Restoration by Energy Minimization

Restoration/representation algorithms are often related to the minimization of an energy function of the form

$$f(x) = \frac{1}{2} \| x - y \|^2_2 + \Pr(x)$$

where:
- $y$: Given measurements
- $x$: Unknown to be recovered

- Bayesian type of approach
- What is the prior? What is the image model?

Thomas Bayes
1702 - 1761
The *Sparseland* Model for Images

- Every column in $D$ (dictionary) is a prototype signal (Atom).
- The vector $\alpha$ contains very few (say L) non-zeros.
What Should the Dictionary $D$ Be?

$$
\hat{\alpha} = \underset{\alpha}{\arg \min} \frac{1}{2} \| D\alpha - y \|_2^2 \text{ s.t. } \| \alpha \|_0^0 \leq L \quad \Rightarrow \quad \hat{x} = D\hat{\alpha}
$$

$D$ should be chosen such that it sparsifies the representations.

One approach to choose $D$ is from a known set of transforms (Steerable wavelet, Curvelet, Contourlets, Bandlets, ...).

Learn $D$:
- Multiscale Learning
- Color Image Examples
- Task / sensing adapted
- Internal structure
What is being learned?

Learning Sparsity
Learning D to reconstruct

Each example is
a linear combination
of atoms from D

Min

\[ \sum_{j=1}^{P} \| D \alpha_j - x_j \|_2^2 \]

s.t. \( \forall j, \| \alpha_j \|_0 \leq L \)

Each example has a sparse representation with no more than L atoms

Field & Olshausen (’96)
Engan et. al. (’99)
Lewicki & Sejnowski (’00)
Cotter et. al. (’03)
Gribonval et. al. (’04)
Aharon, Elad, & Bruckstein (’04)
Aharon, Elad, & Bruckstein (’05)
Ng et al. (’07)
Mairal, Sapiro, Elad (’08)
The K–SVD Algorithm – General

Aharon, Elad, & Bruckstein (`04)

Initialize $D$

Sparse Coding
  Orthogonal Matching Pursuit (or L1)

Dictionary Update
  Column-by-Column by SVD computation over the relevant examples

$X^T$

$D$
Non-uniform noise

\[
\{\alpha_{ij}, \hat{D}, \hat{x}\} = \arg \min_{D, \alpha_{ij}, x} \lambda \|\beta \otimes (x - y)\|_2^2 \\
+ \sum_{i,j} \mu_{ij} \|\alpha_{ij}\|_0 \\
+ \sum_{ij} \|\left(R_{ij, \beta}\right) \otimes (D\alpha_{ij} - R_{ij}x)\|_2^2.
\]
Show me the pictures
Change the Metric in the OMP

\[
\langle y, x \rangle_\gamma = y^T x + \frac{\gamma}{n^2} y^T K^T K x = y^T \left( I + \frac{\gamma}{n} K \right) x,
\]

\[
K = \begin{pmatrix}
J_n & 0 & 0 \\
0 & J_n & 0 \\
0 & 0 & J_n
\end{pmatrix}.
\]
Example: Non-uniform noise
Example: Inpainting
Example: Demoisaiic
Example: Inpainting

Since 1699, when French explorers landed at the great bend of the Mississippi River and celebrated the first Mardi Gras in North America, New Orleans has brewed a fascinating melange of cultures. It was French, then Spanish, then French again, then sold to the United States. Through all these years, and even into the 1900s, others arrived from everywhere: Acadians (Cajuns), Africans, indige-
Multiscale Dictionaries
Learned multiscale dictionary
Color multiscale dictionaries
Extending the Models
Universal Coding and Incoherent Dictionaries

\[ f(X, D, A) = \|X - DA\|_F^2 + \lambda \sum_{j=1}^{N} \sum_{i=1}^{K} \log (|\alpha_{ij}| + \beta) + \zeta \|D^T D - I_K\|_F^2 + \eta \sum_{k=1}^{K} (\|D_k\|_2^2 - 1)^2. \]

- Consistent
- Improved generalization properties
- Improved active set computation
- Improved reconstruction
- Improved coding speed
Group Sparsity
Sparsity + Self-similarity = Group Sparsity

- Combine the two of the most successful models for images
Sparsity + Self-similarity = Group Sparsity
Sparsity + Self-similarity = Group Sparsity

Adobe Camera Raw

Proposed Method
Learning to Sense Sparse Images
Motivation

- **Compressed sensing** (Candès & Tao, Donoho, et al.)
  - Sparsity
  - Random sampling
    - Universality
    - Stability

- Shall the sensing be adapted to the **data type**?
  - Yes! (Elad, Peyre, Weiss et al., Applebaum et al, this talk).

- Shall the sensing and dictionary be learned simultaneously?
Some formulas....

\[
\min_{\psi, \phi, \theta} \left\{ \alpha \| X - \Psi \Theta \|_F^2 + \| Y - \Phi \Psi \Theta \|_F^2 \right\} \quad \text{s.t.} \quad \forall i, \| \theta_i \|_{\ell_0} \leq S 
\]

+ “RIP (Identity Gramm Matrix)”
Design the dictionary and sensing together
Just Believe the Pictures
Just Believe the Pictures
Just Believe the Pictures
- Sparsity should be learned.
- Sparsity should have a goal.
- You should not learn to do A and expect it to do B.