The crew scheduling problem

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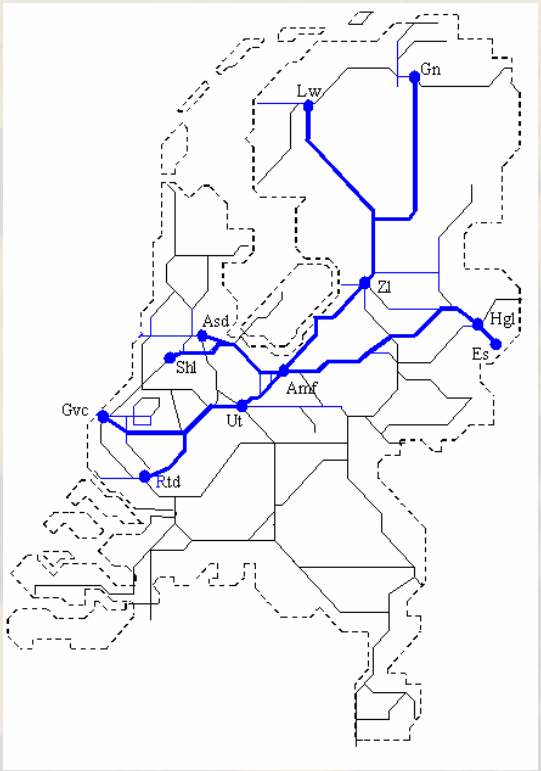
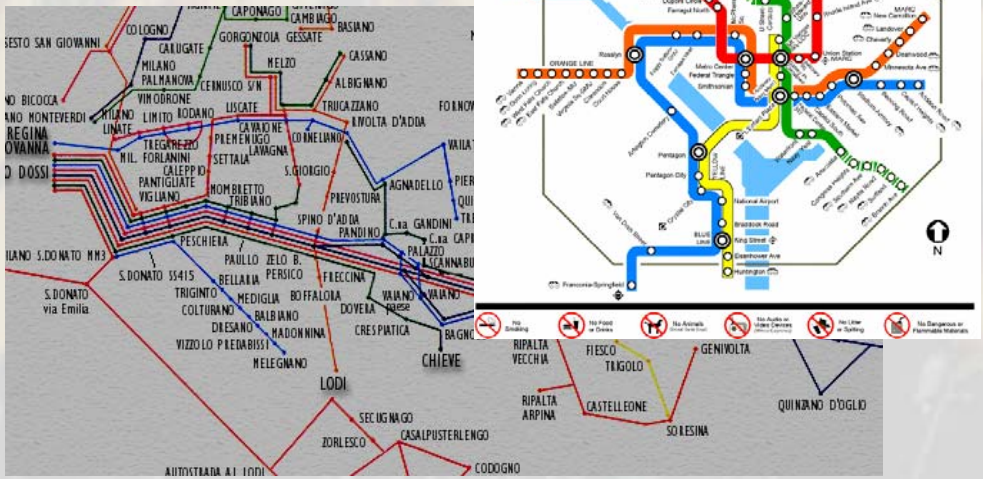
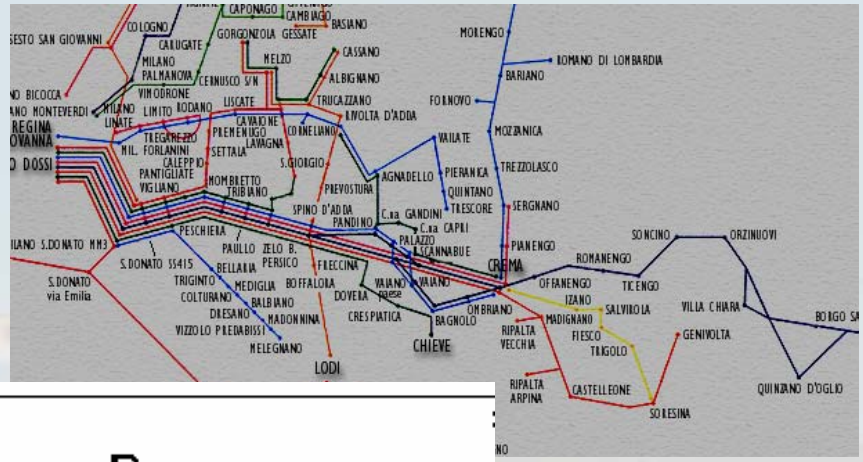
The basic problem

- A **<u>transportation company</u>** (bus, railways, airline, metro, etc.)
- A network of **<u>relief points</u>** where a driver-change can take place





- A **timetable** for a certain day …



F1 (Malè)-Ponte di Legno-Bergamo-Milano

F1	Fer6		Fer6	Scol	Fer6	
Malè					5.45	
Ponte di Legno staz. autolinee	4.05			6.30	7.15	7.15
Edolo p. stazione	4.35			7.00	7.45	7.45
Breno bar Sport	5.10			7.35		8.20
Boario Terme staz. autolinee	5.35	5.35	7.00	7.55		8.45
Costa Volpino bar Sport	5.50	5.52	7.17			9.02
Lovere p. 13 Martiri	5.53	5.55	7.20			9.05
Trescore Baln. rist. Tonale	6.23	6.32	7.57			9.42
Bergamo staz. autolinee	6.45	7.00	8.25			10.10
Sesto S. Giovanni rondo'	7.27	7.42				10.43
Milano p. Castello *a.*	7.45	8.00				11.11

- … and the associated set of **<u>trips</u>** = driving **atoms**, i.e., indivisible time-tabled increments of work to be covered by a driver, each characterized by the **starting** and **ending times** and associated **relief points** (plus additional information)

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Milano p. Castello a.	7.45					

Microsoft Excel - Cartel1.xls

File Modifica Visualizza Inserisci Formato Strumenti Dati Finestra ?

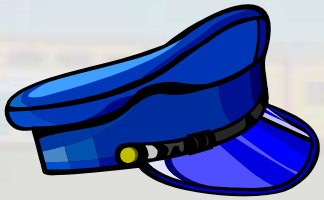
Arial 12 100%

B1 = 5.35.00

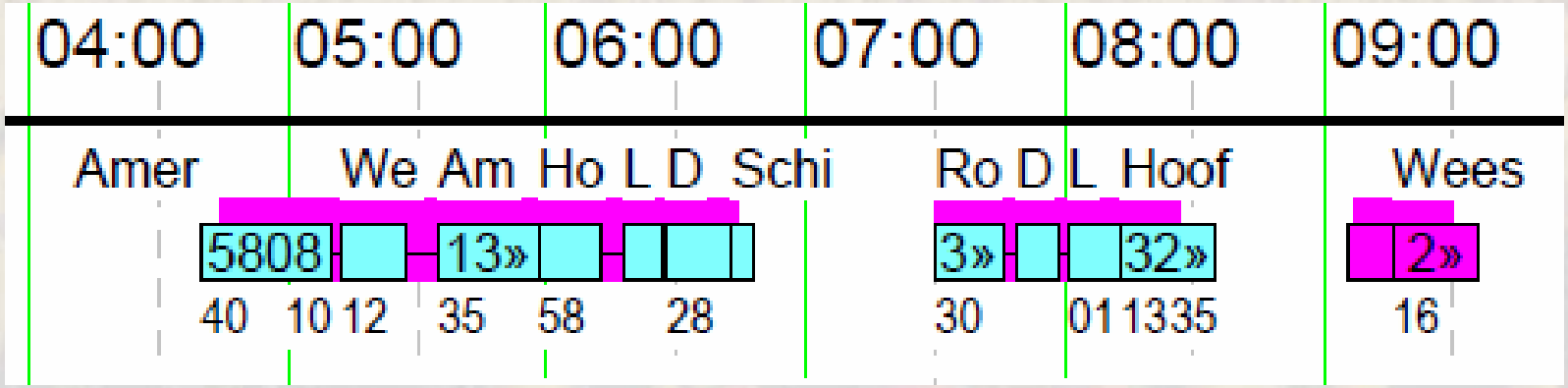
	A	B	C	D
1	4.05	5.35	Ponte di Legno staz. Autolinee	Boario terme staz. Autolinee
2	5.35	6.45	Boario terme staz. Autolinee	Bergamo staz. Autolinee
3	6.45	7.45	Bergamo staz. Autolinee	Milano p. Castello
4	5.35	7.00	Boario terme staz. Autolinee	Bergamo staz. Autolinee
5	7.00	8.00	Bergamo staz. Autolinee	Milano p. Castello
6	7.00	8.25	Boario terme staz. Autolinee	Bergamo staz. Autolinee

Foglio1 Foglio2 Foglio3

Pronto NUM

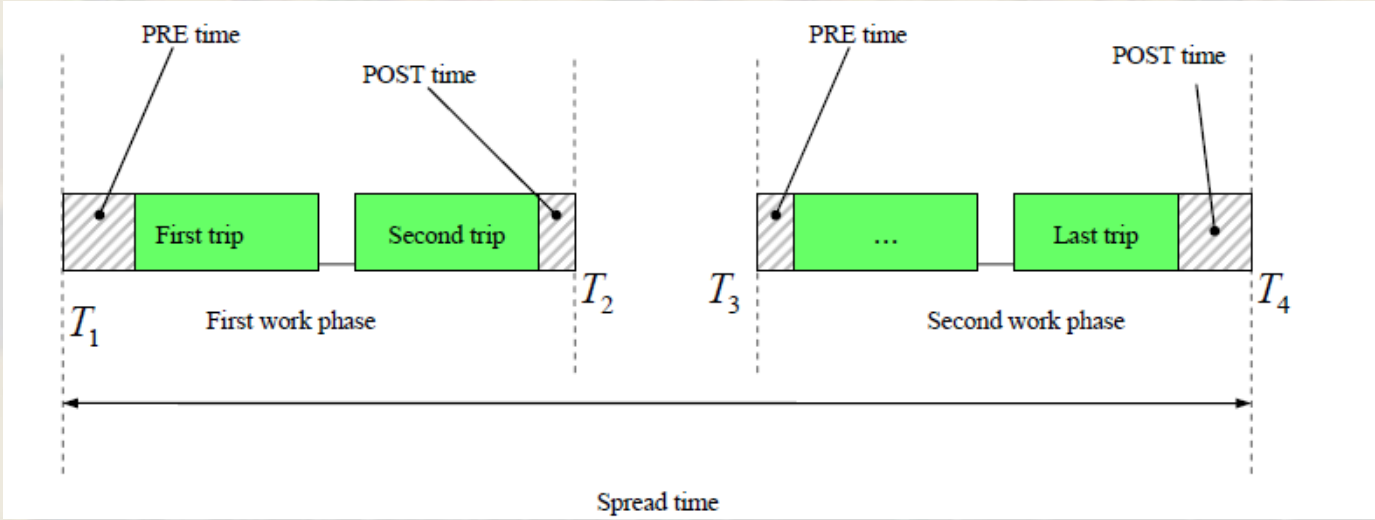


A **duty,** or **shift,** is a sequence of trips to be performed consecutively by a same **driver** (or conductor)



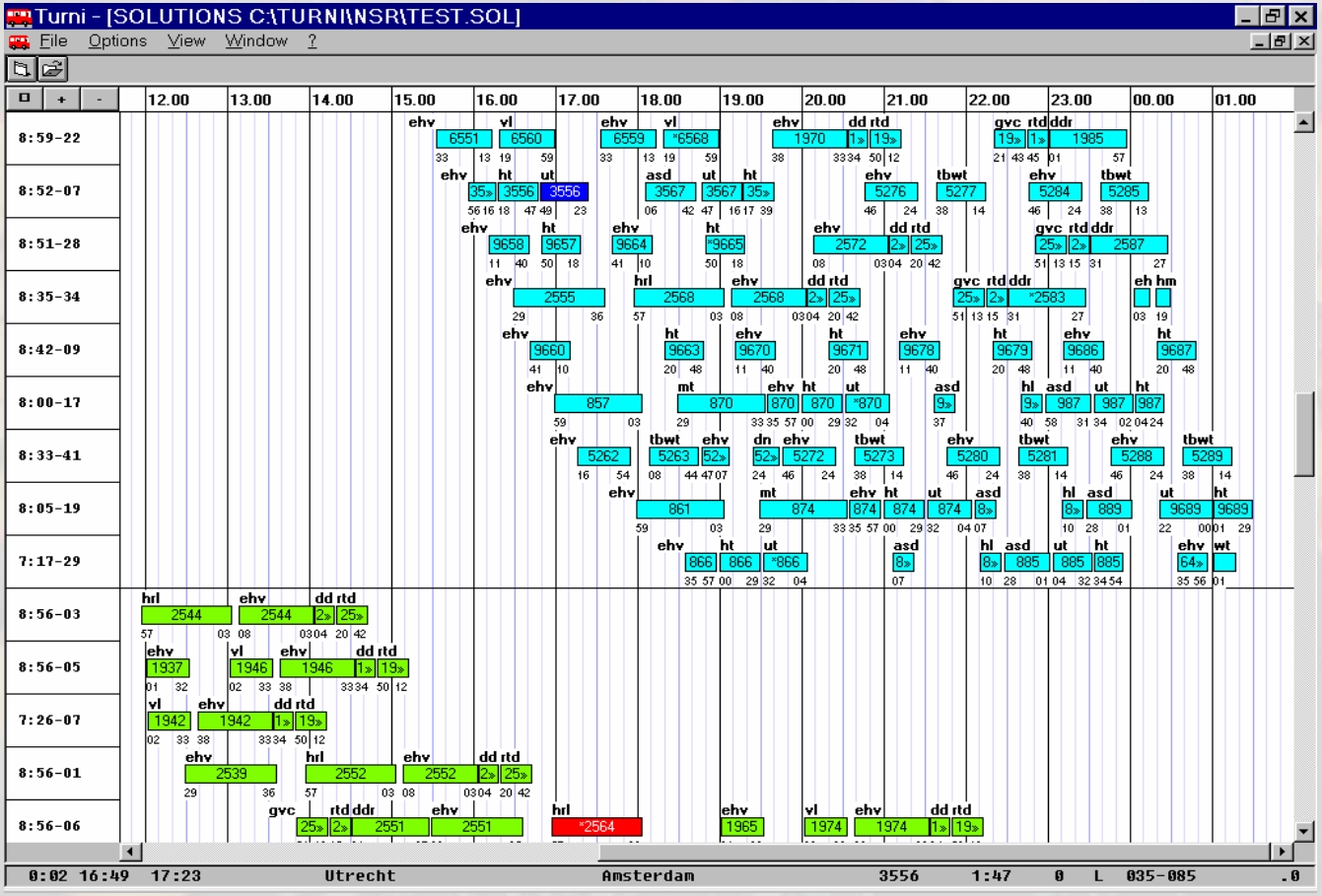
Basic rules for the feasibility of a single duty

- Every duty has to **start** and **end** at the same relief point defining the **home depot** for the duty
- Each trip in the sequence must end at the same relief point from which the **next trip** starts, and the interleaving time between arrival/departure must be large enough to allow for a **safe connection**
- Each work phase starts and ends with so-called PRE and POST times
- Both **working and spread times** in a feasible duty cannot exceed given **bounds**
- **…**



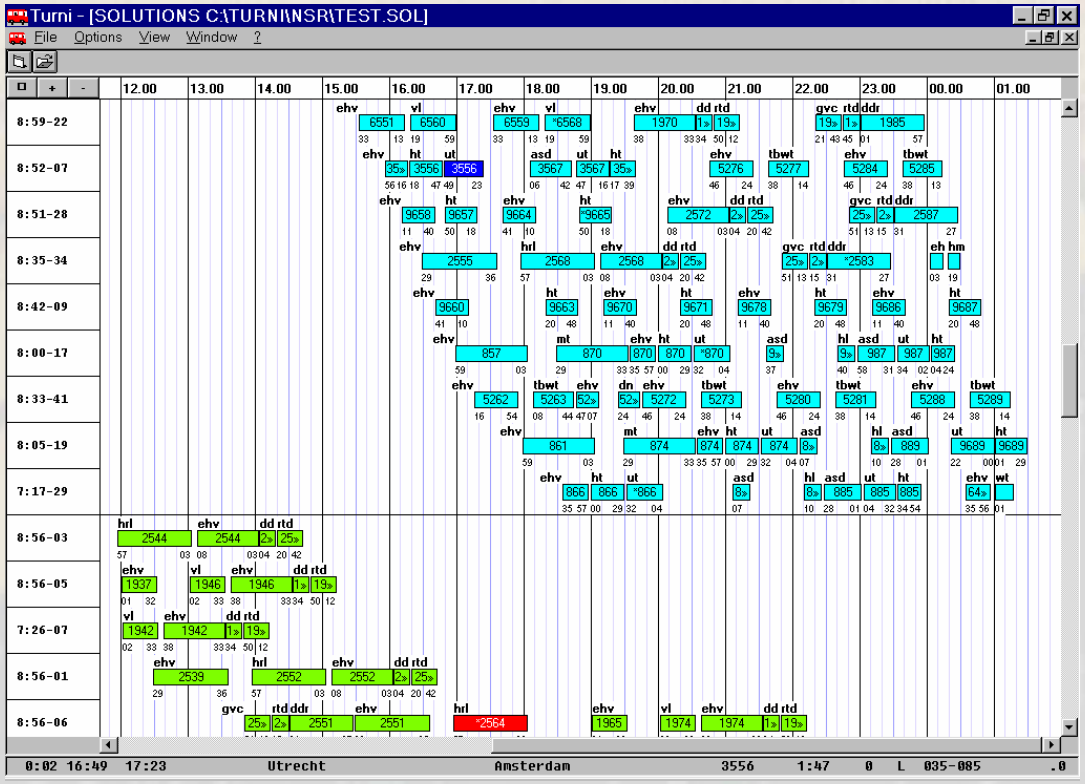
The overall crew schedule

- A **crew scheduling solution** is a set of feasible duties covering all the given trips (a driver is assigned to all trips)



Global constraints

- The **average** working and spread times (as well as some percentages of duties with “**bad**” characteristics) meet pre-defined bounds
- **BASE CONSTRAINTS**: each depot has its own bounds on the average working time, number of duties with a long spread time, etc.
- Passengers penalized (or not allowed)
- …



The problem

- Duties have an associated **cost** (the higher the cost, the less efficient the duty)
- Overall solution cost = sum of the duty costs, plus penalties for “slightly violated” global constraints
- **Find a min-cost crew scheduling solution**

A **very hard** problem!

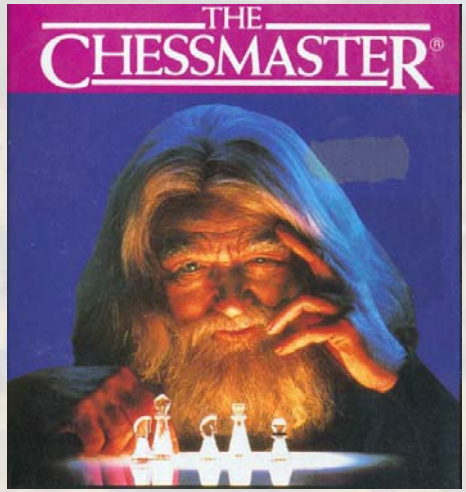
How to solve it?

- **Planners'** approach: start with a "previous-year plan" and modify it by a sequence of trip swaps among duties (local exchanges)
- Kind of art where the **creativity** and **experience** of the planner plays a central role
- Often **smart** solutions are found, but:
  - Time-consuming
  - Local optimizations only (may lose 5-10% efficiency)
  - No quality indicator (how good is the planners' solution?)
  - Poorer solutions with unfamiliar data (bids)
  - Almost impossible to perform a sounded "what-if" analysis
  - A difficult-to-learn art (company know-how loss for retirements)

  → **A key issue for the company is subject to unpredictable events (good planners availability and willingness)**

An automatic solution approach

• **Artificial intelligence**: try to replicate the human approach → unsuccessful…



• Like **chess**, an effective approach cannot mimic the "human masters" → find an alternative approach suited for **computation**

Most powerful solution approach so far → Operations Research with a **generate & select** scheme

The OR approach

1. **Generate** all possible trip combinations that produce a legitimate duty
2. **Select** a min-cost subset of duties covering all trips and satisfying (as a whole) the global constraints

Step 2 can be formalized through a **mathematical model** (kind of "**super-equation"** whose solution gives the required schedule)

Step 1 made possible through a on-line **mathematical pricing scheme** that generate "good" duties only…

$$\min \sum_{d=1}^{D} c_d x_d + \sum_{t=1}^{T} \delta_t s_t + \sum_{h=1}^{H} \vartheta_h \sigma_h \qquad (1)$$

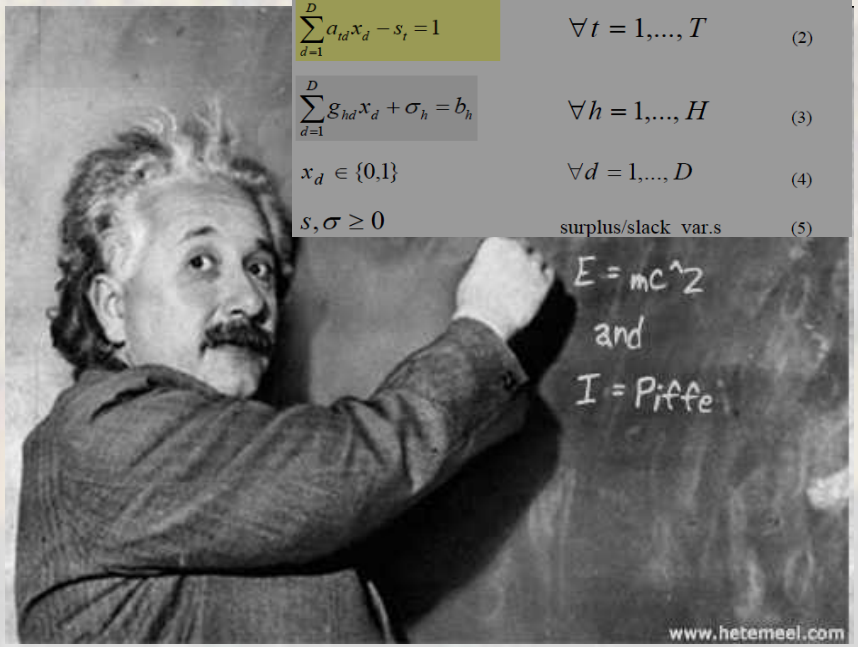
subject to

$$\sum_{d=1}^{D} a_{td} x_d - s_t = 1 \qquad \forall t = 1,\ldots,T \qquad (2)$$

$$\sum_{d=1}^{D} g_{hd} x_d + \sigma_h = b_h \qquad \forall h = 1,\ldots,H \qquad (3)$$

$$x_d \in \{0,1\} \qquad \forall d = 1,\ldots,D \qquad (4)$$

$$s, \sigma \geq 0 \qquad \text{surplus/slack var.s} \qquad (5)$$



Nice, but … does it work in practice?

- **Flexible & fast** computer implementations of the OR approach available on the market
- Provable good results for **airlines** and (more recently) for **railways** and **bus** companies
- **Efficiency** improvements of 2-4% (and sometimes much more)
- A must for **what-if** analysis and **bids**

OR: a technology that works!