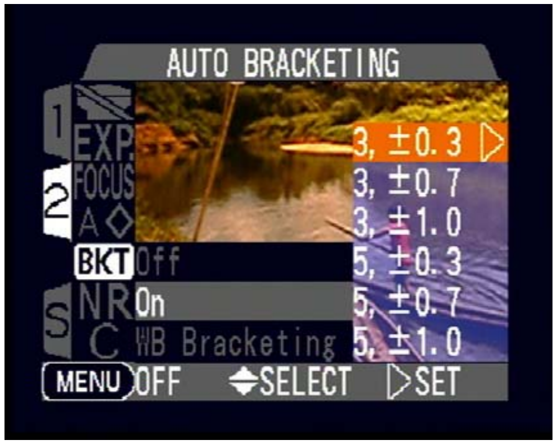
MIP models for MIP heuristics

or

the Bracket-and-Fix method

$$[x_j]$$



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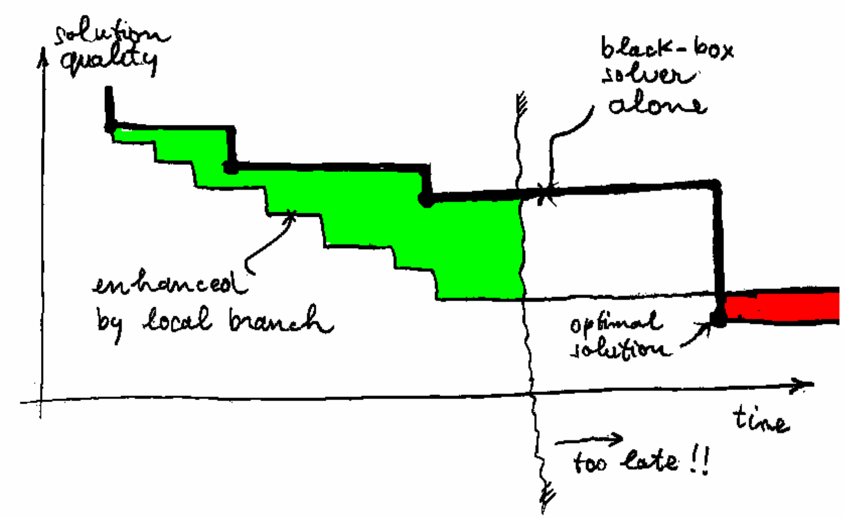
11th Combinatorial Optimization Workshop, Aussois, January 12, 2007

LOCAL BRANCHING

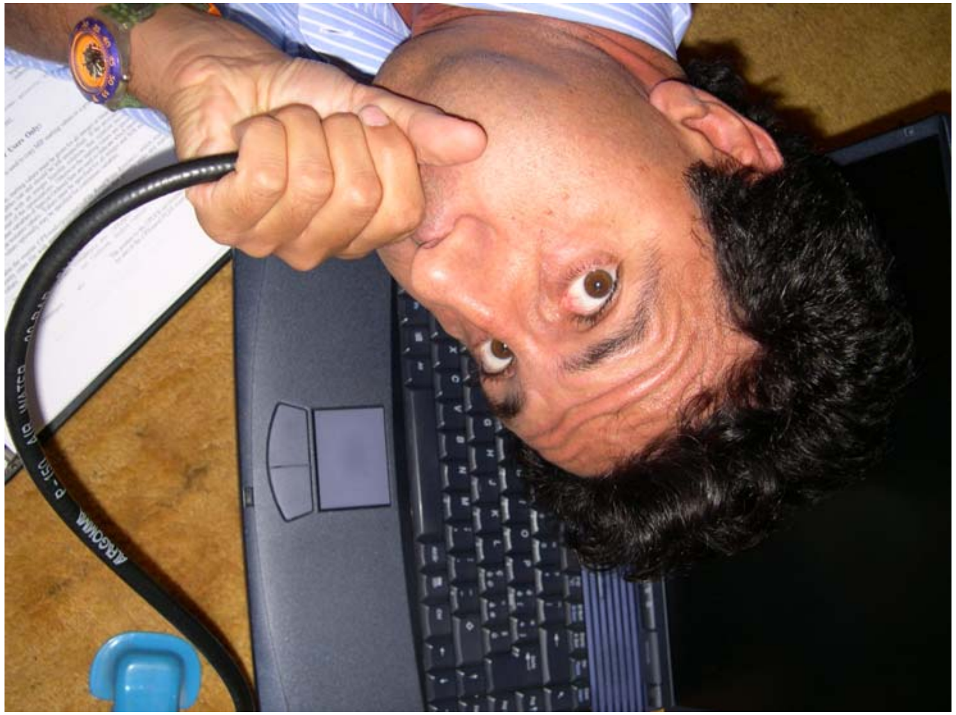
or

How to enhance the heuristic behaviour of your favourite 0-1 MIP solver



AUSSOIS, January 2002

THE FEASIBILITY PUMP



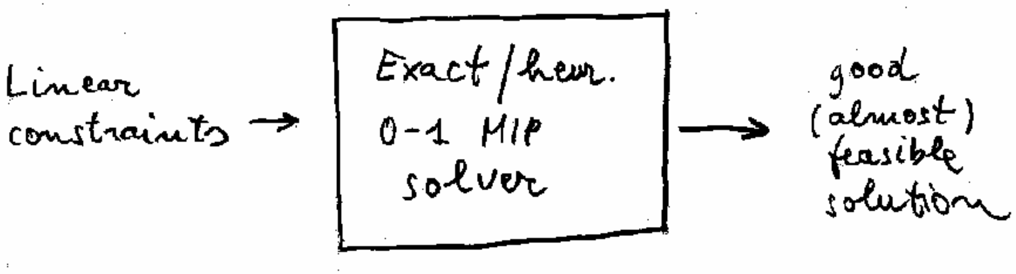
AUSSOIS, January 2004

THE LOCAL BRANCHING FRAMEWORK

- Consider a generic Mixed-Integer Linear Programming Problem (MIP) with 0-1 variables

$$
\begin{aligned}
\min \quad & c^T x \\
& A x \geq b \\
& x_j \in \{0,1\}, \quad \forall j \in \mathcal{B} \neq \emptyset \\
& x_j \text{ integer}, \quad \forall j \in \mathcal{G} \ (\supseteq \emptyset)
\end{aligned}
$$

- We aim at embedding a **black-box** (general-purpose or specific) 0-1 MIP solver within an overall **heuristic framework** that "helps" the solver to deliver improved heuristic solutions



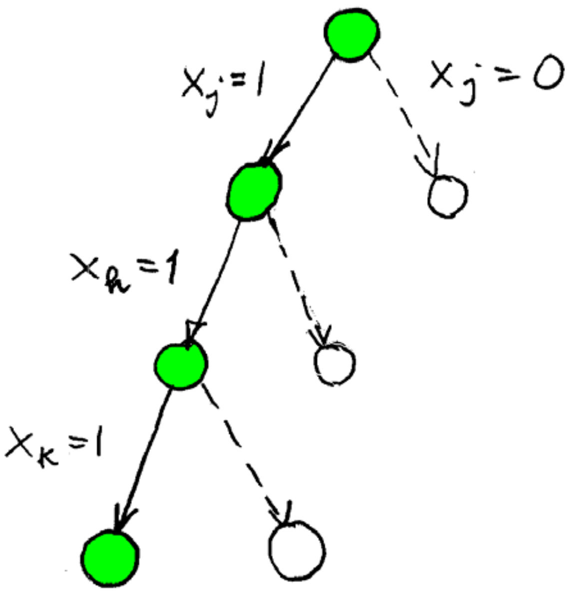
The available black-box module

Hard variable fixing (diving)

- A commonly-used (often quite effective) heuristic framework
- Let x^H be an (almost) feasible "target solution", and let

$$S = \{ j \in B : x_j^H = 1 \}$$

denote its **binary support** (binary var.s at value 1).



- **Heuristic depth-first search** of the branching tree:
  - iteratively **fix to 1** certain "highly efficient" variables in S (**green nodes**)
  - apply the **black-box module** to some **green nodes** only
  - only limited **backtracking** allowed

Advantages:

- Problem size quickly reduced: the black-box solver can concentrate on smaller and smaller "**core problems**"
- The black-box solver is applied over and over on different subproblems (**diversification**)

Disadvantages:

- How to choose the "highly efficient variables" to be fixed?
- Wrong choices at early levels are typically very difficult to detect, even when **lower bounds** are computed along the way

How to reach a sufficiently-deep branching level

with a good lower bound?

Soft variable fixing

- Idea: Given the heuristic solution x^H, don't decide the actual variables to be fixed, but just their **number**

  - Introduce the **local branching** constraint

$$\Delta(x, x^H) := \sum_{j \in B : x_j^H = 0} x_j + \sum_{j \in B : x_j^H = 1} (1 - x_j) \le k$$

so as to define a convenient **k-OPT neighbourhood** $N(x^H, k)$ of the target solution x^H (the larger k, the larger the neighbourhood)

"Akin to k-OPT for TSP"

  - Search $N(x^H, k)$ by means of the **black-box** module

  - Diversification (also modelled through MIP cuts)

RINS

- An effective variable-fixing heuristic proposed in

  E. Danna, E. Rothberg, C. Le Pape. Exploring relaxation induced neighborhoods to improve MIP solutions. Mathematical Programming 102, 71-90, 2005.

- Given a heuristic solution x^H (e.g., the incumbent) and a fractional solution x^* (e.g., the LP optimum at some branching node): **fix** $x_j = x_j^H$ **whenever** $x_j^H = x_j^*$ and solve (heuristically) the resulting sub-MIP through a **black-box MIP solver**.

Advantages

- Fixing can reduce the MIP size (and difficulty) dramatically
- Simple and clever identification of the variables to be fixed
- Easy diversification if applied for different fractional sol.s $\mathbf{x}^*$ (e.g., within Branch & Cut)

Disadvantages

- Little control on the number of fixed var.s (if too few, the method is completely useless)

A new variable-fixing heuristic: bracket and fix

"Hard-fixing the var.s is Ok, but choosing those to fix is a difficult decision → just MIP it!"

- Original MIP

$$
\begin{aligned}
\min c^T x & \\
Ax \leq b & \\
x_j \geq 0 \quad & \forall j \in \mathcal{C} \\
x_j \geq 0 \quad \text{integer} \quad & \forall j \in \mathcal{I} \\
x_j \in \{0,1\} \quad & \forall j \in \mathcal{B}
\end{aligned}
$$

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\end{aligned}
$$

Bracketing MIP

$$
\begin{aligned}
&\min c^T x \\
&Ax \le b \\
&x_j \ge 0 \quad \forall j \in \mathcal{C} \\
&x_j \ge 0 \quad \text{integer} \quad \forall j \in \mathcal{I} \\
&f_j^1 \le x_j \le 1 - f_j^0 \quad \forall j \in \mathcal{B} \\
&\sum_{j \in \mathcal{B}} (f_j^0 + f_j^1) \ge k \\
&f_j^0, f_j^1 \in \{0,1\} \quad \forall j \in \mathcal{B}
\end{aligned}
$$

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Bracketing MIP

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$$\sum_{j \in \mathcal{B}} (f_j^0 + f_j^1) \ge k$$
$$f_j^0, f_j^1 \in \{0,1\} \quad \forall j \in \mathcal{B}$$

sub-MIP with fixed var.s

$$\min c^T x$$
$$Ax \le b$$
$$x_j \ge 0 \quad \forall j \in \mathcal{C}$$
$$x_j \ge 0 \quad \text{integer} \quad \forall j \in \mathcal{I}$$
$$x_j \in \{0,1\} \quad \forall j \in \mathcal{B}$$
$$x_j = 1 \quad \forall j \in \mathcal{B} : \bar{f}_j^1 = 1$$
$$x_j = 0 \quad \forall j \in \mathcal{B} : \bar{f}_j^0 = 1$$

where $(\bar{f}^0, \bar{f}^1)$ is an almost-optimal solution of the bracketing MIP

The bracket-and-fix heuristic

```
Basic Bracket-and-fix heuristic:
 0. run ILOG-Cplex  default until a first feasible solution is found
 1. while (time < timeLimit) and (iteration < iterationLimit) do
       1.a solve the bracketing MIP with a limit of n_B nodes
       1.b define and solve the fixed sub-MIP with a limit of n_F nodes
       1.c modify the bracketing MIP with a nogood cut
           (plus eventually with some random perturbation)
 2. enddo
```

Preliminary implementation (BF1):

- Exploit the incumbent x^H (as in RINS): set

$f_j^0 = 0$ if $x_j^H = 1$ → never fix to 0 a variable having value 1 in x^H

$f_j^1 = 0$ if $x_j^H = 0$ → never fix to 1 a variable having value 0 in x^H

- Hardwired parameters: ; $n_B = 100$, $n_F = 500$, $k = 0.2$ * total number of binary variables

$$\sum_{j \in B} (f_j^0 + f_j^1) \geq 0.2 \, |B|$$

Variants under investigation

- **Variant 1 (BF2)**: As BF1, but in the bracketing MIP invert the role of the objective function and of the cardinality constraint

$$c^T x \qquad \Leftrightarrow \qquad \sum_{j \in B} (f_j^0 + f_j^1) \geq k$$

$$c^T x \leq LB + 0.1 * GAP$$ "percentage gap w.r.t. incumbent value"

"Which is the maximum n. of var.s that I can fix by accepting a 10% lower-bound worsening?"

- **Variant 2 (LB+RINS)**: Instead of the bracketing MIP, solve the **LP relaxation** of the initial problem with a **local branching** constraint

$$\sum_{j \in B : x_j^H = 0} x_j + \sum_{j \in B : x_j^H = 1} (1 - x_j) \leq 0.2 \, |B|$$

"Parameter 0.2 should be better tuned on the fly to reach a sufficient matching between x^H and x^*"

- **Asymmetric versions**: for all variants, forget about 0-fixing (i.e, set $f_j^0 = 0$ for all j) → still to be tested

Preliminary computational results

		Cplex 10.0	RINS		LB+RINS (.20)		
name	$\|\mathcal{B}\|$	value	value	time	fixed	value	time
A1C1S1	192	12,452.4276	11,837.1930	36.43	75	11,791.3993	99.31
biella1	6110	3,680,593.9300	3,630,472.4800	46.92	5526	3,349,537.4100	34.27
CMS750_4	7196	773.0000	364.0000	57.76	4923	503.0000	21.27
dg012142	640	153,388,548.0000	8,232,594.0000	57.91	243	**3,268,726.0000**	143.80
ljb2	681	1.2129	0.6681	2.47	493	**0.6363**	6.48
sp97ar	14101	693,904,022.4000	693,904,022.4000	49.82	13868	**680,750,648.6400**	16.83
tr12-30	360	133,478.0000	130,901.0000	5.82	113	131,033.0000	9.09

Table 1: One iteration: **symmetric** versions, Cplex 10.0 initial, RINS vs LB+RINS

		BF1 (.20)			BF2 (.10)		
name	$\|\mathcal{B}\|$	fixed	value	time	fixed	value	time
A1C1S1	192	38	11,591.1888	89.53	102	**11,563.2272**	49.92
biella1	6110	5524	3,345,029.3600	34.20	5922	**3,250,161.4700**	155.63
CMS750_4	7196	3229	**346.0000**	61.05	4332	404.0000	31.98
dg012142	640	243	3,460,266.7500	137.42	589	16,316,886.9333	15.02
ljb2	681	136	0.7441	6.98	662	0.9432	3.28
sp97ar	14101	13873	684,494,973.1200	11.91	13985	686,543,415.6800	100.36
tr12-30	360	72	**130,815.0000**	5.14	100	130,913.0000	4.81

Table 2: One iteration: **symmetric** versions, variants of BF