Finding and evaluating robust train timetabling solutions

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Robust planning and Rescheduling in Railways

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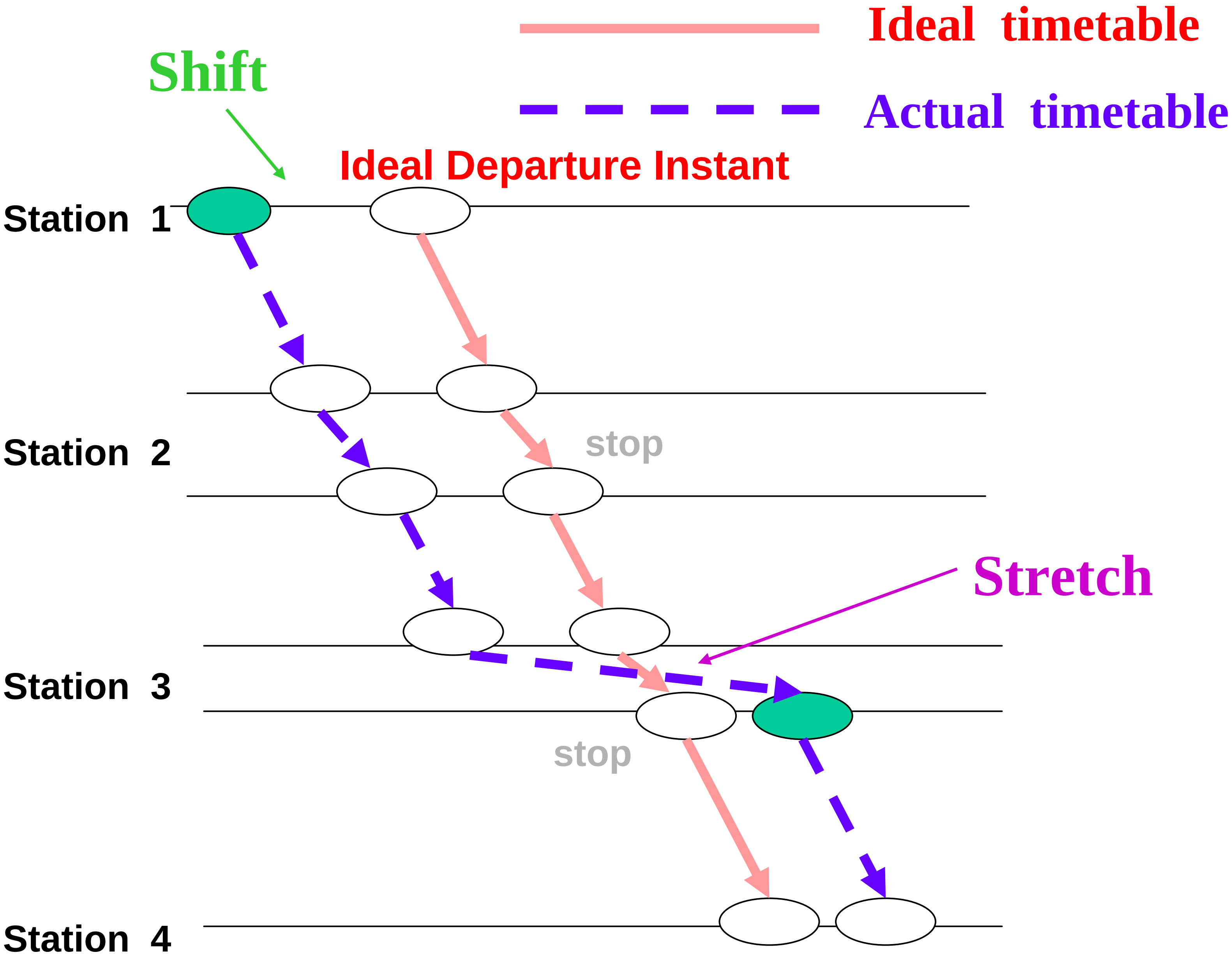
Problem Definition

- single one-way line
- aperiodic daily timetable to be designed

- Minimize the timetable cost computed as follows…


Ideal timetable
Actual timetable
Shift
Ideal Departure Instant
Station 1
Station 2
stop
Stretch
Station 3
stop
Station 4

An event-scheduling MIP model

Variables:

- Arrival and departure times (event times) ………… t_i
- Binary variables modeling event precedences ……… x_{ij}

Constraints:

- Minimum travel times d_{ij}
- Safety constraints:
  - buffer times, no overtaking outside stations, etc.
- Typical constraints of the type: $t_i - t_j \geq d_{ij} - M x_{ij}$

Objectives:

- Minimize the cost of the schedule
- Robustness (whatever it means)

Timetable robustness …

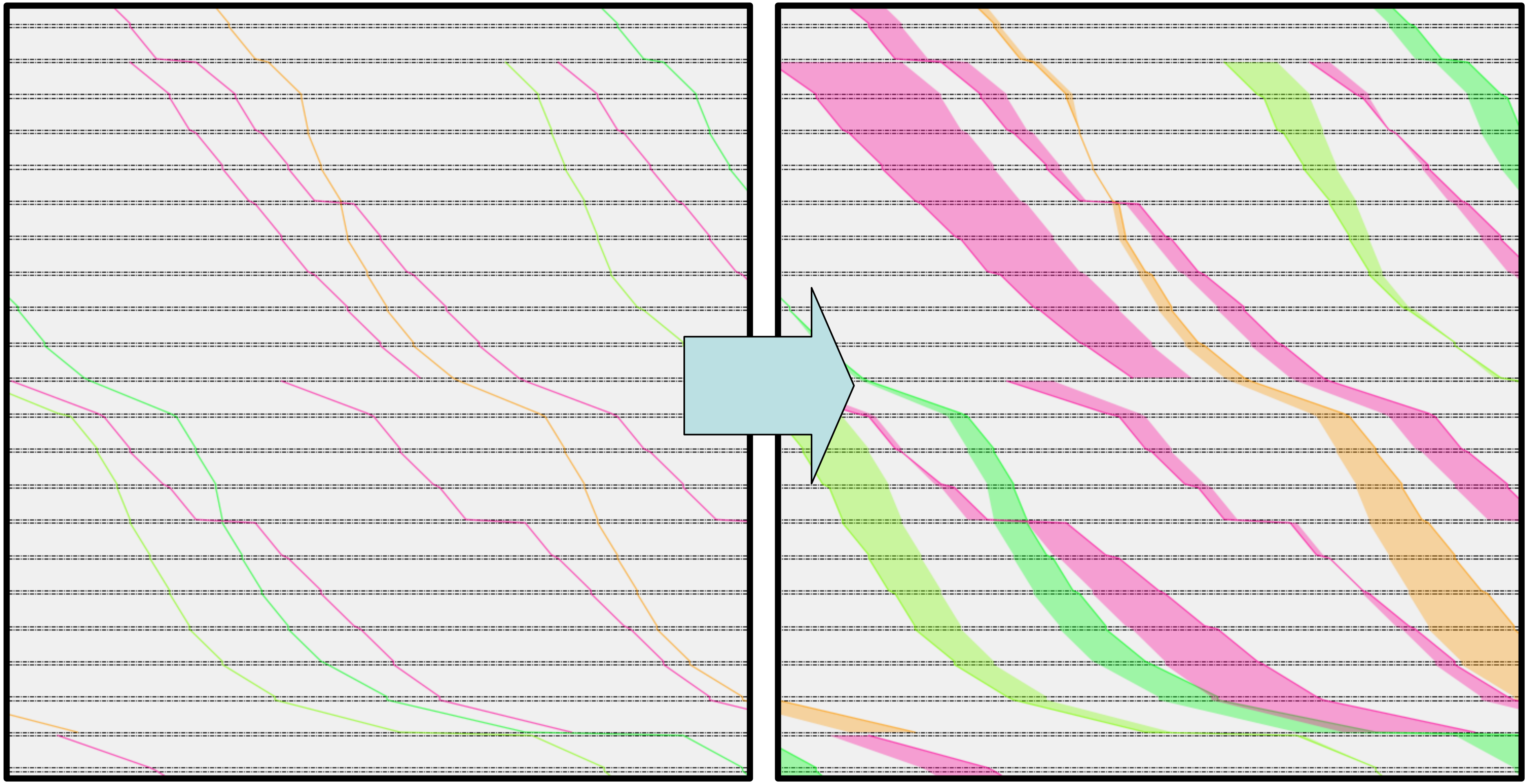
- … is **not** concerned with major disruptions
- … is **not** intended to cope with heavy truck breaks or alike

  - → to be handled by **REAL TIME CONTROL SYSTEMS**

- … is a way to control **delay propagation**
- … has to favor **delay compensation** without heavy actions from the traffic control center

  - → no overtaking allowed to prevent delay propagation
  - → no train cancellation
  - → train precedences unchanged w.r.t. to the planned timetable

Non-robust and robust timetables



Our approach

- Take a **feasible timetable** -> near-optimal solution of the “nominal” timetable problem
- Fix a maximum **price of robustness** → the cost of the robust solution cannot exceed by more than XX% the optimal cost of the nominal problem
- **Fix all train precedences** (binary var.s x_{ij} in the MIP model)
- **Relax the integrality** on the event-time var.s t_i (the only unknowns)
- Enforce **robustness** in the resulting LP by using alternative techniques
- Evaluate the achieved robustness through a common **validation model**
- Compare the results

Pursuing robustness in the LP/MIP context

- **Stochastic Programming**
  - Take first-stage decisions
  - Pay for restoring feasibility afterwards (second-stage recource var.s)
  - Applied successfully by the Kroon’s group to periodic timetabling
  - Very flexible but computationally heavy in scenario-based approaches

- **Robustness à la Bertsimas-Sim**
  - Kind of worst-case analysis of robustness
  - Limits the moves of the adversary (just a few coefficients can change in each constraint)
  - Feasibility deterministic (if adversary behaves as expected) or with high probability (otherwise)
  - Very simple model
  - Unfortunaltely, of no use in the timetable context (infeasible or very inefficient solutions)

- **Light Robustness**
  - “Light” version of Bertsimas-Sim using slack variables for “too conservative” constr.s
  - Linear or quadratic objective function (minimize slack var.s)
  - Very well suited for timetabling → talk in the afternoon…

Stochastic programming model

- Two-stage model with **recourse** var.s (unabsorbed delay)

Nominal constraint

$$t_i - t_j + s_{ij}^{(\omega)} \geq d_{ij} + \delta_{ij}^{(\omega)}$$

Recourse Disturbance

- Deterministic model through **scenario** expansion

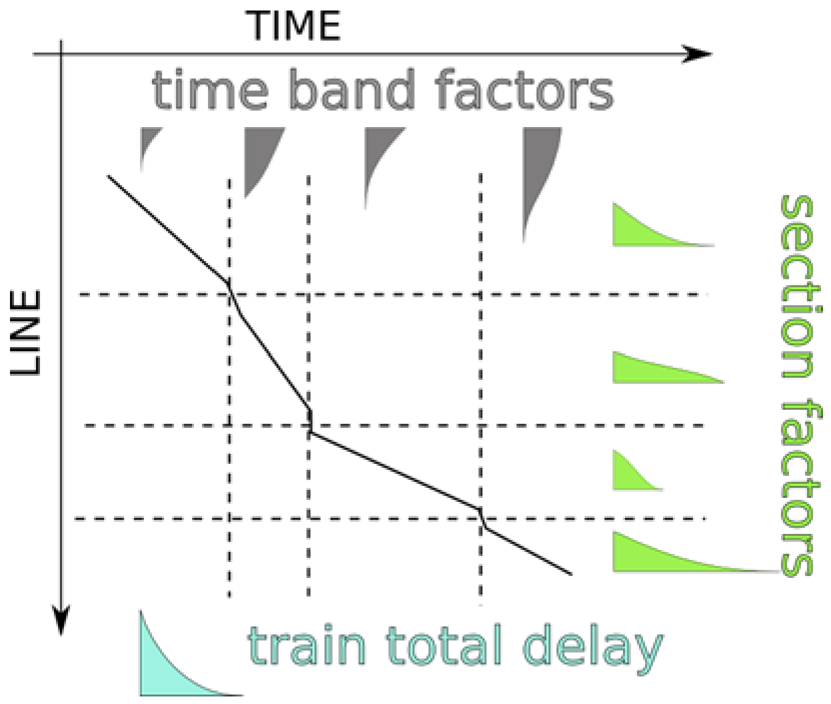
$$t_i - t_j + s_{ij}^{(r)} \geq d_{ij} + \delta_{ij}^{(r)}, \ \forall r \in [1 \ldots N]$$

- Objective function: minimize the average unabsorbed delay

- → **big LP model to be solved** (though each scenario actually introduces just a few new var.s and constr.s)

Scenario Generation

- **Delay model:**
  - Random cumulative train delay
  - Scaled by time band factors
  - Distributed across lines with section factors



Validation model

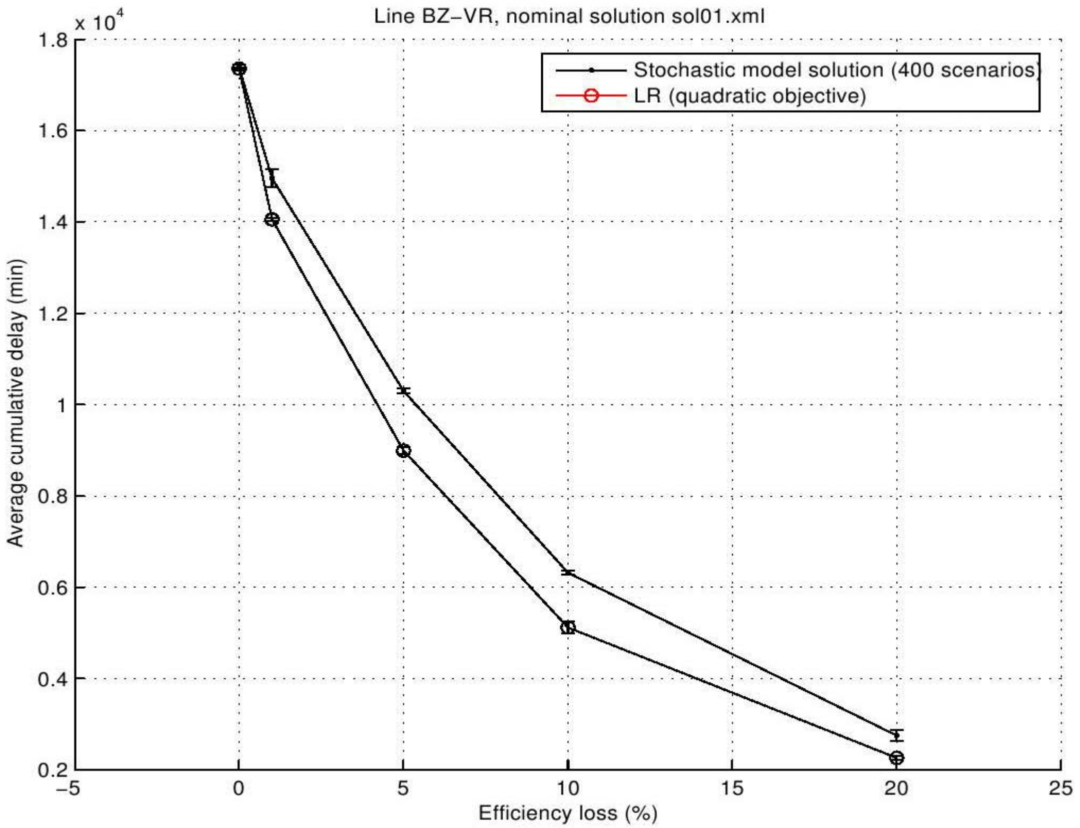
- Simulation tool used to **evaluate** the actual robustness of a given timetable $(\tilde{x}, \tilde{t})$
- Uses information on the line to generate a **delay scenario** for each run
- For each run, solve an **LP model** to *absorbe as much delay as possible*
  - Fixed precedences → $x_{ij} := \tilde{x}_{ij}$
  - Continuous event-time var.s only → t_i = actual times in the delayed schedule
  - Cannot anticipate with respect to the input solution to evaluate → $t_i \geq \tilde{t}_i$
  - Minimize sum of delays (event-time shifts) → $\min \sum_i (t_i - \tilde{t}_i)$
- Gather **statistical information**

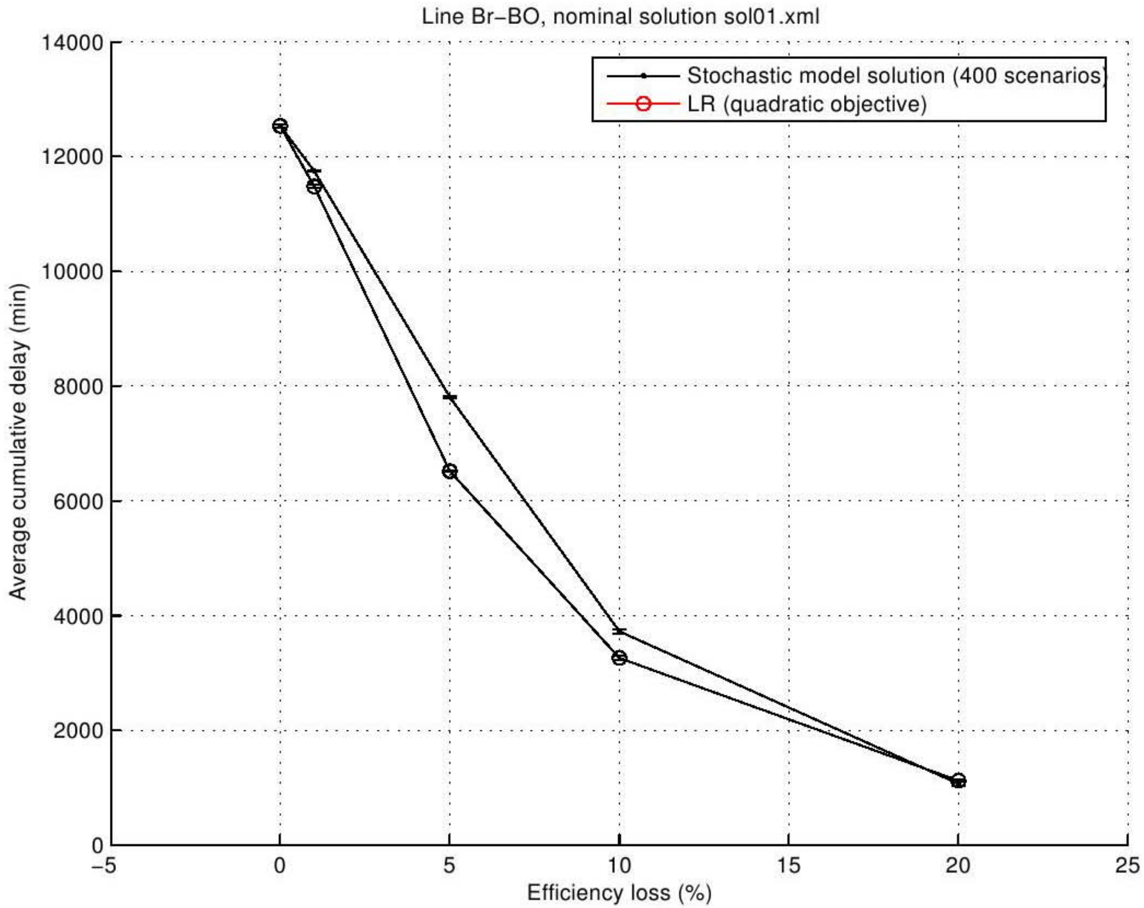
Test bed

- Real world instances from RFI
  - PD-BO: 17 stations, ~35 trains
  - BZ-VR: 27 stations, ~130 trains
  - Mu-VR: 48 stations, ~50 trains
  - Br-BO: 48 stations, ~70 trains

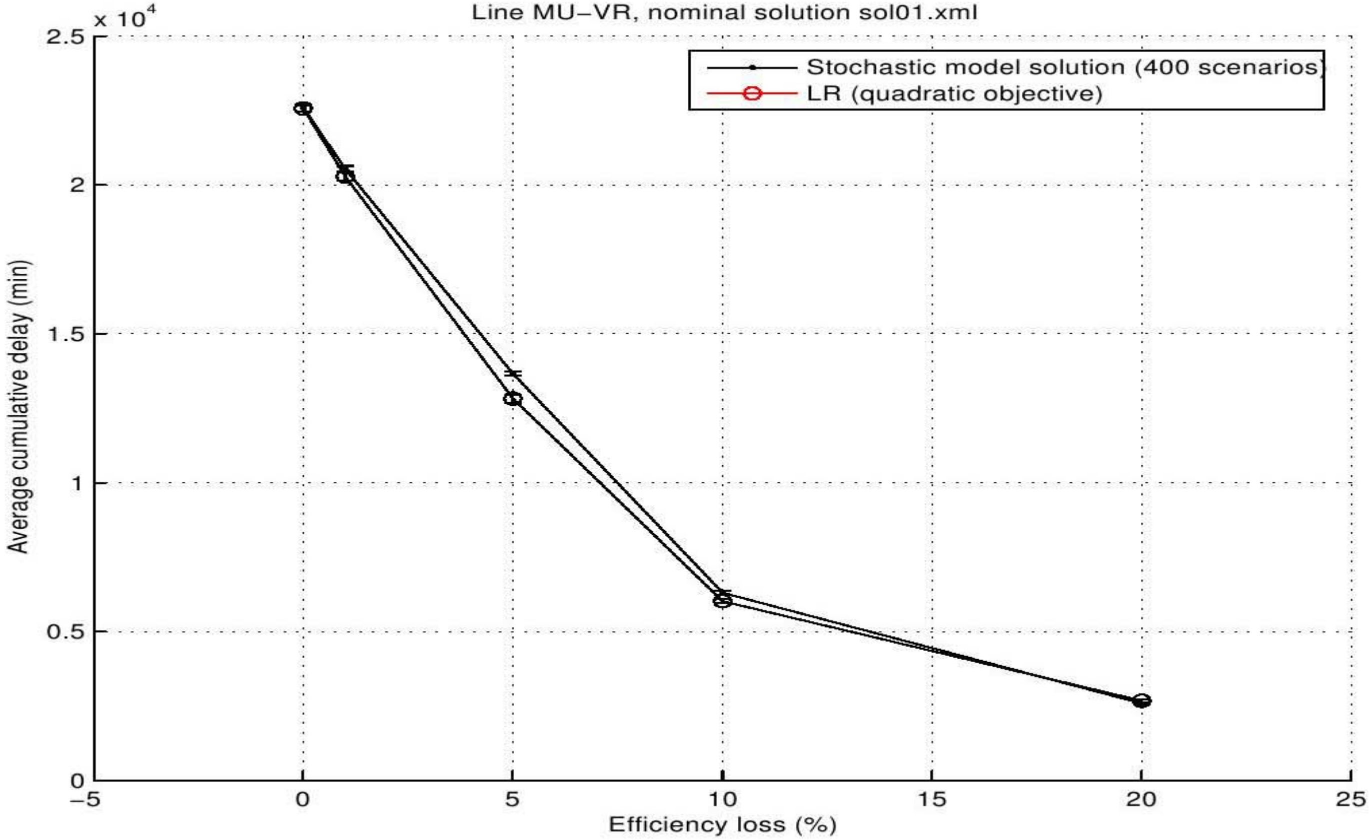
- For each instance, 5 almost-optimal (non-robust) timetables computed by DEIS

Validation results

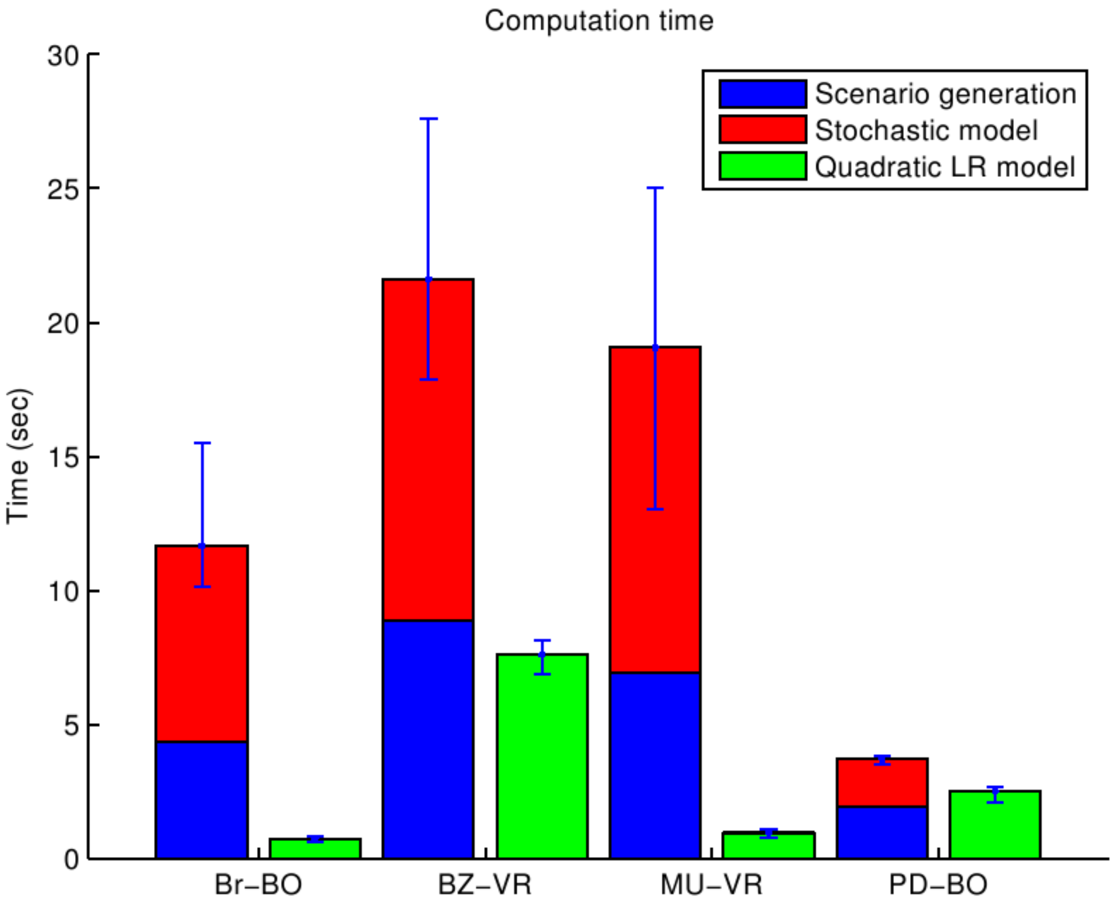




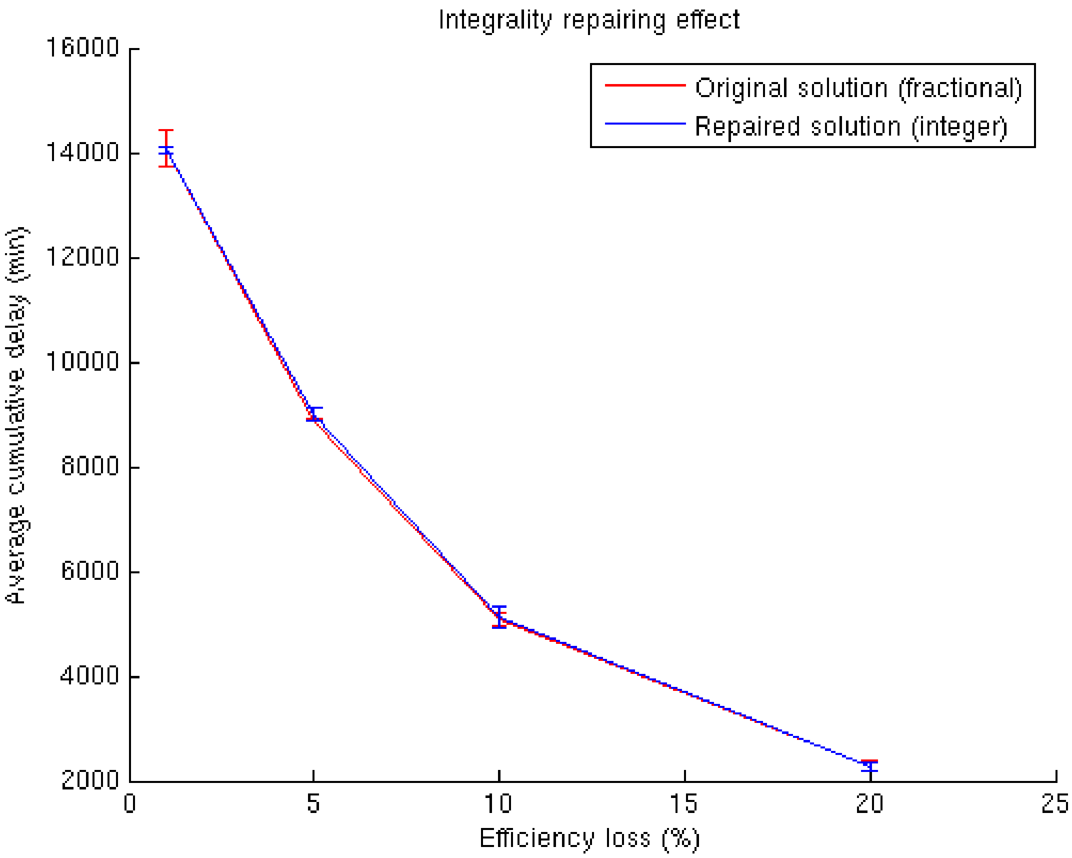
Line Br−BO, nominal solution sol01.xml
Stochastic model solution (400 scenarios)
LR (quadratic objective)
Average cumulative delay (min)
Efficiency loss (%)
14000
12000
10000
8000
6000
4000
2000
0
−5
0
5
10
15
20
25


Line MU−VR, nominal solution sol01.xml
x 10^4
Stochastic model solution (400 scenarios)
LR (quadratic objective)
Average cumulative delay (min)
Efficiency loss (%)
2.5
2
1.5
1
0.5
0
−5
0
5
10
15
20
25

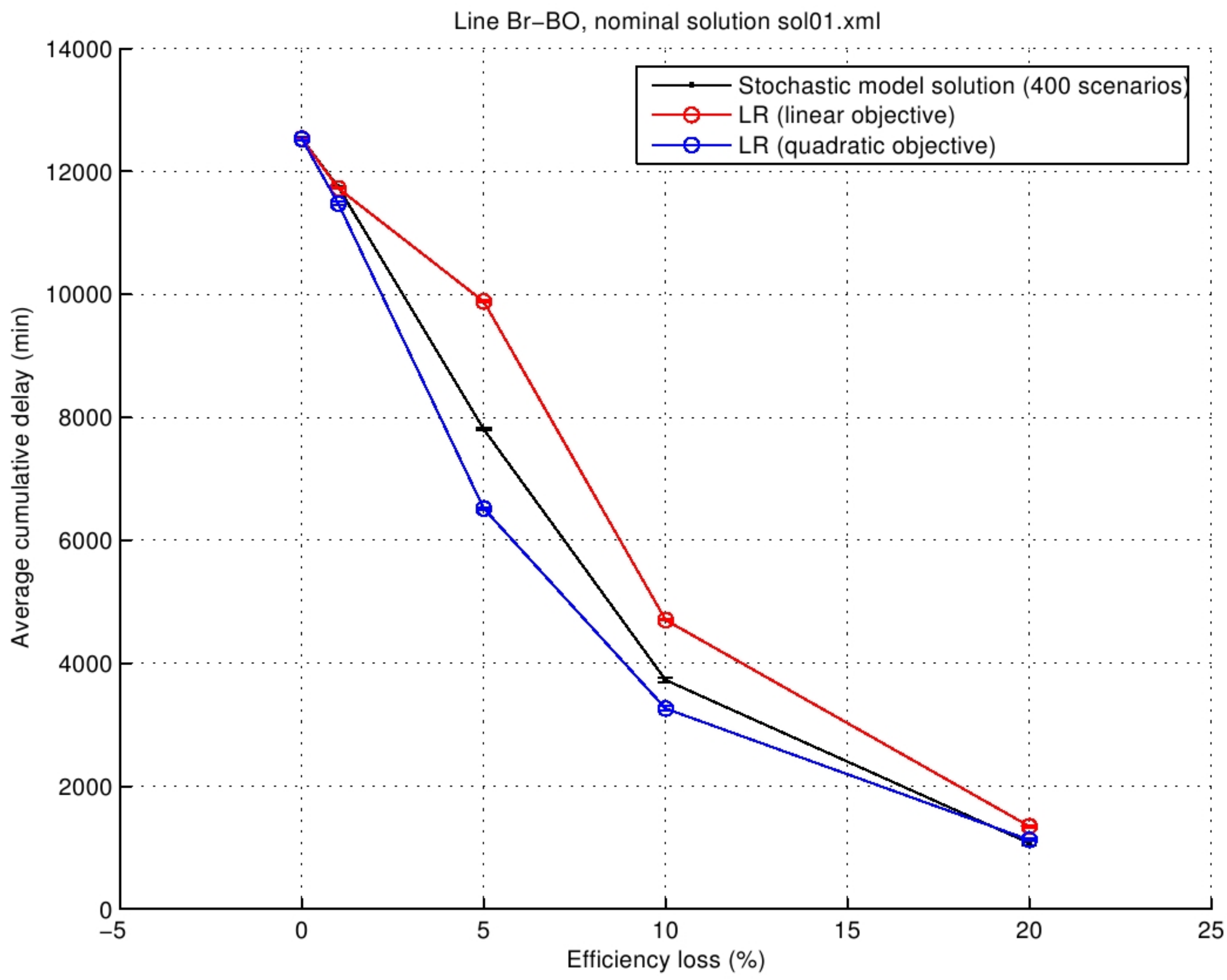
Computing times



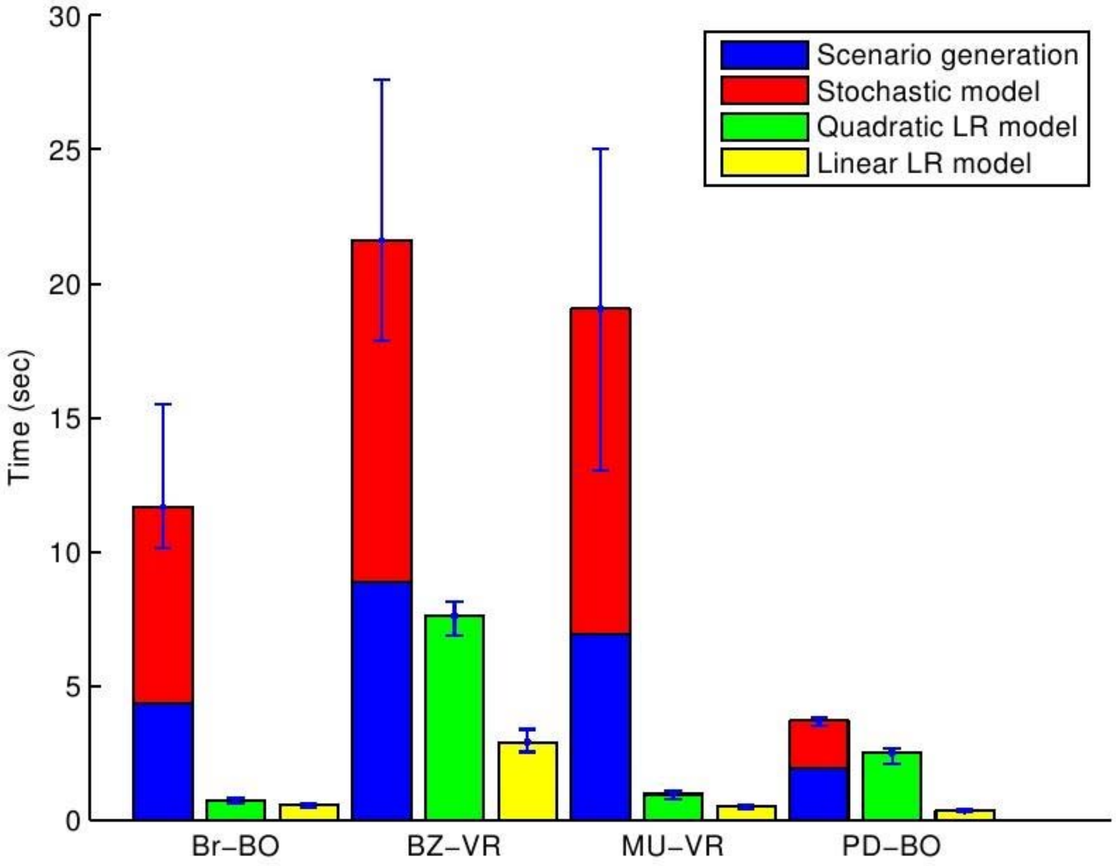
Restoring integrality on the timetable var.s



Importance of the quadratic LR function



Computing times (updated)



Thanks