Urban Hitchhiking

Marco Bressan and Enoch Peserico

Dip. Ing. Informazione, Univ. Padova, Italy
{bressanm,enoch}@dei.unipd.it o

Abstract. You are an urban hitchhiker. All drivers are willing to give you a ride, as long as they do not have to alter their trajectories to accommodate your needs. How (and how quickly) can you get to your destination? We analyze two scenarios, depending on whether hitchhikers have a global picture of who is going where through some information infrastructure, or only a local picture - i.e. they can only ask cars passing by where they are going.

1 Introduction

You are an urban hitchhiker. All drivers are willing to give you a ride, as long as they do not have to alter their trajectories to accommodate your needs. How (and how quickly) can you get to your destination? This is a fundamental question in the area of *dynamic ride-sharing*, which is attracting ever more attention due to soaring oil prices and increasing pollution concerns [1–5, 8, 9]. The answer (not surprisingly) depends mainly on whether you have a global picture of who is going where (most likely through some information infrastructure), or only a local picture - i.e. you can only ask cars passing by where they are going. We examine the two cases respectively in Section 4 and Section 3 after presenting, in Section 2, our city traffic model. Section 5 concludes the paper with a brief analysis of the significance of our results and of the many problems this preliminary work leaves open.

2 A simple city traffic model

There is a vast number of traffic models in the literature (see [6] for a review); however, most of them focus on congestion control and are too complex to be solved analytically at the city level. Our model is much simpler, although it is still sufficiently rich to produce non-trivial results and, we feel, to capture the essence of the problem.

We model the city and its periphery as a circle of radius R (see Figure 2), where the unit of space is the *contact distance* - the maximum distance at which a hitchhiker can signal a car "passing by" to stop and query it for information (indicatively $\approx$ 30 meters, the range of class 2 Bluetooth devices present on most of today's mobile phones and perhaps half a city block). The contact distance is

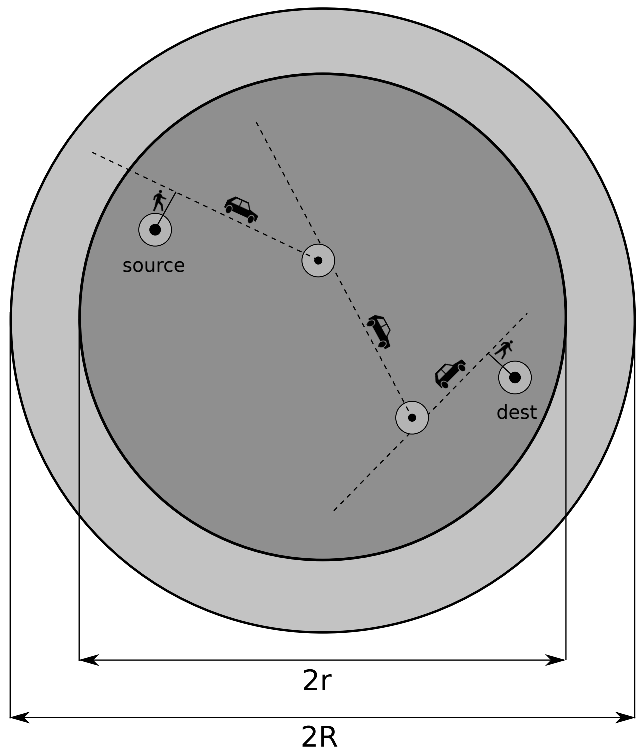

Fig. 1. The city (radius r) containing source and destination, and the city plus the periphery (radius $R > r$). Cars travel in a straight trajectory (dashed line) from a random source to a random destination, and the hitchhiker may hop into them if they pass within unit distance of him. The hitchhiker may also walk (solid line) towards the destination or towards a good ride.

also the minimum distance for which we assume that a hitchhiker may seek a ride instead of simply resorting to walking. We set the time scale so that traveling 1 unit of space takes a car 1 unit of time, and a pedestrian P units.

The city itself is a smaller, concentric circle of radius r. We only consider hitchhiking within the city. The periphery - the annulus around the city proper - simply serves as an abstraction for the sources and sinks of traffic outside of the city. Cars enter the traffic system of the city and its periphery at points chosen uniformly at random within the (larger) circle of radius R according to a Poisson process with intensity of $\frac{1}{G}$ cars per unit of area per unit of time; each car then heads in a straight line towards a destination also chosen uniformly at random. It is easy to verify that the process describing arrivals/departures of cars *in transit* through any unit circle is poissonian, with an intensity $\Theta(\frac{R}{G}) = \Theta(\rho)$ that is within a factor $1 - \frac{r}{R}$ for *any such area* within the city. The rest of the paper assumes $1 - \frac{r}{R} = \Theta(1)$, which is equivalent to the (realistic) assumption that a constant fraction of the city's traffic has a source or a destination outside the city proper.

The hitchhiker wants to move, within the city, from an arbitrary source to an arbitrary destination at distance $d \geq 1$ from the source. While a private car allows the hitchhiker to cover this distance in time d with only one leg, hitchhiking requires, in general, more time (and perhaps more legs). In the rest of the paper, we will often state results in terms of *effective speed*, i.e. the average speed of the hitchhiker from the source to the destination.

3 Hitchhiking with Local Information Only

This section examines the case of a hitchhiker equipped only with local knowledge - i.e. who can only query nearby cars (those within distance 1) about their destination. A strictly optimal strategy is beyond the scope of this work; we can, however, provide a simple strategy that is optimal in terms of effective speed (and among effective speed-optimal strategies, in terms of legs) within a small constant factor.

Let a ride be *good* if it can take the hitchhiker at least twice as close to as he currently is; and let it be *better* than another ride if it can take the hitchhiker closer to the destination. Consider the following three simple hitchhiking rules:

1. Initially, walk towards the destination taking the first good ride encountered.
2. When encountering a new potential ride, take it if and only if a) it is better than any ride seen so far and b) the current ride, if any, is no longer good.
3. As soon as the current ride is no longer making progress towards the destination dismount and start walking towards the destination.

We can prove that:

Theorem 1. *If the hitchhiker's current distance is* $d > \frac{1}{\rho}$*, he will reach within distance* $\frac{1}{\rho}$ *of the destination in* $O(d)$ *time (only* $O(\frac{1}{\rho})$ *of which spent walking) and* $O(1)$ *legs. If the hitchhiker's current distance is* d *with* $\frac{1}{P\rho} < d \leq \frac{1}{\rho}$*, he will* halve *his current distance in expected* $O(\frac{1}{\rho})$ *time and* $O(1)$ *legs, and will reach within distance* $\frac{1}{P\rho}$ *of the destination in* $O(\frac{log(P)}{\rho})$ *time and* $O(\log(P))$ *legs. If the hitchhiker's current distance is* $d \leq \frac{1}{P\rho}$*, he will reach the destination in time* $O(dP)$*.*

Proof. First of all note that, if the hitchhiker is not moving, the probability density (in time) of witnessing a previously unseen car pass by that will take him within distance δ of the destination is within a constant factor (see Section 2) of a function $p(\delta/d)$ that depends solely on the ratio between δ and the current distance d to the destination: $p = \Theta(\rho \cdot \frac{\delta}{d})$, where $\frac{\delta}{d}$ is (within a small constant factor) the ratio between the length of the circumference of radius d centered on the current position and passing through the destination and the length δ of its arc centered on the destination. The expected time to witness a ride which carries the hitchhiker within distance δ of the destination is then $1/p = O(\frac{1}{\rho} \cdot \frac{d}{\delta})$. The same holds, within a small constant factor, when the hitchhiker is walking, since the speed of cars relative to him will be between $1 + 1/P$ (for cars with the same trajectory but opposite direction) and $1 - 1/P$ (for cars with the same trajectory and the same direction).

This is no longer true while riding, since the probability density of witnessing a ride depends on the angle its trajectory makes with the trajectory of the current ride. In particular, the probability density of witnessing a ride with a trajectory at an angle $\alpha < \frac{\pi}{2}$ from that of the hitchhiker is $\rho\ sin(\alpha)$. Let x be the current distance between the hitchhiker and the point of the ride closest to the destination, d' be the distance of this point from the destination. The probability density of witnessing a ride that would take the hitchhiker within d'' of the destination depends on the sine of the angle formed by the current trajectory and the direction of destination (which for $x > 2d'$ can be approximated by d'/x) and the angle formed by the circle of radius d'' (which for $x > 2d'$ can be approximated by d''/x). Therefore, the hitchhiker witnesses approximately $\rho \cdot \frac{d''}{x} \cdot \frac{d'}{x}$ rides per time unit that would carry him within distance d'' of the destination. While the current ride is still good (i.e. for $d \leq x \leq 2d'$), the total number of witnessed rides that would carry within d'' of the destination is $\int_{d'}^{d} \rho \cdot \frac{d''}{x} \cdot \frac{d'}{x} dx = \Theta(\rho d'')$; and thus the best witnessed ride carries the hitchhiker within an expected distance $\Theta(1/\rho)$. Once the ride is no longer good, the hitchhiker (now at distance $\Theta(d')$ from the destination) has to wait or walk $\Theta(\frac{1}{\rho})$ time units in expectation to witness a ride better than any ride seen so far. Therefore, in expected $O(d)$ time and $O(1)$ legs, the hitchhiker reaches within distance $O(1/\rho)$ and, with an additional constant number of good legs (as we will prove now), he reaches within distance $1/\rho$ from the destination.

Once within distance $d = O(1/\rho)$ from the destination, the probability density of witnessing a good ride is $\Theta(\rho)$. Then the expected time to witness a good ride is $O(1/\rho)$, while the ride time is $\Theta(d) = O(1/\rho)$, and therefore wait time dominates travel time. This means that, while riding, the hitchhiker witnesses $O(1)$ good rides, and thus the best ride seen so far would carry him, in expectation, within a distance from the destination which is a constant factor smaller than the current distance. Therefore, the hitchhiker has to wait $O(1/\rho)$ time units in expectation to hop into the next ride, which halves its distance from the destination in $O(1/\rho)$ expected time and $O(1)$ expected legs. The hitchhiker can then reach within distance $\frac{1}{P\rho}$ of the destination in $\Theta(\log(P))$ legs and $O(\log(P)/\rho)$ time.

Once within distance $\frac{1}{P\rho}$ of the destination, walking to it requires time $1/\rho$, within a constant of that required to witness a ride.

In a nutshell, Theorem 1 says that a hitchhiker with only local information can expect to make progress towards the destination at a speed within a constant factor of that achievable with a private car, up to a distance $\frac{1}{\rho}$ from the destination - approximately the expected distance a car can travel before passing by another car. Within that range, the hitchhiker's effective speed drops proportionally with the distance to the destination, until at range $\frac{1}{P\rho}$ it reaches within a constant factor of walking speed.

It is natural to ask whether a hitchhiker aware of all car traffic - rather than only of that passing nearby - can reduce his travel time and/or the number of legs he will need. We prove this is indeed the case in Section 4.

4 Hitchhiking with Global Information

This section examines strategies for a hitchhiker armed with global knowledge of the position and destination of all cars that are currently moving. This could be the case, for example, when a central authority continuously gathers all the traffic information and provides directions to hitchhikers. It turns out that (unlike the case of local information) with global information two leg trips are always asymptotically optimal, and indeed even one leg trips are asymptotically optimal except for a relatively narrow interval of source-destination distances.

4.1 One-leg trips.

In these scenario, the hitchhiker attempts to reach his destination using at most a single ride. His strategy is extremely simple. Armed with global knowledge of car traffic, he can compute for each car the minimum time required to hop onto it (by walking and/or waiting), ride on it, and finally walk to the destination. He then chooses the car yielding the shortest time to the destination - unless, of course, simply walking directly to the destination is faster. We can prove the following:

Theorem 2. *For $d < \frac{1}{\sqrt{P\rho}}$ the hitchhiker moves at the pedestrian's speed $\frac{1}{P}$. For $\frac{1}{\sqrt{P\rho}} \leq d < \frac{P}{\sqrt{\rho}}$, the hitchhiker moves at an effective speed $\Theta(\frac{\sqrt[3]{d^2}}{\sqrt[3]{d^2}+\sqrt[3]{P^2/\rho}})$. For $d \geq \frac{P}{\sqrt{\rho}}$, the hitchhiker moves at an effective speed $\Theta(1)$.*

Proof. Assume the hitchhiker is willing to spend t time units walking or waiting around the source before hopping into a ride, and t time units walking to the destination after hopping off the ride. He can walk a distance $x \leq \frac{t}{P}$ from a in every direction, and then wait $t - x$ time units; thus he witnesses $\frac{t^2\rho}{P}$ distinct rides in expectation, and the fraction of rides that carry the hitchhiker to within distance t/P from the destination, ensuring him a walk time no greater than t, is proportional to $\frac{t/P}{d}$ in expectation. Therefore the expected number of such rides is $\frac{t^2\rho}{P}\frac{t/P}{d} = \frac{t^3\rho}{P^2 d}$; when this quantity is 1, which holds for $t = \sqrt[3]{\frac{P^2 d}{\rho}}$, we have at least a constant probability of witnessing a ride. This gives us the *critical time* $t_c = \sqrt[3]{\frac{P^2 d}{\rho}}$, i.e. the expected time to witness a useful ride.

We can now find the values of d which correspond to "phase transitions" in the effective speed. For d small enough, the critical time t_c dominates the time

dP required to walk to the destination: $\sqrt[3]{\frac{P^2 d}{\rho}} \geq dP$, which gives $d \leq \frac{1}{\sqrt{P\rho}}$. Below this distance, the hitchhiker can at best move at speed $\frac{1}{P}$ by walking.

For d large enough, the traveling time dominates the waiting time: $d \geq \sqrt[3]{\frac{P^2 d}{\rho}}$, which gives $d \geq \frac{P}{\sqrt{\rho}}$. Above this distance, the hitchhiker moves at an effective speed $\Theta(1)$.

Between the two thresholds, i.e. for $\frac{1}{\sqrt{P\rho}} \leq d < \frac{P}{\sqrt{\rho}}$, the effective speed grows as $\Theta(\frac{d}{d+t_c}) = \Theta(\frac{d}{d+\sqrt[3]{P^2 d/\rho}}) = \Theta(\frac{\sqrt[3]{d^2}}{\sqrt[3]{d^2}+\sqrt[3]{P^2/\rho}})$.

According to Theorem 2, as the distance between source and destination increases, the hitchhiker's expected speed grows from walking speed to within a constant factor of the unit speed achievable with a private car. In fact, we can prove a stronger result: as the distance increases, the hitchhiker's effective speed tends to 1 with probability that also tends to 1:

Theorem 3. *The hitchhiker moves at an effective speed* $\geq 1 - k\sqrt[3]{\frac{P^2}{d^2\rho}}$ *with probability* $\geq 1 - e^{-k^3}$.

Proof. The total time spent by the hitchhiker is the sum of the traveling time (which is approximately d) and the walking/waiting time (which is approximately t). When the former dominates the latter, the effective speed is approximately $\frac{d}{d+t} = 1 - \frac{t}{d}$ which, for $t = kt_c$, becomes $1 - \frac{k\sqrt[3]{\frac{P^2 d}{\rho}}}{d} = 1 - k\sqrt[3]{\frac{P^2}{d^2\rho}}$. According to the proof of the previous theorem, waiting a time t gives a probability of witnessing a ride of at least $1 - e^{-\frac{t^3\rho}{P^2 d}} = 1 - e^{-k^3\frac{t_c^3\rho}{P^2 d}} = 1 - e^{-k^3}$.

4.2 Two-leg trips.

In this scenario, the hitchhiker can employ up to two rides to reach the destination. Again, the strategy is very simple. Armed with global knowledge of car traffic, the hitchhiker computes for every pair of cars the shortest total time required to reach the first, ride on it, walk to/wait for the second car, ride on it and finally walk to the destination. It turns out that this does not reduce - compared to the single ride case - the critical distance $\frac{1}{\sqrt{P\rho}}$ at which the hitchhiker is better off walking. However, at distances greater than that, two-leg trips yield better effective speeds than one-leg trips. More specifically, we show that:

Theorem 4. *For* $d < \frac{1}{\sqrt{P\rho}}$ *the hitchhiker moves at the pedestrian's speed* $\frac{1}{P}$. *For* $\frac{1}{\sqrt{P\rho}} \leq d < \frac{\sqrt{P}}{\sqrt{\rho}}$, *the hitchhiker moves at an effective speed* $\Theta(\frac{d}{d+\sqrt{P/\rho}})$. *For* $d \geq \frac{\sqrt{P}}{\sqrt{\rho}}$, *the hitchhiker moves at an effective speed* $\Theta(1)$.

Proof. Assume the hitchhiker is willing to spend t time units walking or waiting around the source before hopping onto a car, t time units waiting for the second ride once he leaves the first, and t time units walking to the destination after dismounting from the second ride. As in the case of Theorem 2, the hitchhiker witnesses $\frac{t^2\rho}{P}$ distinct rides around the source in expectation, and a constant fraction of these carry him within a constant angle from the source-destination axis. Therefore the expected number of such rides is $\frac{t^2\rho}{P}$; when this quantity is 1, which holds for $t = \sqrt{P/\rho}$, we have at least a constant probability of witnessing a ride. This gives us the *critical time* $t_c = \sqrt{P/\rho}$, i.e. the expected time to witness a useful ride. Note that this holds also for a ride which travels towards the destination, and thus with constant probability there will be two rides which spatially intersect and give a feasible 2-leg solution.

We can now find the values of d which correspond to "phase transitions" in the effective speed. For d small enough, the critical time t_c dominates the time dP required to walk to the destination: $\sqrt{\frac{P}{\rho}} \geq dP$, which gives $d \leq \frac{1}{\sqrt{P\rho}}$. Below this distance, the hitchhiker can at best move at speed $\frac{1}{P}$ by walking. For d large enough, the traveling time dominates the waiting time: $d \geq \sqrt{\frac{P}{\rho}}$, which gives $d \geq \sqrt{P/\rho}$. Above this distance, the hitchhiker moves at an effective speed $\Theta(1)$. Between the two thresholds, i.e. for $\frac{1}{\sqrt{P\rho}} \leq d < \sqrt{P/\rho}$, the effective speed grows as $\Theta(\frac{d}{d+t_c}) = \Theta(\frac{d}{d+\sqrt{P/\rho}})$.

Again, we can easily prove that, as the travel distance grows beyond the minimum distance yielding effective speed $\Theta(1)$, the effective speed of the hitchhiker converges to 1 with probability that also converges to 1 - and this convergence is slightly faster than in the case of single rides:

Theorem 5. *The hitchhiker moves at an effective speed* $\geq 1 - k\sqrt[3]{\frac{P}{d^2\rho}}$ *with probability* $\geq 1 - e^{-k^3}$.

Proof. The total time spent by the hitchhiker is the sum of the waiting time t and the traveling time $\leq d(1+\alpha)$, where α is the angle that the trajectories of the legs form with the source-destination axis. We want $d\alpha \approx t$. This yields an effective speed of approximately $\frac{d}{d+t} = 1 - \frac{t}{d} = 1 - \frac{k\sqrt[3]{\frac{Pd}{\rho}}}{d} = 1 - k\sqrt[3]{\frac{P}{d^2\rho}}$. The expected number of rides within the angle is then $\frac{t^2\rho}{P}\frac{t}{d} = \frac{t^3\rho}{Pd}$, and the critical time to witness at least one becomes $t_c = \sqrt[3]{\frac{Pd}{\rho}}$; therefore, when $t = kt_c$, the probability of witnessing at least one such ride is $1 - e^{\frac{k^3 t_c^3 \rho}{Pd}} = 1 - e^{-3}$.

Figure 4.2 summarizes the results of this Section, plotting effective speed as a function of the distance between source and destination, depending on whether the hitchhiker is constrained to use at most one ride, or at most two rides. Note that, when global traffic information is available, using three or more legs cannot asymptotically improve effective speed: the time spent by the "two-leg" hitchhiker (see Theorem 4 is the sum of two terms — the first asymptotically equal to the time d necessary for a private car to reach the destination, and the other asymptotically equal to the time $\Theta(\sqrt{\frac{P}{\rho}})$ necessary (even with full knowledge of future car traffic) to find *any car at all*.

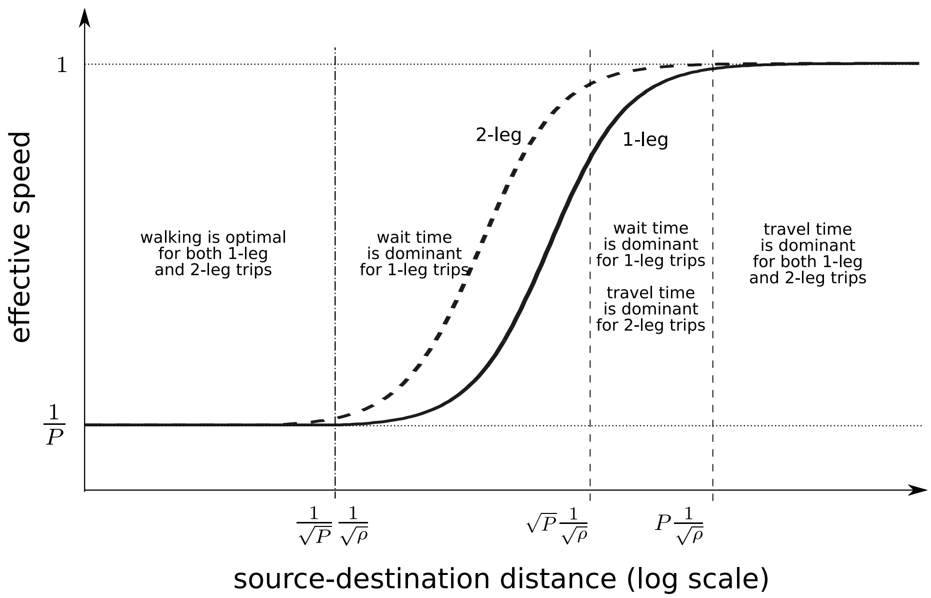

Fig. 2. The hitchhiker's expected effective speed as a function of the source-destination distance in the case of 1-leg trips (solid) and 2-leg trips (dashed).

5 Conclusions

Urban hitchhiking - assuming a wide base of collaborating drivers - can be extremely efficient, particularly for long distances. A centralized infrastructure is beneficial, but even without it a hitchhiker can advance towards the destination at an effective speed within a small constant factor of that achievable with a private vehicle, changing car only $O(1)$ times - at least until he reaches within a distance $\frac{1}{\rho}$ of the destination, i.e. within the average distance a car can drive before encountering another car. Below this threshold the effective speed of a hitchhiker with purely local information decreases proportionally to the distance to the destination, until it reaches walking speed, and car changes become more frequent.

An infrastructure providing information about all moving cars improves a hitchhiker's effective speed, allowing him to travel to the destination almost as fast as with a private vehicle - and, importantly, changing cars at most once

- as long as he starts at a distance from the destination that is at least $\sqrt{\frac{P}{\rho}}$. This distance is mean proportional between the average distance $\frac{1}{\rho}$ one can drive before encountering another car, and the average distance P one can drive in the time to walk the few meters of a "contact distance". Thus, under realistic choices of P and ρ, access to global traffic information allows one to travel almost as efficiently through hitchhiking as through the use of a private vehicle when moving more than a few hundred meters.

This brief, preliminary work can be expanded in many directions. First, it might be interesting to enrich the model to take into account that the source-destination pairs might not be chosen uniformly, but perhaps according to some "small world" power law distribution [7]; and that cars do not travel along straight lines, but tend instead to concentrate along major traffic arteries. At a first analysis, neither change significantly appears to affect our results, at most making hitchhiking, especially without an infrastructure, slightly more efficient. Much more interesting, however, would be validating our model and conclusions on real traffic data; these are unfortunately hard to obtain, since very few systems keep track of the trajectories of large numbers of *individual* cars. Of course, the ultimate test of our results would be an implementation of a dynamic ride-sharing system based on them!

Acknowledgments. This work was supported in part by MIUR under PRIN AlgoDEEP and by Univ. Padova under Strategic Project AACSE. Marco Bressan was supported in part by a Univ. Padova Research Fellowship.

References

1. S. Abdel-Naby and S. Fante. Auctions negotiation for mobile rideshare service. In *Proc. IEEE Second International Conference on Pervasive Computing and Applications (ICPCA07)*, 2007.
2. R. Buliung, K. Soltys, C. Habel, and R. Lanyon. Driving factors behind successful carpool formation and use. *Transportation Research Record: Journal of the Transportation Research Board*, 2118, 2009.
3. R. W. Calvo, F. de Luigi, P. Haastrup, and V. Maniezzo. A distributed geographic information system for the daily car pooling problem. *Comput. Oper. Res.*, 31(13):2263–2278, 2004.
4. S. Hartwig and M. Buchmann. Empty seats traveling: Next-generation ridesharing and its potential to mitigate traffic and emission problems in the 21st century. Technical report, Nokia, 2007.
5. K. Kelley. Casual carpooling–enhanced. *Journal of Public Transportation*, 10(4), 2007.
6. A. Klar, R. Kuehne, and R. Wegener. Mathematical models for vehicular traffic. *Survey on Mathematics for Industry*, 1996.
7. J. Kleinberg. The Small-World Phenomenon: An Algorithmic Perspective. In *Proceedings of the 32nd ACM Symposium on Theory of Computing*, 2000.
8. J. Morris. Saferide: Reducing single occupancy vehicles. Technical report, 2008.
9. P. Murphy. The smart jitney: Rapid, realistic transport. *New Solutions Journal*, (4), 2007.