Markov Models for Electric Vehicles: the Role of Battery Parameters and Charging Point Frequency

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***Abstract*—Electric vehicles represent a promising choice to decrease pollution and reducing fossil fuel consumption. However, their limited autonomy poses a challenge that prevents them from being suitable for many car users. Several practical solutions are sought to avoid this problem, in particular one may think of enhancing the battery capacity, reducing the charging time, or increasing the frequency of charging points. In this paper, we discuss how these aspects can be integrated by a proper Markov model, thereby offering a neat analytical solution to investigate all these problems. Some preliminary results are also shown to demonstrate the descriptiveness of the model. Further investigation can frame the proposed contribution within an optimization framework, maintaining an analytical context.**

***Index Terms*—Electric vehicles, Markov chains, stochastic models, renewable energy, range anxiety.**

I. Introduction

ALARMING pollution levels and increasing world-wide oil demand are two major circumstances that lead many nations to search for alternate energy sources and decrease carbon dioxide emissions, especially from car transportation, which makes use almost exclusively of internal combustion vehicles (ICVs). The problem is especially heavy in the United States, causing 30% of world greenhouse gases emissions, and also in Europe, where the 20-20-20 directive set a goal for the reduction of fossil fuel usage of 20% and replacement with renewable energies (increased by 20%) by year 2020 [1].

In this context, electric vehicles (EVs) are a technological solution not only to decrease pollution, but also to reduce usage costs for the end user. EVs can diminish carbon dioxide emissions by 50% and have four to six times lower per-km cost than traditional ICVs [2]. EVs can be integrated with the Internet of Things (IoT); this allows to prevent vehicle thefts, or to detect and possibly avoid traffic congestion, car accidents, and similar problems by means of monitoring systems based on the IoT technology [3]. Yet, some hurdles to a widespread diffusion of EVs are represented by limited autonomy, long recharging times, and scarceness of recharging points.

These aspects result in a limited applicability of EVs, so that a vicious circle arises. The diffidence of the customers towards their practicality especially due to their limited autonomy, a phenomenon known as *range anxiety* [4], leads to a niche market. As a result, public investments for a better service are discouraged, and this, in turn, further decreases the palatability of EVs for the average customer. Indeed, preliminary socio-logical studies have shown that EV users often limit the usage and do not fully exploit the vehicle autonomy, fearing they will not find a recharging point before the battery runs out of charge. Also, long recharging times are distasteful to the users, since they prevent an intense usage and independence in other daily activities.

From a modeling standpoint, there are several intervention points where these aspects can be tackled. One may think of improving the battery efficiency, either increasing its capacity, or making it faster to recharge, or both. At the same time, more charging points can be deployed, thus increasing their availability. With present-day technology, the cost-benefit relationship of these actions can be extremely variable, being also connected with the subjective perception of the end users. Investigating them with an adequate level of detail would involve several related economic and social aspects that are out of the scope of our study. Nevertheless, we aim at giving a preliminary contribution for what concerns the technical evaluation of these elements, by discussing the capability of stochastic models and, in particular, of Markov chains to integrate all of them in a comprehensive description.

Markov models have been used for quantitative evaluation of urban transportation practices. Especially, there are studies proposing a Markov model to characterize EV behaviors in urban mobility scenarios [5]. Inspired by these descriptions, we propose instead an enlarged Markov chain where also battery management can be integrated. In particular, we will give the following contributions. First of all, we will discuss the accuracy of the model, and show that even a loose time granularity may still correctly reflect average performance of the EV, especially for the metrics of interest related to range anxiety. Moreover, we show how the model can be properly tuned to investigate the impact of three key aspects mentioned above, i.e.: (i) autonomy of the battery; (ii) speed of battery recharging; (iii) frequency of recharging points.

The proposed Markov model can be further extended to several more advanced investigations, still conserving a fully analytical formulation that gives a powerful description, but without requiring an excessive computational cost. In particular, more advanced behavioral patterns of the EV users can be considered, as well as more traffic dynamics and intensities over the day. Also, complex battery effects can be introduced instead of the simple model where the battery charge increases when the EV is parked in a charging point and decreases as the EV moves, by considering leakage or stabilization effects. Finally, the model can be integrated within stochastic optimization frameworks [6], so as to devise intelligent algorithms for maximum energy efficiency and/or autonomy.

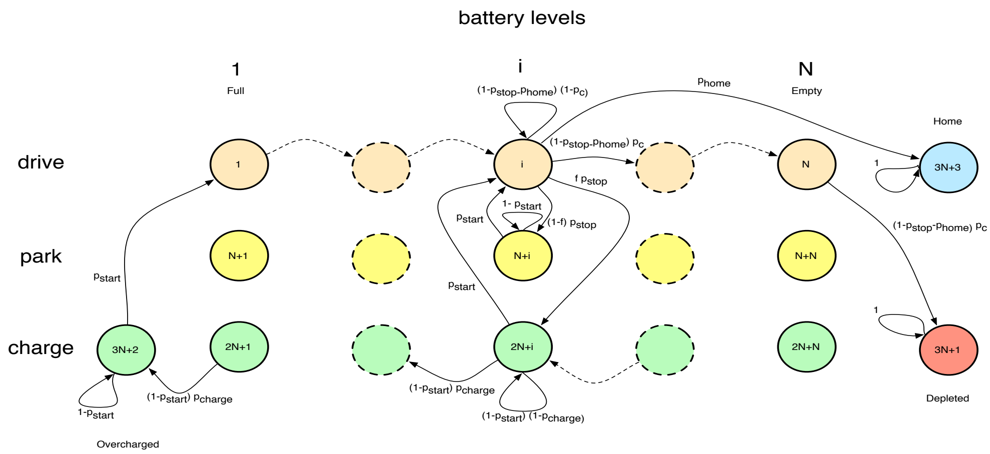

Fig. 1. The resulting system Markov Chain with $3N+3$ states.

II. System Model

To represent the daily activity of an EV, we consider the following simplified model. We assume that the EV drives through a urban scenario during a day, cyclically alternating being parked and on the move. The EV is considered to always leave from and return to the owner's home, where a charging point is available, in the morning and at the end of the day, respectively. Thus, at the beginning of each day cycle, its battery is fully charged. The duration of the entire daily activity is random, but its expected value is a working day period, so as to model the condition where the vehicle returns home at night after a determined average time. Also, some (but not all) of the parking area within the city have a charging point where the EV's battery can be replenished. To relate this analysis to the study of range anxiety, our objective is to study the probability of running out of all the charge in the battery before the day ends, called p_d. This probability depends, among other aspects, on the length of the chosen path for that day, the number of driving routines performed, the battery models, and the fraction of parking spots that are charging points. Also, we want to study especially the impact on p_d of increasing the charging speed or the battery capacity.

We model this scenario with a discrete Markov Chain, making a transition every Δt seconds, in which we consider that the EV can operate under three conditions: *Drive*, that is, the EV moves consuming battery; *Charge*, whenever the EV stops in a parking spot that is equipped with a charging point, so that the battery is recharged; and finally *Park*, if the EV stops in a parking position without any charging. Moreover, we describe the battery as an *energy queue*, having multiple possible charge levels, which are a quantization of the battery charge. We set N as the number of levels, which means we consider levels from 1 to N, with 1 representing the highest charge, i.e., 100% of the battery.

The transitions between the three conditions happen according to an independent underlying Markov process, for which several characterizations exist in the literature [5]. Note that this part of the model can be enlarged with additional conditions describing different ways of operating the EV, but such an extension would add little to the description made above. Thus, we can define transitions from i j, with $i, j \in \{\mathrm{D}, \mathrm{C}, \mathrm{P}\}$, where D, C, P stand for *Drive*, *Charge*, and *Park*, respectively. These three conditions further define macro-blocks of the Markov chain, in which we combine the condition (D, C, or P) with the charge level k.

It is worth noting that the charge level can only increase (or otherwise stay constant) if the condition is C; conversely, it can only decrease (or again stay constant) in condition D. When the system condition is P, we assume that neither discharge nor charge of the battery take place. It would be possible to also include other effects, such as charge balancing or leakage, but their effect is marginal with respect to charge and discharge caused by actual stationing at a charging point and driving, respectively, so we leave their inclusion for future work.

Under these assumptions, the Markov Chain would consist of $3N$ states, but we consider three additional states: the *Depleted* state identifies the situation in which an EV is in condition D with an empty battery and is therefore forced to stop, a condition that causes serious dissatisfaction to the user; the *Overcharged* state, representing a scenario in which the EV is in condition C with the battery fully charged, and therefore may have a higher probability of leaving the charging spot; and finally, the *Home* state, which corresponds to the situation in which the EV eventually arrives home at the end of the day if its operation is not terminated sooner by encountering the *Depleted* state. In other words, the *House* state describes the service termination for the day in the ideal usage conditions. From the modeling stand point, it is just one of the possible exit states from a D condition, which corresponds to the EV heading back home instead of temporarily parking. Thus, the Markov chain has $3N+3$ states, with *Depleted* and *Home* being absorbing states. Fig. 1 shows the resulting diagram of the Markov chain and its transitions.

The *Depleted* state is considered to characterize the range anxiety, since the owner of the EV would like to complete its daily route without being absorbed into it. For this reason, since the chain is ultimately absorbed by either the *Depleted* or the *Home* state, we define as p_d the probability that the former occurs, whereas $1 - p_d$ describes the probability of ending in the *Home* state, as is desirable.

For the sake of simplicity, we consider the transitions among the system conditions to be independent of the EV charge level. In other words, we assume that the driver is actually oblivious to the state of the battery, which does not influence his/her behavior. It would be easy to improve the description of the model by considering a higher likelihood of moving towards a charging point when the charge level is low. Such a characterization would surely be possible within a Markov approach akin to our model, but it would require some assumptions on the driver's behavior (e.g., his/her risk aversion and/or the desire to avoid the *Depleted* state as much as possible), which in turn would need some supporting experimental data. Since all these characterizations are related to the range anxiety of the users, we prefer to keep this concentrated in the simple evaluation of p_d with independence between the system condition and the battery charge level.

As a result, we can describe the entire system through few simple parameters. In particular, we define p_{start} as probability that when the system is in conditions C or P, the engine is turned on and a corresponding state with the same charge level, but under condition D, is entered. As said above, this value is the same regardless of the actual charge level of the battery. Also, we assume that this is also the exit probability from state *Overcharged*, even though it would be easy to change this, for example, by assuming that a fully charged EV leaves the charging spot immediately with probability 1.

Quite similarly, we define the probability that the EV leaves condition D as a constant term p_{stop}; in this case, the system condition becomes P or C, which in turn depends on how broad is the fraction f of electrified parking. We assume that charging stations are uniformly distributed across the areas where the EV can park, thus condition D is left towards C with probability fp_{stop}, and towards P with probability $(1-f)p_{\text{stop}}$. However, another exit options exists from condition D, which is the probability to enter the absorbing *Home* state, which happens with probability p_{home}. Note that this parameter can be set depending on the desired average duration of a daily cycle not interrupted by absorption to the *Depleted* state; in particular, in the following we chose it so that the EV stays out for an average of 12 hours. Finally, the two remaining parameters are the probabilities that the EV battery charge decreases or moves to a higher level, things that may happen when the system condition is D or C, and which are denoted by symbols p_{c} and p_{charge}, respectively.

Such a Markov chain can be solved very easily through standard stochastic process analysis to find, among other metrics of interest, the value of p_d. The number of states N does not heavily impact on the complexity of the solution, as will be further discussed in the next section. Technically speaking, this happens because the transition matrix related to the Markov model is sparse; indeed, the process can be seen as a Quasi-Birth-and-Death process similar to those in [7]. In the following, we discuss the application of our model and the extraction of p_d as a range anxiety analysis. We preliminary investigate the quantization granularity required to have an accurate model, and afterwards we show that our proposed model is able to keep into account the impact of battery capacity, recharge speed, and frequency of the charging spots.

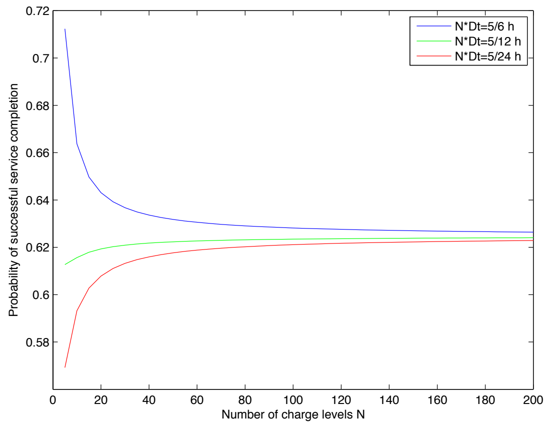

Fig. 2. Probability of successful service completion within the day (i.e., the battery is not fully depleted), varying discretization parameters Δt and N.

III. Discussion

First of all, we consider the impact of quantization parameters in the model proposed in the previous section. The model accuracy heavily depends on the choice of Δt and N, which do not refer to real characteristics of the EV system, but are just chosen so as to make the model tractable. Also, in order to have a better understandable model, we introduce further parameters that characterize the system more descriptively, as opposed to the transition probabilities of the Markov chain.

The chosen parameters are the EV autonomy in km from a full battery, denoted as A; the average recharge time, denoted as T_{charge}, to reach the state *Overcharged* from an empty battery assuming no change of condition occurs; the average length of the paths that the EV drives and their average number within a day, denoted as L and n, respectively; the duration of the daily hours, denoted as H; the average EV speed v.

In particular, we can introduce $H^* = H(1 - \frac{Ln}{Hv})$ as the time fraction in which the EV is not driving. Thus, we can write the following relationships, which directly follow from their definitions.

$$\begin{aligned} p_{\text{start}} &= \frac{n\Delta t}{H^*} & p_{\text{stop}} &= \frac{v\Delta t}{L}\frac{n-1}{n} \\ p_{\text{home}} &= \frac{v\Delta t}{L}\frac{1}{n} & p_{\text{c}} &= \frac{v\Delta t}{A/N} \\ p_{\text{charge}} &= \frac{\Delta t}{T_{\text{charge}}/N}. \end{aligned}$$

In the following, if not specified otherwise, we assume these values, that can be seen as a reasonable choice: $A = 100$ km, $T_{\text{charge}} = 2$ h, $L = 25$ km, $n = 8$, $H = 12$ h, $v = 50$ km/h, $f = 0.5$. These are sample values useful to give a proof of concept for the model; more realistic results can be used if available. For what concerns the evaluations of the quantization accuracy, similar results have actually been obtained in a wide range of parameters. Thus, we can plot in Fig. 2 the probability of successful service completion, in other words, the probability that the EV at the end of the day is not found in the *Depleted* state, i.e., $1 - p_d$. This is shown by considering different choices of Δt and N. Regardless of the actual value of the metric, from the figure it is evident that, when the number of levels N is sufficiently

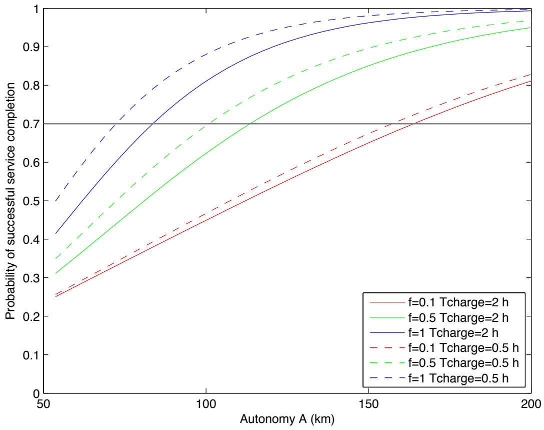

Fig. 3. Probability of successful service completion within the day (i.e., the battery is not fully depleted), varying system parameters A, f, and T_{charge}.

high, granularity in time only has a marginal effect. This is likely to happen because the considered time interval of a daily cycle is sufficiently large to make a time quantization interval of several seconds acceptable. Conversely, from the figure it appears that a sufficiently high value of N is required. As a result, in the following we use $N = 100$ and $\Delta t = 15$ seconds.

Now, we explore some building aspects of the model in more depth. The impact of the battery capacity on the range anxiety is related to the width of the Markov chain: the higher the number L of battery levels, or conversely, the slower the transitions towards the *Depleted* state, the less likely that the EV usage unsuccessfully ends in that state. An increase of the charge speed implies considering a stronger transition probability towards states with higher energy levels in condition C. Finally, another relevant element is the frequency of charging points, which also directly captured by our model. We also stress that these aspects all have a direct translation into three system parameters, namely, the autonomy A, the time to charge T_{charge}, and the parameter f.

To show a sample result, in Fig. 3 we report the probability of successful service completion $1 - p_d$ as a function of the battery autonomy A, but also considering different values of f and T_{charge}. One can notice that A is the key factor to avoid range anxiety, since a reasonably high probability of service completion (in the figure, we highlighted a threshold of $1 - p_d = 70\%$) can be achieved only if A is large enough. However, also the frequency of charging points impacts on this result, since if f is very low, the required autonomy might be too high: for example, $f = 0.1$ causes $1 - p_d$ to be below 70% unless A is very large.

Finally, we also focus on the impact of T_{charge}. From the considered model, this impact is less relevant, but still significant. According to the figure, a faster charging cycle can decrease the required autonomy by 10% or more; however, this happens only when f is high enough. This happens because a faster charge can keep battery depletion sufficiently far even in the cases where the EV stops at the charging point only for a short while.

In addition, we must consider that the Markov chain presented in this paper has only absorption to state *Depleted* as a (large) penalty, i.e., total failure. However, we can extend this point by introducing a reward process over the Markov chain, according to which the EV could cumulate a payoff depending on the condition it is in. In this case, we can consider an additional state, in which we force the vehicle's charging; this state involves no reward for the EV. More in general, several other investigations can exploit the proposed model, either expanding it, or including experimental evaluations to derive meaningful system parameters.

IV. Conclusions

We proposed a Markov chain for assessing and possibly improving the performance of EVs, with particular reference to the evaluation of their autonomy and the resulting range anxiety in the customers. Our chain builds up on a behavioral model of the EV, including possible actions such as "drive" and "park," as well as the battery status modeled as an energy queue. In particular, we showed how the model can easily integrate several elements of interest in the performance evaluation, namely the battery capacity, the density of the charging points, and the speed of charge for the battery.

As possible extension of this work, we could consider a relationship between the behavior of the users in the EVs and the charge level of the battery through a Markov decision process (MDP). We can also take into account different conditions of motion; in particular, we can consider multiple states related to various motion kinds, each of which could have different levels of battery charge, and also possibly a different reward in an MDP. Moreover, in our analysis we did not treat situations in which a deterioration of the battery or some malfunctions could imply a decrease in the EV autonomy over time. In future works, we can take care of these aspects considering a longer timescale. Furthermore, some additional battery effects (such as leakage) can be considered by defining other parameters to describe the probability to move to lower level of charge even under *Park* conditions. Finally, an optimization framework for this reward collection can be formulated [6], so as to transparently implement intelligent algorithms for battery management.

References

[1] P. Richardson, D. Flynn, and A. Keane, "Optimal charging of electric vehicles in low voltage distribution systems," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 268–279, 2012.

[2] H.Y. Mak, Y. Rong, Z.J.M. Shen, "Infrastructure planning for electric vehicles with battery swapping," *Manage. Sci.*, vol. 59, pp. 1557–1575, 2012.

[3] Z. Liu, W. Tao, L. Jiang, and C. Zhu, "Design and application on electric vehicle real-time condition monitoring system by Internet of Things technology," *Proc. IEEE Int. Conf. on Softw. Eng. and Serv. Sc. (ICSESS)*, pp. 744–747, 2014.

[4] S. Skippon, M. Garwood, "Responses to battery electric vehicles: UK consumer attitudes and attributions of symbolic meaning following direct experience to reduce psychological distance," *Transp. Res. Part D*, vol. 16, pp. 525–531, 2011.

[5] F. J. Soares, J. A. Peças Lopes, P. M. Rocha Almeida, C. L. Moreira, L. Seca, "A stochastic model to simulate electric vehicles motion and quantify the energy required from the grid," *Proc. Power Systems Computation Conference*, Stockholm, Sweden, 2011.

[6] E. B. Iversen, J. M. Morales, H. Madsen, "Optimal charging of an electric vehicle using a Markov decision process," *Applied Energy*, vol. 123, pp. 1–12, 2014.

[7] L. Badia, "On the impact of correlated arrivals and errors on ARQ delay terms," *IEEE Trans. Commun.*, vol. 57, no. 2, pp. 334–338, 2009.