Non Orthogonal Spectrum Sharing and Scheduling Strategies in Next Generation Mobile Networks

Francesco Guidolin, Luca Anchora, Leonardo Badia, and Michele Zorzi
Dept. of Information Engineering, University of Padova, via Gradenigo 6/B, 35131 Padova, Italy
email: {`francesco.guidolin,luca.anchora,badia,zorzi`}`@dei.unipd.it`

***Abstract*—Spectrum sharing has been recently proposed as a promising paradigm to improve the efficiency of resource usage in next generation mobile networks. In particular, non orthogonal spectrum sharing allows the operators to re-use the available frequencies at the cost of higher interference at the receivers. In this paper, we mathematically analyze the performance of this technique and how it is statistically related to the channel coefficients. Moreover, we compare different kinds of schedulers that exploit various aspects of non orthogonal spectrum sharing. Finally, the resulting system performance are assessed, at first through an exact statistical framework and then by simulating the schedulers in an LTE scenario with the open-source network simulator ns3.**

***Index Terms*—Next Generation Networking; MIMO; 4G mobile communication; statistical distributions; interference suppression; scheduling algorithm; computer simulation.**

I. INTRODUCTION AND RELATED WORKS

The increasing demand for transmission capacity in NGMN (Next Generation Mobile Networks) has pushed the operators to develop new paradigms of cooperation to provide high Quality of Experience (QoE) to the end users. The NGMN Alliance [1] identified the increase of spectral efficiency and the reuse of existing infrastructure as key requirements for the wireless networks of the future. In this paper, we focus on the downlink side of the Long Term Evolution (LTE) technology of the Universal Mobile Telecommunication System (UMTS) [2], where user multiplexing is based on Orthogonal Frequency Division Multiple Access (OFDMA) and multi-antenna technologies are introduced, so as to realize a Multiple Input Multiple Output (MIMO) system. Exploiting those features of LTE and properly designing the physical and the medium access layers allows different operators to jointly use the spectrum resources. This scenario has been investigated by the EU-funded project SAPHYRE [3] with the aim to quantify the gain obtained by sharing resources from the points of view of technology and business.

The simultaneous usage of the spectrum resources belonging to the operators can be exploited orthogonally to increase the multi-user diversity of the system, thereby improving the resource utilization efficiency [4]. In [5], this very approach is used to minimize the cell blocking probability by using the sharing frequencies to enlarge the available bandwidth. Orthogonal sharing is relatively simple to implement, but only achieves a relevant gain in asymmetric scenarios, i.e., whenever the amounts of traffic in the operators buffers are unbalanced. In fully loaded scenarios, the gain is given only by the increased multi-user diversity, and it results marginal if the number of users is large [6].

A further improvement can be obtained by exploiting non-orthogonal spectrum sharing (NOSS) that allows the operators to use the shared spectrum resources at the same time. This configuration allocates multiple users to the same frequency simultaneously, thereby causing a degradation of the Signal-to-Interference-plus-Noise-Ratio (SINR) at the intended receivers. The interferences have to be controlled through the use of multiple antenna at the BS and proper mitigation techniques, such as beamforming [7].

Moreover, the sharing paradigm can be extended to the infrastructure through the use by the operators of the same communication point (IS-NOSS) with further improvement in terms of capital and operational expense (CAPEX, OPEX) costs [8]. In both cases the performance can be improved with the combined use of scheduling algorithms that consider the users related to the different operators as a unique pool, so as to exploit their channel characteristics, or to preserve the priority of the spectrum owner when selecting the users. As illustrated in Fig. 1, colocation and non-colocation of the base stations (BSs) pertaining to the operators result in two different arrangements of the channel coefficients. These configurations can be exploited by the scheduler for user selection in each resource block.

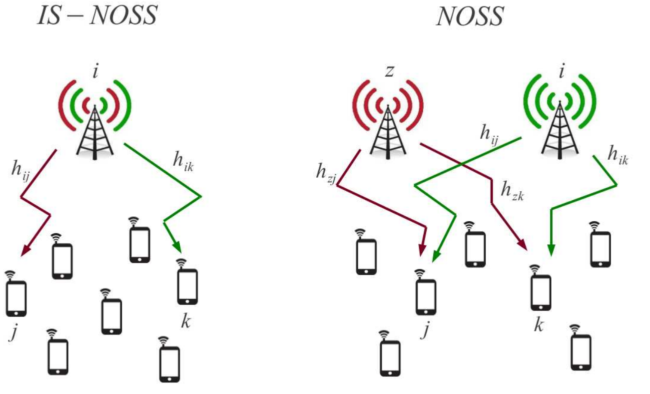

Fig. 1: IS-NOSS and NOSS scenarios

The paper presents the following contributions.

First, to characterize the degradation of the SINR in the NOSS scenarios we introduce a *novel performance indicator* called the Interference Suppression Ratio parameter (ISR) defined as the ratio among the the SINR perceived in the NOSS case and Signal to Noise Ratio (SNR) perceived when orthogonal scheduling is applied.

Moreover, we define *three different schedulers* that exploit in different ways the channel characteristics and the way the resources are shared, and we compare their performance through a statistical analysis of the ISR obtained.

The paper is organized as follows. Section II gives some background on beamforming techniques and their implementation in NGMNs. Section III presents an analytical study of the NOSS systems and the related scheduler algorithms. In Section IV the performance of the algorithm is evaluated via simulation. Finally, conclusions are drawn in Section V.

Remark: Throughout the paper, we use boldface letters for vectors, we denote with $\|\cdot\|$ the norm of a vector and with $(\cdot)^H$ the conjugate transpose.

II. BACKGROUND

We consider a Multi-Input Single-Output (MISO) system where the BSs use multiple antennas in transmission while the users are equipped with a single antenna in reception. Moreover, we assume a perfect knowledge at the BSs of the channel coefficients as a column vector $\mathbf{h}_{ij}$, where indices i and j refer to the BS and the user, respectively. The use of multiple antennas in transmission permits to spatially steer the power to the receiver according to linear precoding vectors $\mathbf{w}_{ij}$ determined through the channel coefficients [10].

If a spectrum resource is used exclusively by one user, which is refer to as Orthogonal Spectrum Sharing (OSS), the linear precoding vector $\mathbf{w}_{ij}$ can be computed to maximize the useful signal level perceived by the user in the absence of interference through the Maximum-transmission Ratio Technique (MRT) [11]:

$$\mathbf{w}_{ij}^{(mrt)} = \frac{\mathbf{h}_{ij}}{\|\mathbf{h}_{ij}\|} \tag{1}$$

If instead a spectrum resource is shared non-orthogonally (either NOSS or IS-NOSS) among the operators the MRT techniques is inefficient due the large amount of interference created by the BSs to the users. In this scenario, it is more convenient to exploit the cooperation among the two operators. The Zero Forcing (ZF) approach [12] permits to cancel the interferences generated by the other link computing the unit-norm beamforming vector that is orthogonal to the channel of the second user, and which at the same time maximizes the product $|\mathbf{w}_{ij}^H\mathbf{h}_{ij}|$. Thus, we can determine the vector $\mathbf{w}_{ij}$ for ZF beamforming as

$$\mathbf{w}_{ij}^{(zf)} = \frac{\mathbf{P}_{\mathbf{h}_{ik}}\mathbf{h}_{ij}}{\|\mathbf{P}_{\mathbf{h}_{ik}}\mathbf{h}_{ij}\|} \tag{2}$$

where $\mathbf{P}_{\mathbf{h}_{ik}}$ is defined as:

$$\mathbf{P}_{\mathbf{h}_{ik}} \triangleq \mathbf{I} - \mathbf{h}_{ik}(\mathbf{h}_{ik}^H\mathbf{h}_{ik})^{-1}\mathbf{h}_{ik}^H$$

and k is the index related to the other user that is sharing the same spectrum resource.

The SNR perceived by the user in the non spectrum sharing case ($SNR_j^{(nsh)}$) is then

$$SNR_j^{(nsh)} = \frac{p_{ij}|\mathbf{w}_{ij}^{(mrt)H}\mathbf{h}_{ij}|^2}{\sigma^2} \tag{3}$$

where p_{ij} is the power transmitted to user j and σ^2 is the noise power. In the NOSS case and in the IS-NOSS case the values of the SINR perceived by the user j ($SINR_j^{(noss)}$, $SINR_j^{(isnoss)}$) are, respectively,

$$SINR_j^{(noss)} = \frac{p_{ij}|\mathbf{w}_{ij}^{(zf)H}\mathbf{h}_{ij}|^2}{\sigma^2 + p_{zj}|\mathbf{w}_{zk}^{(zf)H}\mathbf{h}_{zj}|^2} \tag{4}$$

$$SINR_j^{(isnoss)} = \frac{p_{ij}|\mathbf{w}_{ij}^{(zf)H}\mathbf{h}_{ij}|^2}{\sigma^2 + p_{zj}|\mathbf{w}_{zk}^{(zf)H}\mathbf{h}_{ij}|^2} \tag{5}$$

where z is the index related to the other BS that is sharing the same spectrum resource. From (5) we can notice how the precoding matrix cancels the interference at the receiver but also reduces the power of the useful signal. This degradation is due to the non-perfect orthogonality among the channel matrices used for the construction of the beamforming matrices.

III. ANALYTICAL EVALUATION

To study the degradation of the SINR perceived by the user in the NOSS case in relation with the SNR obtained in the OSS scheme we define the parameter ISR of user j as:

$$ISR_j = \frac{SNR_j^{(sh)}}{SNR_j^{(nsh)}} \tag{6}$$

where $SNR_j^{(sh)}$ can be considered as $SNR_j^{(noss)}$, see (4), or $SNR_j^{(isnoss)}$, see (5).

We introduce another parameter, the Degree of Orthogonality (ρ) that describes the compatibility of the users which can be selected to share the same spectrum resource assigned by BS i related to their channel coefficients:

$$\rho_{jk} = 1 - \frac{|\mathbf{h}_{ik}^H\mathbf{h}_{ij}|}{\|\mathbf{h}_{ik}\|\|\mathbf{h}_{ij}\|} \tag{7}$$

where j and k are the indices related to the users.

Note that a small ρ_{jk} represents an inefficient coupling among the users while as $\rho_{jk} \to 1$ the losses due to the simultaneous usage of the frequency resources are reduced.

As demonstrated in (8), it is possible to express (6) as a function of the coefficients ρ_{jk}, which gives

$$ISR_j = 1 - (\rho_{jk} - 1)^2 \tag{9}$$

Through (9) it is possible to obtain the statistical behavior of ISR from the probability distribution of ρ, that in turn depends on the choice of the scheduler. Therefore, we need to consider which scheduling policies can be adopted to select the users that share the allocation.

$$ISR_j = \frac{\frac{p_{ij}|\mathbf{w}_{ij}^{(zf)H}\mathbf{h}_{ij}|^2}{\sigma^2+p_{zj}|\mathbf{w}_{zk}^{(zf)H}\mathbf{h}_{zk}|^2}}{\frac{p_{ij}|\mathbf{w}_{ij}^{(mrt)H}\mathbf{h}_{ij}|^2}{\sigma^2}} \simeq \frac{|\mathbf{w}_{ij}^{(zf)H}\mathbf{h}_{ij}|^2}{|\mathbf{w}_{ij}^{(mrt)H}\mathbf{h}_{ij}|^2} = \frac{\left|\left(\frac{\mathbf{P}_{\mathbf{h}_{ik}}\mathbf{h}_{ij}}{\|\mathbf{P}_{\mathbf{h}_{ik}}\mathbf{h}_{ij}\|}\right)^H \mathbf{h}_{ij}\right|^2}{\left|\frac{\mathbf{h}_{ij}}{\|\mathbf{h}_{ij}\|}^H \mathbf{h}_{ij}\right|^2} = \frac{\left| \frac{\left(\left(\mathbf{I}-\mathbf{h}_{ik}\left(\mathbf{h}_{ik}^H\mathbf{h}_{ik}\right)^{-1}\mathbf{h}_{ik}^H\right)\mathbf{h}_{ij}\right)^H}{\|\left(\mathbf{I}-\mathbf{h}_{ik}\left(\mathbf{h}_{ik}^H\mathbf{h}_{ik}\right)^{-1}\mathbf{h}_{ik}^H\right)\mathbf{h}_{ij}\|}\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|^2} \tag{8}$$

Since $\mathbf{h}_{ik}^H\mathbf{h}_{ik} = \|\mathbf{h}_{ik}\|^2$ and applying the conjugate transpose operator we obtain that:

$$\begin{aligned} ISR_j &= \frac{\left|\mathbf{h}_{ij}^H\left(\mathbf{I}-\frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H}{\|\mathbf{h}_{ik}\|^2}\right)\mathbf{h}_{ij}\right|^2}{\|\left(\mathbf{I}-\frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H}{\|\mathbf{h}_{ik}\|^2}\right)\mathbf{h}_{ij}\|^2} \cdot \frac{1}{\|\mathbf{h}_{ij}\|^2} \\ &= \frac{\left|\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - \mathbf{h}_{ij}^H\mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\|^2 - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} \cdot \frac{1}{\|\mathbf{h}_{ij}\|^2} = \frac{\left|-\mathbf{h}_{ij}^H\mathbf{h}_{ik}\right|^2}{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2} \cdot \frac{\left|\frac{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2}{-\mathbf{h}_{ij}^H\mathbf{h}_{ik}} + \mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}}{\|\mathbf{h}_{ik}\|}\|^2} \\ &= (\rho_{jk}-1)^2 \cdot \frac{\left|\frac{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2}{-\mathbf{h}_{ij}^H\mathbf{h}_{ik}} + \mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \frac{\mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}}{\|\mathbf{h}_{ik}\|}\|^2} = (\rho_{jk}-1)^2 \cdot \left|\frac{\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2}{\mathbf{h}_{ij}^H\mathbf{h}_{ik}} - \mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2 \cdot \frac{\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} \\ &= (\rho_{jk}-1)^2 \cdot \frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - |\mathbf{h}_{ik}^H\mathbf{h}_{ij}|^2\right)}{|\mathbf{h}_{ij}^H\mathbf{h}_{ik}|^2} \cdot \frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - |\mathbf{h}_{ik}^H\mathbf{h}_{ij}|^2\right)'\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} \\ &= (\rho_{jk}-1)^2 \cdot \left(\frac{1}{(\rho_{jk}-1)^2} - 1\right) \cdot \frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - \left|\mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2\right)'\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} = 1-(\rho_{jk}-1)^2 \end{aligned}$$

since:

$$\frac{\left(\|\mathbf{h}_{ij}\|^2\|\mathbf{h}_{ik}\|^2 - \left|\mathbf{h}_{ik}^H\mathbf{h}_{ij}\right|^2\right)'\|\mathbf{h}_{ik}\|^2}{\|\mathbf{h}_{ij}\|\mathbf{h}_{ik}\| - \mathbf{h}_{ik}\mathbf{h}_{ik}^H\mathbf{h}_{ij}\|^2} = 1$$

We call Max SNR scheduler (M-SNR) the scheduling policy where the allocation is based on the SNR perceived by the users in the case of no-sharing without considering the ρ parameter. In particular, for every spectrum resource the operators select from the overall pool of users those with the best SNR, exploiting the multi-user diversity derived from a larger number of users.

Assume a unit-variance Rayleigh fading, i.e., $\mathbf{h}_{ij} \sim CN(\mathbf{0}, \mathbf{I})$, where $\mathbf{0}$ is an all-zero matrix and $\mathbf{I}$ is an identity matrix; thus, the cumulative distribution function (cdf) of ρ is given by the regularized incomplete beta function $I_x(\alpha, \beta) = B_x(\alpha, \beta)/B(\alpha, \beta)$, where $B_x(\alpha, \beta)$ is the incomplete beta function and $B(\alpha, \beta)$ is the (complete) beta function [13]. The shape parameters α and β are obtained by simulation; we found that $\alpha = 1, \beta = 2$ are suitable values. This analysis can be extended to the case of a NOSS or IS-NOSS network for different scheduling policies.

We consider two other different schedulers in addition to the M-SNR scheduler: a max-ISR scheduler (M-ISR) and a priority scheduler (PS). The former considers, as in the M-SNR case, the users belonging to the two operators as a unique pool but the allocation is based only on the ISR parameter. In particular, for every spectrum resource the two users that mutually achieve the highest ISR are chosen without considering the quality of the channel perceived by the single user from the BS.

In the PS case the operators allocate their users separately, but the "owner" of the resource, i.e., the operator that would exploit the spectrum resource in case of no sharing, has priority over the other. This means that this operator selects the user in its pool with the best no-sharing SNR, then, the other operator chooses the user achieving the best ρ with the aim to preserve the utility of the selection performed by the owner operator. In a game theoretic context, this framework would be akin to that of a Stackelberg games [16].

In the IS-NOSS case, since the operators are sharing the same infrastructure, the channel coefficients and the ρ perceived from the operators are identical. Thus the probability density function (pdf) for the values of ρ achieved by the different schedulers is given, as explained in Appendix II, by the pdf of the maximum of n standard beta variables. This leads to

$$f_{isnoss}(x) = n\left(\frac{B_x(\alpha, \beta)}{B(\alpha, \beta)}\right)^{n-1} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)} \tag{10}$$

where n is equal to the number of possible pairs in the network, i.e.:

- $\binom{N_i+N_z}{2}$ for the M-ISR;
- N_i if the owner operator is i or N_z if the owner operator is z for the PS.

where N_i and N_z are the number of users for the operators i and z.

In the NOSS case the BSs are not colocated and the ρ perceived from the operators are different. Then the objective of the scheduler is not to maximize a single value of ρ but rather the sum of the values perceived by the base stations when a given pair is selected. According to the results presented in [14], the pdf of the sum of the ρ values achieved for the different scheduler in the NOSS case is (see Appendix II)

$$f_{noss}(x) = n\left(\frac{B_x(\alpha,\beta,a,b)}{B(\alpha,\beta)}\right)^{n-1}\frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}} \quad (11)$$

where n is equal to the number of possible pairs available in the network as per (10) and $\frac{B_x(\alpha,\beta,a,b)}{B(\alpha,\beta)}$ is a general beta variable with parameters $\alpha = \frac{7}{3}, \beta = \frac{14}{3}, a = 0, b = 2$. Also these numerical values are computed as per [14].

Figs. 2 and 3 depict the cdf of the ISR based on the ρ statistics described above in the case that 5 users per operator are active in the network. In the NOSS case, the value of the sum of ρ that is computed statistically is divided using a uniform distribution among the users. As expected, since the ISR perceived by the selected users is the same, the IS-NOSS configuration obtains better results than the NOSS configuration. The markers are related to the simulation results and will be explained in Section IV.

IV. SIMULATION RESULTS

In the first part of this section we validate the analytical results obtained in Section III by simulating the proposed scheduler algorithms. In particular, we simulate a 2×1 MISO system with 5 users per operator and a unit-variance Rayleight fading, i.e., $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Thus, we compute the ρ for each possible pair of users and select one pair according to the different scheduling policies; finally, the ISR of the scheduled users is computed. Figs. 2 and 3 compare the cdfs obtained by simulation and by statistical analysis. In the IS-NOSS case the statistical curve shows a very good fit with the simulation results, while in the NOSS case the fit is slightly degraded (but still acceptable) due to the assumption of uniform distribution of the ρ among the users.

To evaluate the performance of the scheduling algorithms proposed in an LTE system, we apply the traces obtained from the statistical analysis through a lookup table within the ns3 simulator [**?**].

The ns3 simulator is an open-source simulation environment for networking research based on object oriented programming and spanning the entire protocol stack, from the physical layer up to the application. Its modular structure allows to develop new algorithms within the protocol stack of different standard of communication. In particular, we extended an existing implementation for the evaluation of the OSS system performance [15] developed within the LTE module of ns-3. Starting from this framework, we developed the NOSS system and the algorithms of scheduling proposed.

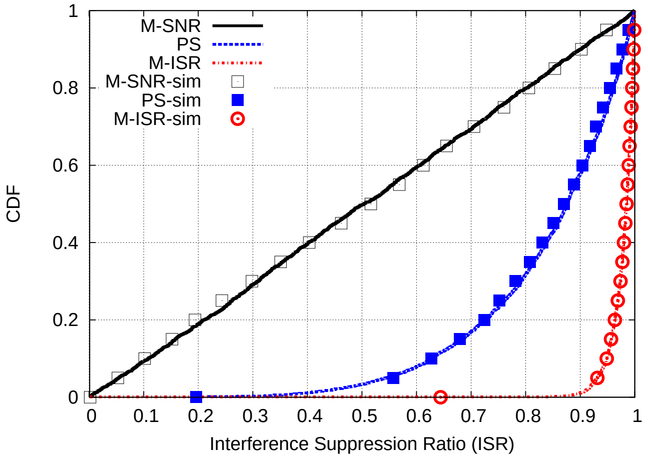

Fig. 2: ISR cdf and validation curves for the IS-NOSS case

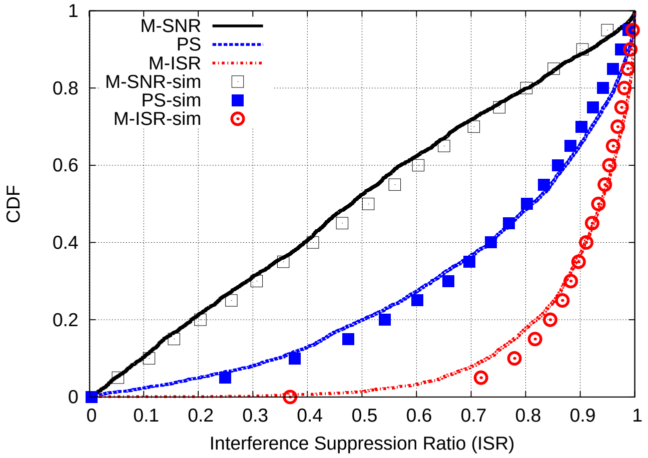

Fig. 3: ISR cdf and validation curves for the NOSS case

Through this approach it is possible to evaluate the impact of the ISR parameter together with the SNR level perceived by the users on the downlink spectral efficiency. We compare the results also with: (i) an optimal OSS scheduler that chooses for every RB the user in the overall pool with the best SNR and (ii) the optimal NOSS scheduler that selects the pair of users that achieve the best spectral efficiency for every RB.

The scenario consists of two BSs, which may either be colocated (IS-NOSS scenario) or non-colocated (NOSS scenario), and are equipped with two antennas, plus a variable number of mobile users equipped with one antenna and randomly (with uniform distribution) positioned in a cell with a radius of 3 km. The downlink bandwidth available per operator of 5 Mhz is divided into 25 RBs. Moreover, we assume a fully loaded scenario, i. e. the downlink traffic saturates each BSs buffer, so all the RBs are used during each frame. Each operator has a total downlink power of 43 dBm that is equally divided among the used RB. The detailed system parameters are reported in Table I.

If we compare the IS-NOSS and the NOSS cases, we notice

Parameter	Value
1-st sub-channel frequency	2110 MHz
Downlink Bandwidth per operator	5 MHz
Sub-Carrier Bandwidth	15 kHz
Doppler Frequency	60 Hz
Resource block bandwidth	180 kHz
Resource block carriers	12
Resource block OFDM symbols	7
BS downlink TX power	43 dBm
Noise spectral density	-174 dBm/Hz
Macrosopic Pathloss (distance R)	$128.1 + (37.6 \cdot \log(R))dB$
Shadow fading	log-normal, σ = 8 dB
Wall penetration loss	10 dB
Frame duration	10 ms
TTI (sub-frame duration)	1 ms
Target Bit Error Rate	5×10^{-5}
Cell coverage	3 km
BS distance (NOSS case)	50 m
Number of UEs per BS	1, 2, 5, 10, 20, 50

TABLE I: Main system parameter

that the former achieves slightly better results; thus, sharing the infrastructure can give to the operators a further improvement in terms of spectral efficiency for the scheduling algorithms that we investigated.

V. CONCLUSION AND FUTURE WORKS

We investigated the NOSS techniques through a statistical analysis of the ISR and a simulation analysis of the spectral efficiency obtained with the use of several scheduling techniques in an LTE network. NOSS appears to be a promising technique for the performance improvement in NGMN, and a joint user scheduling among the operators can give further improvements in terms of spectral efficiency. From the results obtained, it also appears that an additional sharing concerning the infrastructure can, depending on the scheduling algorithm, improve the overall performance of the network. As a possible extension of the present work, the same approach can be applied to further beamforming techniques, and also extended to scenarios with multiple cells.

APPENDIX I

Consider a set of n identically independent random variables. The cdf of the maximum value of those variables is given by:

$$
\begin{aligned}
&P\{\max(y_1, y_2, ..., y_n) < \alpha\} = \\
&P\{y_1 < \alpha, y_2 < \alpha, ..., y_n < \alpha\} = \\
&P\{y_1 < \alpha\}P\{y_2 < \alpha\}, ..., P\{y_n < \alpha\} = \\
&\prod_{i=1}^{n} P\{y_i < \alpha\} = [P\{y < \alpha\}]^n
\end{aligned}
$$

In our case, the cumulative distribution is described by a regularized incomplete beta function $I_x(a,b)$=$B_x(a,b)/B(a,b)$, so the related pdf can be obtained as

$$
\begin{aligned}
&\frac{\mathrm{d}\,[I(x))]^n\, tbh}{\mathrm{d}x} = \frac{\mathrm{d}\,[B_x(a,b)/B(a,b)]^n}{\mathrm{d}x} = \\
&n\left(\frac{B_x(a,b)}{B(a,b)}\right)^{n-1} \frac{\mathrm{d}B_x(a,b)/B(a,b)}{\mathrm{d}x} = \\
&n\left(\frac{B_x(a,b)}{B(a,b)}\right)^{n-1} \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}
\end{aligned}
$$

A similar approach can be used for the case of a cdf that is a general beta variable, i.e.,

$$
\frac{B_x(\alpha, \beta, a, b)}{B(\alpha, \beta)}.
$$

By repeating the same procedure applied above, (11) can be derived.

REFERENCES

[1] "Next Generation Mobile Networks Beyond HSPA & EVDO" *NGMN Alliance White Paper 3.0*, December 2006.

[2] LTE physical layer - general description, TS 36.201 (V9.1.0), 3GPP, March 2010.

[3] The Saphyre Project(FP7-ICT-248001), http://saphyre.eu.

[4] E. A. Jorswieck, L. Badia, T. Fahldieck, E. Karipidis, J. Luo, "Spectrum sharing improves the network efficiency for cellular operators," submitted to *IEEE Comm. Mag.*, 2013.

[5] B. Aazhang, J. Lilleberg, G. Middleton, "Spectrum sharing in a cellular system," *Proc. IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA),* pp. 355- 359, 30 Aug.-2 Sept. 2004.

[6] L. Anchora, L. Badia, H. Zhang, T. Fahldieck, J. Zhang, M. Szydelko, M. Schubert, E. Karipidis, and M. Haardt, "Resource allocation and management in multi-operator cellular networks with shared physical resources," *Proc. ISWCS 2012,* pp. 296-300, Aug. 2012.

[7] R. Mochaourab and E. A. Jorswieck, "Optimal beamforming in interference networks with perfect local channel information," *IEEE Trans. Signal Process.*, vol. 59, no. 3, pp. 1128-1141, Mar. 2011.

[8] J. N. Laneman and G. K. Womell, "Energy-efficient antenna sharing and relaying for wireless networks, *WCNC 2000 IEEE,* vol. 1, no. 7-12, 2000.

[9] L. Anchora, L. Canzian, L. Badia, and M. Zorzi, "A characterization of resource allocation in LTE systems aimed at game theoretical approaches," *Proc. IEEE CAMAD*, pp. 47-51, Dec. 2010.

[10] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Trans. on Signal Proc.*, vol. 56, no. 10, pp. 5292-5296, Oct. 2008.

[11] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications.* Cambridge, U.K.: Cambridge Uni. Press, 2003.

[12] E. G. Larsson and E. A. Jorswieck, "Competition versus collaboration on the MISO interference channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1059-1069, Sep. 2008.

[13] Y. Taesang and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE Journal on Selected Areas in Communications,* vol. 24, no. 3, pp. 528- 541, Mar. 2006.

[14] T. G. Pham and N. Turkkam, "Reliability of a standby system with beta-distributed component lives," *IEEE Transactions on Reliability*, vol. 43, no. 1, pp. 71-75, Mar. 1994.

[15] L. Anchora, M. Mezzavilla, L. Badia, and M. Zorzi, "A performance evaluation tool for spectrum sharing in multi-operator LTE networks", *Computer Communications,* Vol. 35, pp. 2218-2226, Nov. 2012.

[16] L. Canzian, L. Badia, and M. Zorzi, "Promoting cooperation in wireless relay networks through Stackelberg dynamic scheduling," *IEEE Trans. on Commun.*, vol. 61, no. 2, pp. 700–711, Feb. 2013.