordinated energy sources is a relatively unexplored field that can be interesting to investigate in light of the strong interest received by energy harvesting as a way to achieve self-sustainable transmission and green networking.

III. System Model

We consider a system with a single queue and two candidate servers, server 1 and server 2, which transmit packets arriving to the system with rate λ. Each server can either be *active* or *inactive* and has a certain probability μ_1 or μ_2, respectively, of correct packet transmission when it is *active*; thus, service has Bernoulli distribution with rate μ_j. This aspect generalizes various cases of outage, for example related to energy unavailability, collisions, and/or channel errors. We consider that, when a packet arrive to the system, it can be transmitted if an active server succeeds in transmitting it, otherwise it is dropped (discarded). The cost of being *active* is the same for each server and is some $c \in (0, 1)$. Instead, dropping the packet, either because the server is *inactive* or because it is *active* but not able to ensure a successfully transmission, causes a cost d for both servers. The packet is not dropped if at least one of the servers takes care of its transmission. Without loss of generality, we set $d = 0$. [1]

The expected payoff obtained by each server depends on the action chosen by the other server and the probability of success of the packet transmission. We assume that successfully and incorrect transmission of a packet involve a profit of 1 and 0, respectively, for both servers (even the one not performing the transmission). If both servers are active, the packet will be served by the one with higher rate (however, the additional active server does not bring any benefit).

We can model this problem as a game, in which the two servers are the players. The set of *actions* that players can take is {*active*, *inactive*}; every pair of actions of the servers yields an arbitrary quantification of the goodness coming from choosing those actions. Based on the payoff associated to each action, each player can choose a best move. According to the state of the players, we have 4 different cases. If both servers are *inactive*, neither attempts packet transmission and both payoffs are 0. In the cases in which server i is *active* and server j is *inactive*, the expected payoff of the former is $\mu_i(1-c)+(1-\mu_i)(-c) = \mu_i - c$ and that of the latter is μ_i. If both servers are *active*, their payoffs are $\max(\mu_1,\mu_2)\cdot(1-c)+(1-\max(\mu_1,\mu_2))\cdot(-c) = \max(\mu_1,\mu_2)-c$.

This game can be represented in an equivalent normal form using a 2×2 matrix shown in Table I. Each entry indicates the respective payoffs, for player 1 and player 2, associated to the pair of actions chosen, on rows and columns for player 1 and 2, respectively.

		Player 2	
		active	**inactive**
Player 1	**active**	$\max(\mu_1,\mu_2)-c$, $\max(\mu_1,\mu_2)-c$	$\mu_1 - c$, μ_1
	inactive	μ_2, $\mu_2 - c$	0, 0

TABLE I
NORMAL-FORM (MATRIX) OF THE GAME

IV. NASH EQUILIBRIUM COMPUTATION

We consider that each server i has a Bayesian type that is the probability of successfully transmitting the packet μ_i. These types follow a given joint distribution, that is common knowledge among the players, but the actual values may not be known. Actually, each player is always fully informed on its own type, but not on the opponent's. This value can be communicated, or the players can form beliefs about it. Our purpose is to compare systems with and without cooperation and/or communication between servers. For this reason, we evaluate the following four scenarios.

A. Scenario 1: Distributed service without signaling

In this scenario, each player does not know the opponent's type. In this context, a pure strategy s_i for player i identifies the choice of being either *active* or *inactive* for every type μ_i.

[1]The case $d > 0$ can be included by just rescaling the utilities.

Server i will be surely inactive and surely active if $\mu_i = 0$ and $\mu_i = 1$, respectively. In the intermediate cases, server i will be *active* if its expected payoff when *active* is greater than when it is *inactive*. We can also state that, looking at a single server i, if a given value of μ_i involves transmission for that server, then any other $\tilde{\mu_i} > \mu_i$ will allow the transmission to the server. This is due to the monotonicity of the expected payoff expression weighed on the type distribution as a function of μ_i. Indeed, single server i will transmit if $\mu_i > c$, therefore if μ_i satisfies the inequality, also $\tilde{\mu_i} > \mu_i$ satisfies it. As a consequence, we can conclude that the optimal strategy for a server follows a threshold policy. From the definition of best response to a belief, it is immediate to prove the following.

Proposition. The best response of player i will be to be *active* and transmit if and only if

$$\mu_i \geq \frac{c}{1-\rho_{-i}} \tag{1}$$

where ρ_{-i} is the probability that server $j \neq i$ is *active*. If the types of the players are uniformly distributed in $[0,1]$, then (1) becomes $\mu_i = c/\hat{\mu}_j$.

If j is using threshold $\hat{\mu}_j$ so that j is *active* if and only if $\mu_j \geq \hat{\mu}_j$ then it follows that j's probability of being *active* corresponds to the probability that its type is greater than or equal to $\hat{\mu}_j$, therefore $1-\rho_{-i} = \hat{\mu}_j$. If i believes that $\hat{\mu}_j < c$ then its best response is to never transmit, that is, to set threshold $\hat{\mu}_i = 1$. Otherwise, its best response is to choose $\hat{\mu}_i = c/\hat{\mu}_j$. The following expression describes the best response threshold strategy $\hat{\mu}_i$ of player i given that player j is using threshold $\hat{\mu}_j$

$$BR_i(\hat{\mu}_j) = \begin{cases} \frac{c}{\hat{\mu}_j}, & \text{if } \hat{\mu}_j \geq c \\ 1, & \text{if } \hat{\mu}_j < c \end{cases}. \tag{2}$$

If we plot the best response curves of the two players in this scenario varying the possible values that μ_1 and μ_2 can take, we observe that the curves coincide for $c \leq \mu_i \leq 1$ and $i = \{1,2\}$. As a consequence, all pairs (μ_1, μ_2) satisfying $\mu_1\mu_2 = c$ with $c \leq \mu_i \leq 1$ and $i = \{1,2\}$ are NEs.

For the computation of the expected payoff of each player we averaged on the areas shown in Fig. 1 weighing on

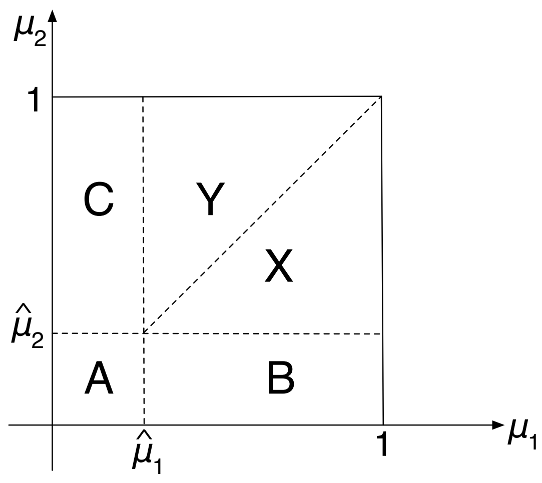

Fig. 1. Areas for the players expected payoff computation.

the types distribution and summing the various contributions. Considering player 1, the contributions for the expected payoff evaluation are derived, after some integrals, as $A = 0$ and

$$B = \hat{\mu}_2 \left[\frac{1-\hat{\mu}_1^2}{2} - c(1-\hat{\mu}_1) \right] \tag{3}$$

$$C = \hat{\mu}_1 \left[\frac{1-\hat{\mu}_2^2}{2} \right] \tag{4}$$

$$X = \frac{(1-\hat{\mu}_1^3)(\hat{\mu}_2-1)}{3(\hat{\mu}_1-1)} + c\hat{\mu}_1(1-\hat{\mu}_2) - \frac{(1+\hat{\mu}_1)}{2}(c+\hat{\mu}_1)(1-\hat{\mu}_2) \tag{5}$$

$$Y = \frac{(1-\hat{\mu}_2^3)(\hat{\mu}_1-1)}{3(\hat{\mu}_2-1)} + c\hat{\mu}_2(1-\hat{\mu}_1) - \frac{(1+\hat{\mu}_2)}{2}(c+\hat{\mu}_2)(1-\hat{\mu}_1) \tag{6}$$

The expected payoff of player 1 is computed as $A + B + C + X + Y$; the expected payoff of player 2 is also obtained similarly, with just minor changes in the integrals, which basically imply to swap B with C and X with Y.

B. Scenario 2: Worst case allocation (lazy server)

This scenario refers to the situation in which one of the two servers is always *inactive*. For this reason, we considered this scenario divided in two sub-cases: case 2a corresponds to the situation in which server 1 is actually the only one, as server 2 is *inactive*, case 2b corresponds to the opposite in which server 1 is *inactive*. This scenario represents the worst case in which there is neither cooperation nor communication between the servers.

For the sake of simplicity, in the following analysis we focus on case 2a, taking into account that the same results are obtained for case 2b after reversing the server roles. Considering that player 2 is always *inactive* the best strategy of player 1 is to be *active* if $\mu_1 - c > 0$. Therefore, in case 2a we have a single NE that is $(\hat{\mu}_1, \hat{\mu}_2) = (c, 1)$ and the expected payoff evaluation involves an unidimensional computation. As a consequence, player 1 expected payoff is

$$\int_0^{\hat{\mu}_1} 0 \; d\mu_1 + \int_{\hat{\mu}_1}^1 (\mu_1 - c) \; d\mu_1 = \frac{1-c^2}{2} - c(1-c) \tag{7}$$

considering that $\hat{\mu}_1 = c$. Whereas, for player 2 we obtain

$$\int_0^{\hat{\mu}_1} 0 \; d\mu_1 + \int_{\hat{\mu}_1}^1 \mu_1 \; d\mu_1 = \frac{1-c^2}{2}. \tag{8}$$

Thus the two cases of scenario 2 are two special cases of the scenario 1; they are obtained considering scenario 1 at the equilibria $(\hat{\mu}_1, \hat{\mu}_2) = (c, 1)$ and $(\hat{\mu}_1, \hat{\mu}_2) = (1, c)$.

C. Scenario 3: Distributed service with signaling

We consider scenario 3 as a modified version of scenario 2. In this case, we allow communication between servers when they are both *active*; indeed, they can exchange information about their types in order to decide which of them is better to be *active* so that it transmits, and which of them should turn *inactive*. Therefore, the analysis shown for scenario 1 holds also for this environment. As a consequence, as well as for scenario 1 in which we found an infinite number of NEs, also for scenario 3 we have infinite NEs.

The computation of the contributions A, B, C, and X for the expected payoff of player 1 is the same as previously discussed for scenario 1; the only difference lies in the fact that for the evaluation considering area Y in Fig. 1 the integrand is now μ_2 because players realize that server 2 has the highest rate and therefore server 1 is forced to inactivity in region Y. Therefore, in this scenario, the contribution for region Y is

$$\int_{\hat{\mu}_2}^1 \left(\int_{\hat{\mu}_1}^{\frac{\hat{\mu}_1-1}{\hat{\mu}_2-1}\mu_2 - \frac{\hat{\mu}_1-\hat{\mu}_2}{\hat{\mu}_2-1}} \mu_2 \; d\mu_1 \right) d\mu_2. \tag{9}$$

Similar observations can be made for computation of the expected payoff of player 2, which is identical to the previous case but for region X where 2 is turned inactive.

D. Scenario 4: Coordinated service

Scenario 4 represents the situation in which players have complete knowledge about the game and, therefore, the values of their opponent type. It describes a situation in which both cooperation and communication are allowed to the servers. In this case, the best response threshold strategy $\hat{\mu}_i$ of player i given that player j is using the threshold $\hat{\mu}_j$ is

$$BR_i(\hat{\mu}_j) = \begin{cases} c, & \text{if } \hat{\mu}_j \le c \\ 1, & \text{if } \hat{\mu}_j > c \end{cases}. \tag{10}$$

Indeed, for example focusing on player 1, since it is known that $\mu_1 > \mu_2$, then player 1 will be *active* if $\mu_1 - c > 0$, otherwise if $\mu_2 > \mu_1$ it will be *inactive*. A similar reasoning holds for player 2 behavior. As a consequence, this scenario has a unique NE (c, c) and lower thresholds $\hat{\mu}_1 = \hat{\mu}_2 = c$.

Our purpose is to compare the case of non-cooperation, i.e., scenario 2, with the best case in which everything is known, i.e., scenario 4, to compute the PoA and measure the inefficiency suffered by the system due to the lack of cooperation. On the other hand, we also compute the PoS to measure the price paid by the systems that consists of servers which cooperate but could not exchange information.

V. Numerical results

We take the distribution of types as uniform in $(0, 1)$. For scenario 1 and scenario 3 we consider these NEs: $(c, 1)$, $(c^{\frac{2}{3}}, c^{\frac{1}{3}})$, $(c^{\frac{1}{2}}, c^{\frac{1}{2}})$, $(c^{\frac{1}{3}}, c^{\frac{2}{3}})$, and $(1, c)$. Fig. 2 and Fig. 3 show the social welfare for scenario 1 and scenario 3, respectively, for the different NEs previously listed. In our analysis, given a certain scenario, the social welfare is defined as the sum of the expected payoff obtained for the two players. As it can be noted from these figures, it is reasonable to focus simply on $(c^{\frac{1}{2}}, c^{\frac{1}{2}})$ as a NE because it gives the highest total expected payoff, that is the highest social welfare.

Fig. 4 shows the expected payoff of player 1 in the various scenarios considered; the values of cost vary from 0.02 to 0.5 because for higher values of c the performance degrades in all cases. The results shown are computed at points $(c^{\frac{1}{2}}, c^{\frac{1}{2}})$ for

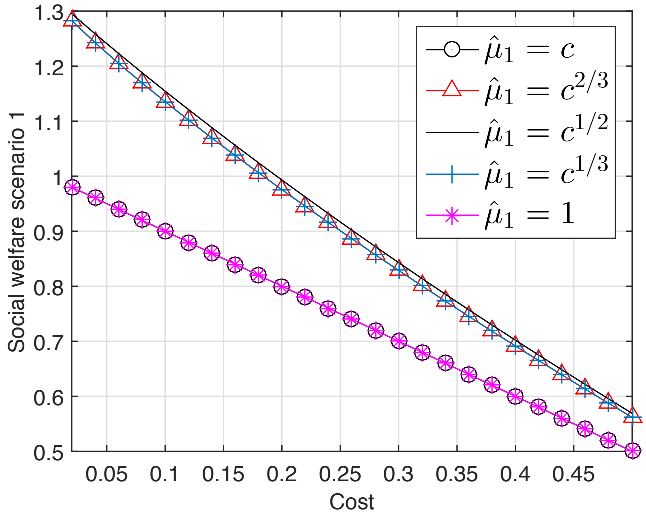

Fig. 2. Total expected payoff for the distributed case without signaling.

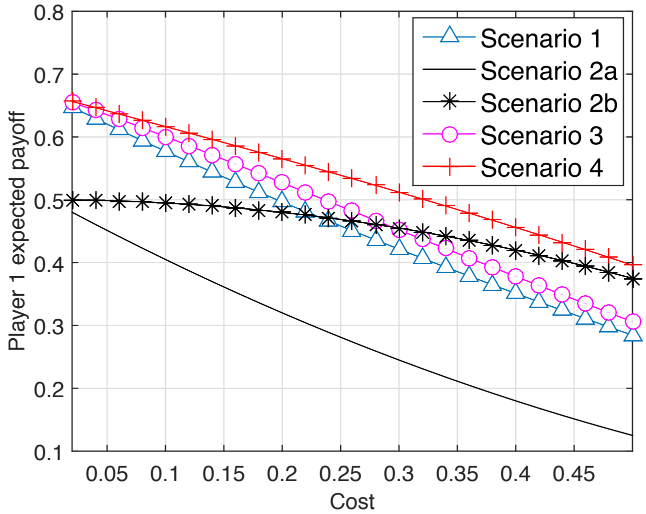

Fig. 4. Player 1 expected payoff considering the NE $(c^{\frac{1}{2}}, c^{\frac{1}{2}})$.

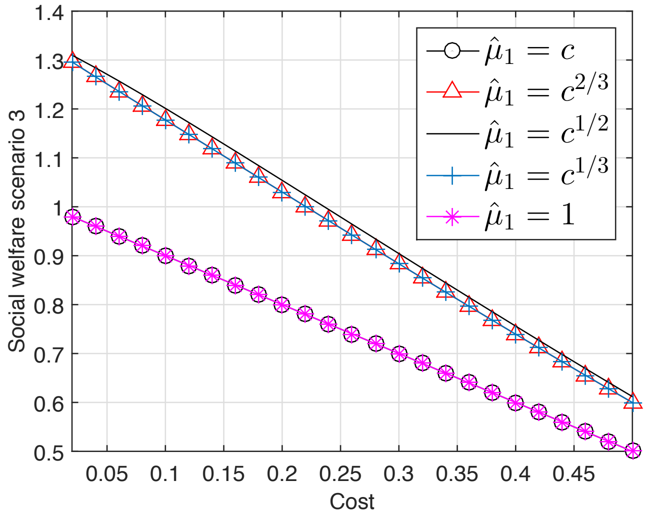

Fig. 3. Total expected payoff for the distributed case with signaling.

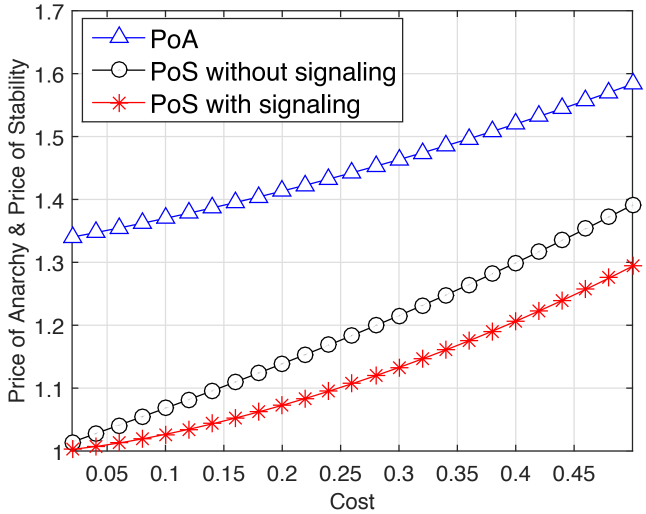

Fig. 5. Prices paid due to lack of cooperation/communication.

scenario 1 and scenario 3, (c, c) for scenario 4, and $(c, 1)$ and $(1, c)$ for scenario 2a and 2b.

The payoff for scenario 4 is an upper bound due to the assumption of complete knowledge, instead the payoff for scenario 2a is a lower bound; Fig. 4 confirms this theoretical result. Moreover, by looking at case 2b, for small values of c it is not advantageous to be always *inactive*, indeed the player could have a higher expected payoff being *active* and cooperating with the other player. Conversely, for higher values of c, case 2b gives a higher expected payoff with respect to scenario 1 and scenario 3. Another important remark is that the expected payoff related to scenario 3 is slightly higher than its counterpart in scenario 1; this is due to the addition of partial information through signaling. For symmetry reasons, the expected payoffs for player 2 are the same as those shown for player 1 in Fig. 4.

Given the expected payoff of each player in each of the cases considered, we evaluate how much does the social welfare decrease due to the lack of cooperation and communication. In Fig. 5 we show the PoA computed as the ratio between the social welfare found in the best case (scenario 4) and the social welfare in the worst case (scenario 2). An increase of the cost c implies an increase in the PoA, meaning that increasing the transmission/service cost the social welfare is most affected by non-cooperation in terms of payoff. Moreover, as c goes to 0, the PoA approaches $\frac{4}{3}$ (from above); this lower bound on the PoA is the same found for example in [16] in the evaluation of selfish routing, as well as other classical mathematical models involving self-interested users.

In the same figure, Fig. 5, we illustrate the price that the system has to pay because of the lack of communication, quantified through the PoS. The curves corresponding to PoS evaluation without or with signaling are computed as the ratio between the social welfare in scenario 4 and the social welfare reached considering the best NE in scenario 1 for the case without signaling and the social welfare obtained considering the best NE in scenario 3 for the case with signaling. Also in this evaluation, an increase of c implies a price increase, i.e., in this case, a higher PoS. Moreover, as it can be noted, for scenario 1, in which no communication between players is considered, the increase of the price is more pronounced respect to scenario 3. This means that PoS could be improved with signaling, for example through carrier sense mechanisms. Moreover, as it can be observed, in both cases in which we computed the PoS, as c goes to 0, PoS goes to 1 and its values are always lower than the PoA values.

VI. Conclusions

We considered a queueing system with candidate strategic servers that transmit packets paying an individual cost. We analyzed this system under different cooperation and communication assumptions and we applied game theory to evaluate the servers' behavior. We quantified the impact of the lack of cooperation and of communication among the servers on the social welfare.

Considering the individual expected payoff, as the service cost increases, the payoff of the player always *active* is a lower bound. The behavior of always staying *inactive* is not advantageous for small values of c. An increase of the cost implies a higher PoA. On the other hand, we obtained that also the lack of communication affects the social welfare; we found that an increase in the cost c involves an increases in the PoS. Moreover, comparing the PoS values obtained considering the scenarios with or without signaling, we found that the lack of communication involves a higher PoS.

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