Pure Cutting Plane Methods for ILP: a computational perspective

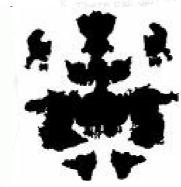
Matteo Fischetti, DEI, University of Padova

The negative of the inverse of the (n in l₆ row of A' and all the unit rows of The dual simplex method consists of resulting in a sequence of matrices A "trial solution" obtained by setting the equal to zero, and then choosing the s satisfy the equations, i.e.,

 $x_1^n = a_{1,0}^n$ The matrix and trial solution are usually so $a_{1,1}^n = 0$ are all nonnegative. They are is the $a_{1,0}^n = 0$ are nonnegative. If A_i^n is of the associated trial solution is the solution the associated trial solution is the solution problem, since the primal feasible proper negative and, since







GOVA steps results in a succession of energy a intinue number of these since there are only a inof nonbasic variables, and any choice uniquely sequently the process must stop. This can happen egative elements in the current α_{0}^{k} , or if there as negative columns $(1/\alpha_{1,0}) \sigma_{1}^{k}$. In the first can obtained, in the second h is easily seen that the phase hargest value that the variable xig can attain being in nonnegative numbers exists.

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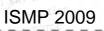
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with the lexicographical dual simplex method
obtain a new "trial solution" of ^{k+1}
obtain a new "trial solution" of ^{k+1}

the some lower bound is known for they'd known that if an integer solution exists why large negative) Norme to the



Rorschach test for OR disorders: can you see the tree?



Wrst Method of Fro

Outline

- 1. Pure cutting plane methods for ILPs: motivation
- 2. Kickoff: Gomory's method for ILPs (1958, fractional cuts)
- 3. Bad (expected) news: very poor if implemented naively

4. Good news: room for more clever implementations

qual to zero, and then or satisfy the equations, i.e.,

Based on joint work with Egon Balas and Arrigo Zanette

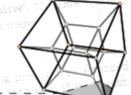


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ical dual simplex method his simplex method need the proof. It is simply th rather long and tedious p

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Motivation

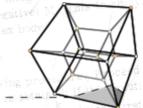
Modern branch-and-cut MIP methods are heavily based on Gomory cuts \rightarrow reduce the number of branching nodes to optimality

However, <u>pure</u> cutting plane methods based on Gomory cuts alone are typically **not used in practice**, due to their poor convergence properties

Branching as a **symptomatic cure** to the well-known drawbacks of Gomory cuts — saturation, bad numerical behavior, etc.

From the cutting plane point of view, however, the cure is even worse than the disease — it hides the **trouble source**!

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The pure cutting plane dimension

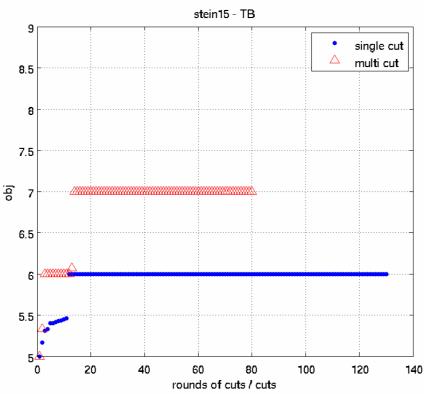
- **Goal**: try to come up with a viable **pure** cutting plane method (i.e., one that is not knocked out by numerical difficulties)...
- ... even if on most problems it will not be competitive with the branch-and-bound based methods
- This talk: Gomory's fractional cuts (FGCs), for several reasons:
 - simple tableau derivation
 - reliable LP validity proof (runtime cut-validity certificate)
 - all integer coefficients → numerically more stable than their mixed-integer counterpart (GMIs)

Rules of the game: cuts from LP tableau

- Main requirement: reading (essentially for free) the FGCs directly from the optimal LP tableau
- Cut separation heavily **entangled** with LP reoptimization!
- Closed loop system (tableau-cut-tableau) without any control valve: highly unstable!
- Intrinsically different from the recent works on the first closure by F. & Lodi (Chvatal-Gomory closure) and Balas & Saxena and Dash, Gunluk & Lodi (GMI/split closure) where separation is an external black-box decoupled from LP reoptimization

Bad news: Stein15 (LP bound) actived to the some row to (b) of the s

The negative of the inverse of the ne is row of A and all the unit ro The dual simplex method consist resulting in a sequence of matrice "trial solution" obtained by settin equal to zero, and then choosing t satisfy the equations, i.e.,



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idded to α_0 , it follows that $\alpha_0 < \alpha_0$, s results in a succession of strictly decreas e number of these since there are only a im onbasic variables, and any choice uniquely ently the process must stop. This can happe we elements in the current α_k^0 , or if there igative columns $(1/\alpha_{1,0}) \sigma_1^R$. In the first cusied, in the second if is eas by seen that the largest value that the variable X_{ij} can attain in homnegative numbers exists.

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z-value \ge some known (possibly large negative) ase if we are dealing with a bounded convex body

Toy set covering instance from MIPLIB; LP bound = 5; ILP optimum = 8

The multi-cut vers. generates rounds of cuts before each LP reopt.

the top down, is positive. Negative is defined similarly.

is given above excel

thle if all/column

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The tip of the iceberg select from these that column

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Bound saturation is just the tip of the iceberg

Let's have a look under the sea...





... with our brand-new 3D glasses

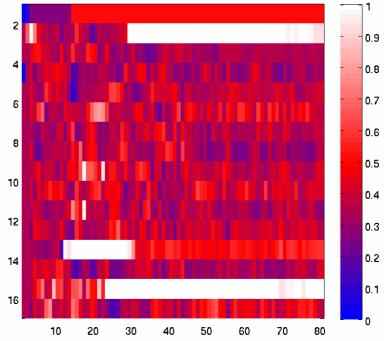




Bad news: Stein15 (LP sol.s)

If simplex method process is some row l_0 ($l_0 \neq 0$) (a) for which $(1/al_0)$ (a) (b) to the least possible in a

| t | t=2 | t=1 | t=0 | iter. |
|----------------------------------|-------|---------|---------|-----------------------|
| x [*] ₁ (t) | 0.433 | 0.499 | 0.500 | X ₁ |
| | | ALDIN V | aipoj | 5 |
| | 1 - 1 | atris | a di in | -1.34 |
| | 0.111 | 0.333 | 0.333 | X_2 |
| x* ₂ (t) | | 1 | | , El |
| | 0.220 | 0.222 | 0.250 | |
| | | | | |
| $\mathbf{x}_{j}^{*}(\mathbf{t})$ | 0.231 | 0.123 | 0.311 | x _j |
| | 0.201 | 0.196 | 0.171 | |
| | | | | |



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Fractionality spectrography: color plot of the LP sol.s (muti-cut

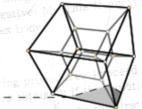
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the **Vers.)** the primal feasible proproblem and, since

• After few iterations, an almost-uniform red plot (very bad...)

We will diso assume that some <u>lower</u>. We will diso assume it is known that if a Inst is we assume it is known that if a z-value ≥ some known (possibly larger ase if we are dealing with a bounded cor-

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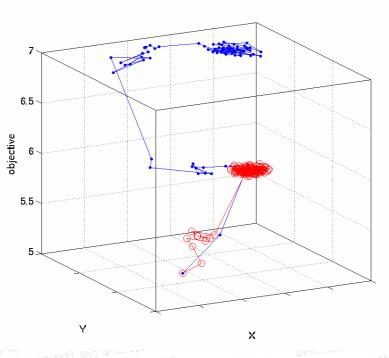


Bad news: Stein15 (LP sol.s)

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We added to α_0 it follows that $\alpha_0 < \alpha_0$, steps results in a succession of strictly decreas inite number of these since there are only a imof nonbasic variables, and any choice uniquely isequently the process must stop. This can happe regative elements in the current α_0^k , or if there no negative columns $(1/a_{1,j}) \alpha_1^k$. In the first curpotained, in the second it is eas by seen that the obtained in the second it is eas by seen that the obtained in nonnegative numbers exists.

OFS

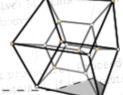
• Plot of the LP-sol. trajectories for **single-cut** (red) and **multi-cut** (blue) versions (multidimensional scaling)

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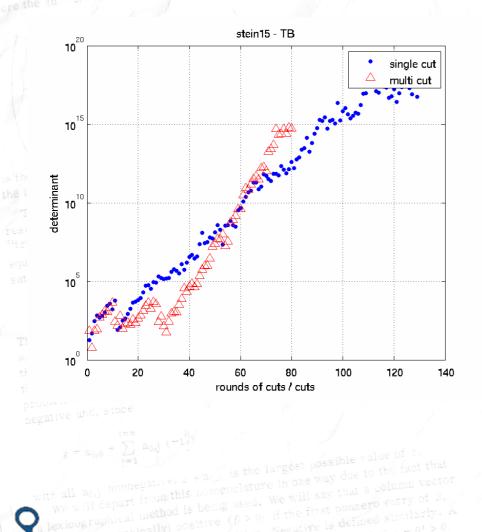
• Both versions collapse after a while \rightarrow no more fuel?



is known for the values of solution exists.



Bad news: Stein15 to the polition in the following way: choose some row lo (1/4)/1 of the negative, and server and server



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• Too much fuel !!



is that α_{0}^{k} is that α_{0}^{k} is the strictly decreased ince there are only a infinite any choice uniquely ast stop. This can happed urrent α_{0}^{k} , or if there $\alpha_{1,2}^{k}$ or if there $\alpha_{1,2}^{k}$ or α_{1}^{k} . In the first can be easily seen that the covariable $x_{1,2}^{k}$ can attain

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a (lexicographically) due pivot steps we choose piv nical dual simplex method i solution? α_0^{k+1} which i products sor α_0^k .

ind is known for the valu integer solution exists, i



Cuts and Pivots

- Very long sequence of cuts that eventually lead to an optimal integer solution \rightarrow cut **side effects** that are typically underestimated when just a few cuts are used within an enumeration scheme
- A must! Pivot strategies to keep the optimal tableau clean so as generate clean cuts in the next iterations
- In particular: avoid cutting LP optimal vertices with a **weird fractionality** (possibly due to numerical inaccuracy)
 - \rightarrow the corresponding LP basis has a large determinant (needed to describe the weird fractionality)
 - → the tableau contains weird entries that lead to weaker and weaker Gomory cuts

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Role of degeneracy

- Dual degeneracy is an intrinsic property of cutting plane methods
- It can play an important role and actually can favor the practical convergence of a cutting plane method...
 - ... provided that it is <u>exploited</u> to choose the cleanest LP solution (and tableau) among the equivalent optimal one
- Unfortunately, by design, efficient LP codes work against us!
 They are so smart in reducing the n. of dual pivots, and of course they stop immediately when primal feasibility is restored!
 → The new LP solution tends to be close to the previous one
 → Small changes in the LP solution imply large determinants
 → Large determinants imply unstable tableaux and shallow cuts
 → Shallow cuts induce smaller and smaller LP solution changes
 → Hopeless!

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Dura lex, sed lex ..

- In his proof of convergence, Gomory used the **lexicographic (dual)** simplex to cope with degeneracy \rightarrow lex-minimize ($x_0 = c^T x, x_1, x_2, ..., x_n$)
- Implementation: use a modern LP solver as a black box:
 - Step 0. Minimize $x_0 \rightarrow optimal value x_0^*$
 - Step 1. Fix $x_0 = x_0^*$, and minimize $x_1 \rightarrow optimal value x_1^*$
 - Step 2. Fix also $x_1 = x_1^*$, and minimize $x_2 \rightarrow 0$ optimal value x_2^*
 - <u>**Key point:**</u> at each step, instead of adding equation $x_j = x_j^*$ explicitly... ... just fix out of the basis all the nonbasic var.s with nonzero reduced cost
 - \rightarrow Sequence of fast (and clean) reoptimizations on smaller and smaller degeneracy subspaces, leading to the required lex-optimal tableau

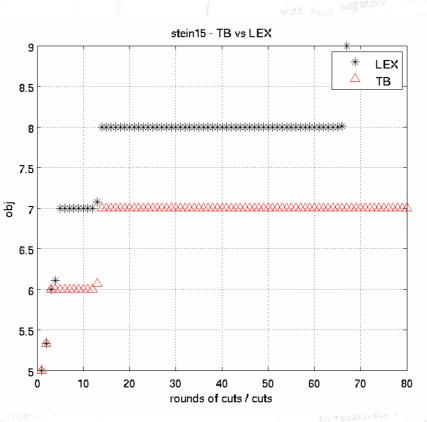
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• Lex-min useful for the convergence proof, but ... also in practice?

Good news #1: Stein15 (LP bound)

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The matrix and trial solution ara(1, j =0 are all nonnegative. 1 the al, b i =0 are nonnegative. the associated trial solution is problem, since the primal feasib negative und, since



LP bound = 5; ILP optimum = 8 and that in all succeeding pivot steps we choose pivot steps we choose pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps we choose pivot steps and that in all succeeding pivot steps are choose pivot steps and that in all succeeding pivot steps we choose pivot steps are choose pivot steps and that in all succeeding pivot steps are choose pivot steps are choose

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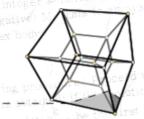
80 11 pse the lexicographical dual simplex method 11 is not implied that this simplex method need 14 is necessary to the proof. It is simply the 15 reduced the original rather long and tedious pro-15 s. 16 is necessary to the proof. It is simply the 16 is simple the original rather long and tedious pro-16 is sume then that we have obtained a (lexicographically) dealers

TB = "Text-Book" **multi-cut** vers. (as before)

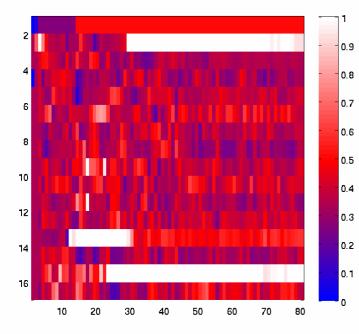
LEX = single-cut with lex-optimization

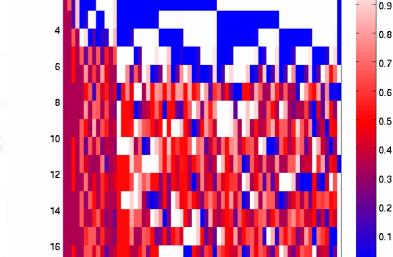
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Good news #1: Stein15 (LP sol.s) are still positive and





0.8

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0.4

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TB = multi-cut vers. (as before)

LEX = single-cut with lex-optimization

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Fractionality spectrography

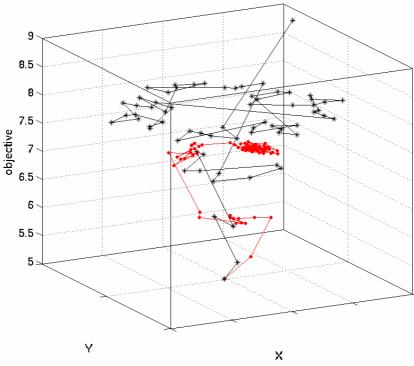
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Good news #1: Stein15 (LP sol.s) for which (1/4) of the least begative, and solution the following way choose solution which (1/4) of the least begative.

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the lexicographical dual simplex metho lot implied that this simplex method nee is necessary to the proof. It is simply t ed the original rather long and tedious t

Plot of the LP-sol. trajectories for TB (red) and LEX (black) versions

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given above choel

elements in according After each pivot then we obtain a new "trial solution" u

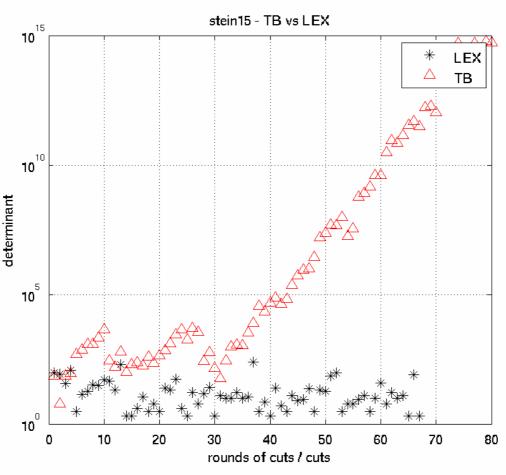
(X,Y) = 2D representation of the x-space (multidimensional scaling)

We will also we assume it is known that it and a z-value ≥ some known (possibly large negative zase if we are dealing with a bounded convex case if we are dealing with a bounded convex

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Good news #1: Stein15 (determinants)



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TB = multi-cut vers. (as before) exicographical method is being used. We will say that a column vect

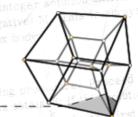
the top down, is positive. Negative is defined similarly. teater than another column vector β^* if $\beta = \beta^* > 0$.

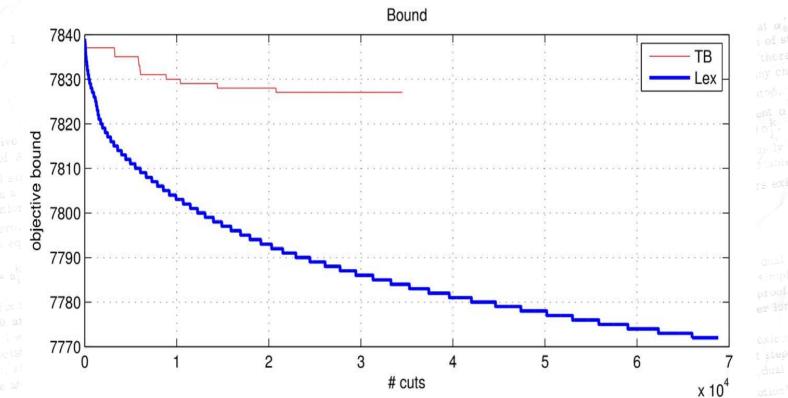
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LEX = single-cut with lex-opt.

Inst is we assume it is known that if an integer solution exists. z-value > some known (possibly large negative) case if we are dealing with a bounded convex bod





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TB = multi-cut vers. (as before)

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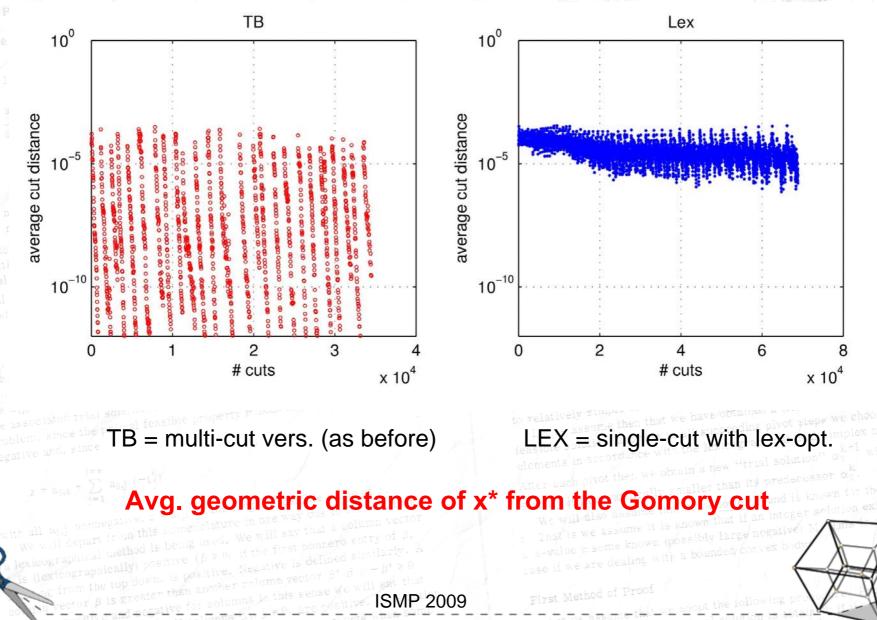
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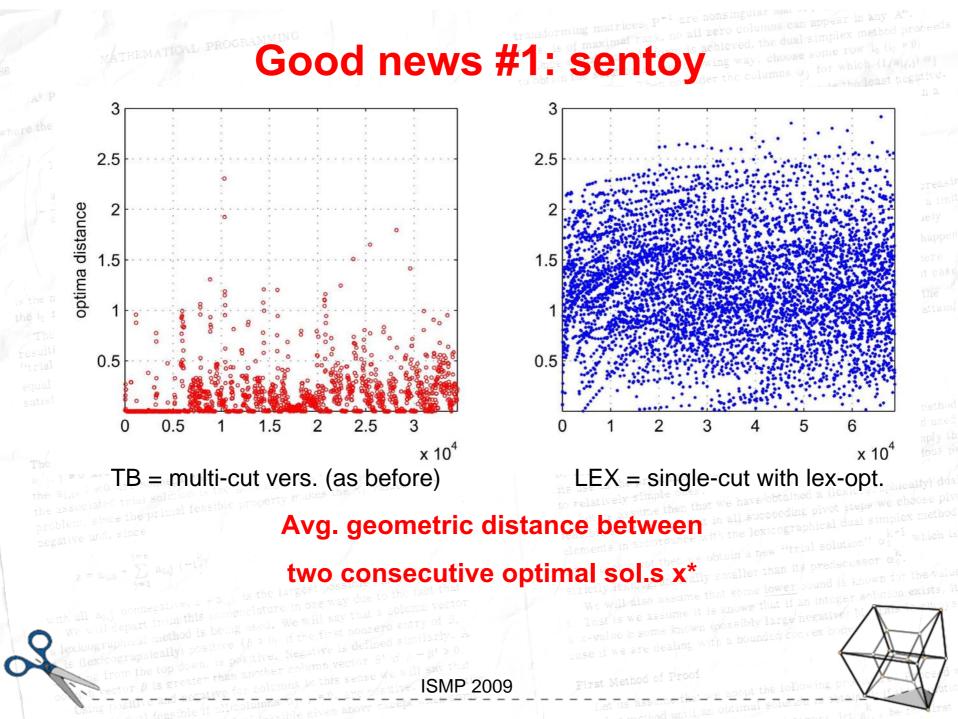
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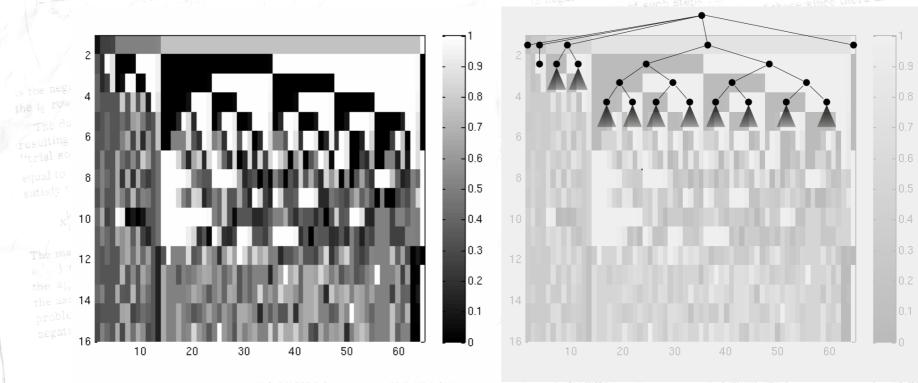
Good news #1: sentoy





Ok, it works ... but M

Enumerative interpretation of the Gomory method (Nourie & Venta, 1982)



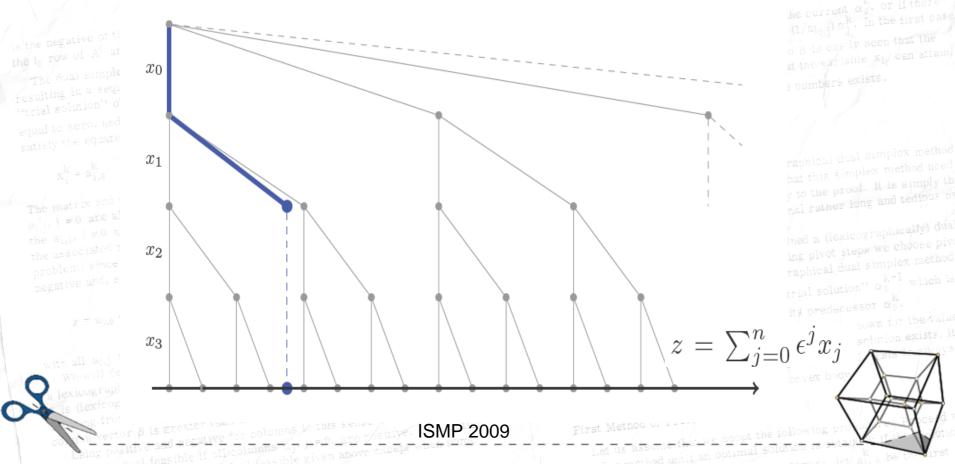
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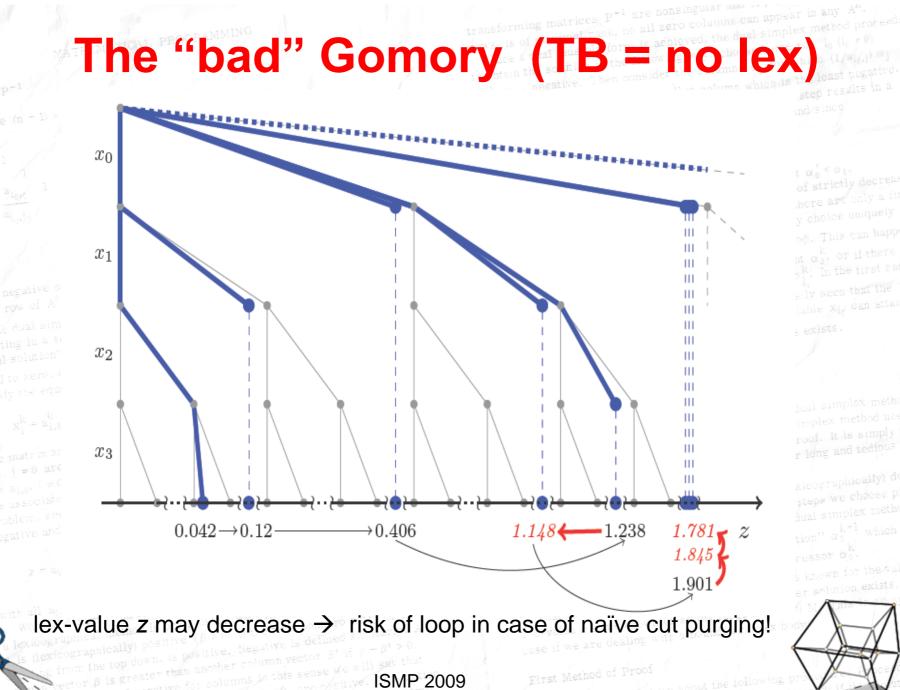
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The underlying enumeration tree

- Any fractional solution x^* can be visualized on a lex-tree
- The structure of the tree is fixed (for a given lex-order of the var.s)
- Leaves correspond to integer sol.s of increasing lex-value (left to right)





le given above excep

| | | | | | X _h | | | | | | X _j | rix A" | in whic | X ₁₀ | | | X _{2!} | 5 | X |
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| X [*] _h | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | - | | + | | + | + | | | + | | 0 |
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basic var.s $\rightarrow \leftarrow$ RHS ←

from the top down, is positive. Negative is defined similarly.

nonbasic var.s

to obtain the solution in the following way: choose some row i_0 $(i_0\neq \emptyset)$

Ingr'is we assume it is known that if an intoger z-value ≥ some known (possibly large negative) case if we are dealing with a bounded convex body

consider the columns of for which (1/al()) of

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<u>Green row:</u> nonbasic "+" var. x_i increases \rightarrow a basic var x_k with k < h increases rictly lexicographically

The key FGC property for convergence

Take the tableau row associated with the (lex) **first** fractional var. x_h^*

$$x_h + \sum_{j \in J^-} \overline{a}_{ij} x_j + \sum_{j \in J^+} \overline{a}_{ij} x_j = \overline{a}_{i0} (= x_h^*)$$

where $J^{-} = \{ j : \overline{a}_{ij} < 0 \}$ and $J^{+} = \{ j : \overline{a}_{ij} > 0 \}$

We want to lex-increase the optimal value \rightarrow add a FGC in its \geq form:

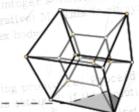
$$x_h + \sum_{j \in J^-} \lceil \overline{a}_{ij} \rceil x_j + \sum_{j \in J^+} \lceil \overline{a}_{ij} \rceil x_j \ge \lceil x_h^* \rceil$$

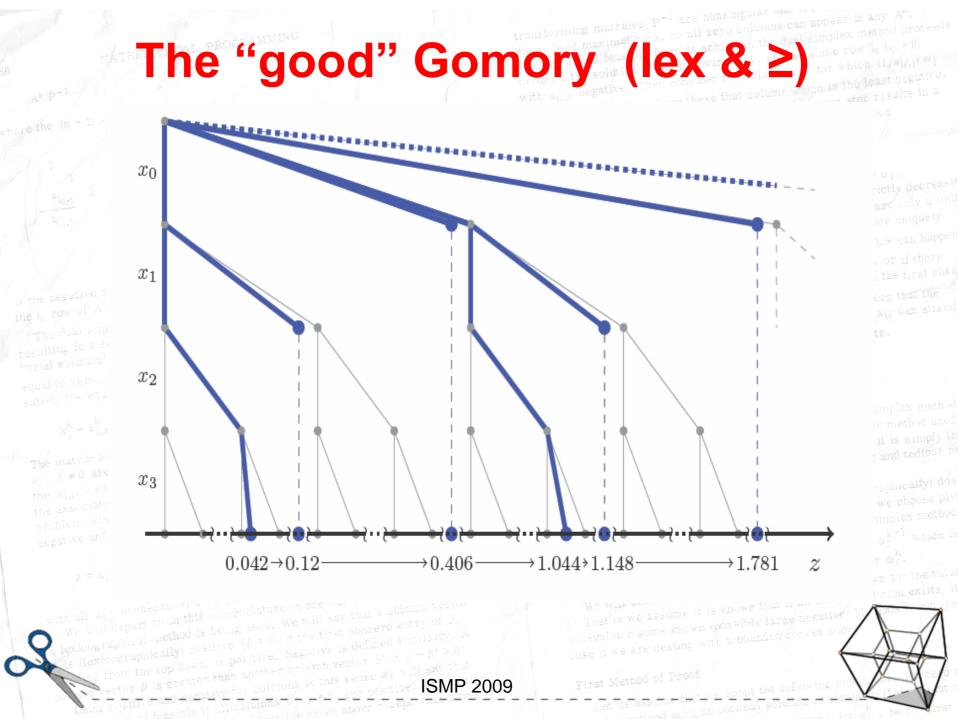
(a FGC in its \leq form will not work!). Two cases for the new LP-opt. x

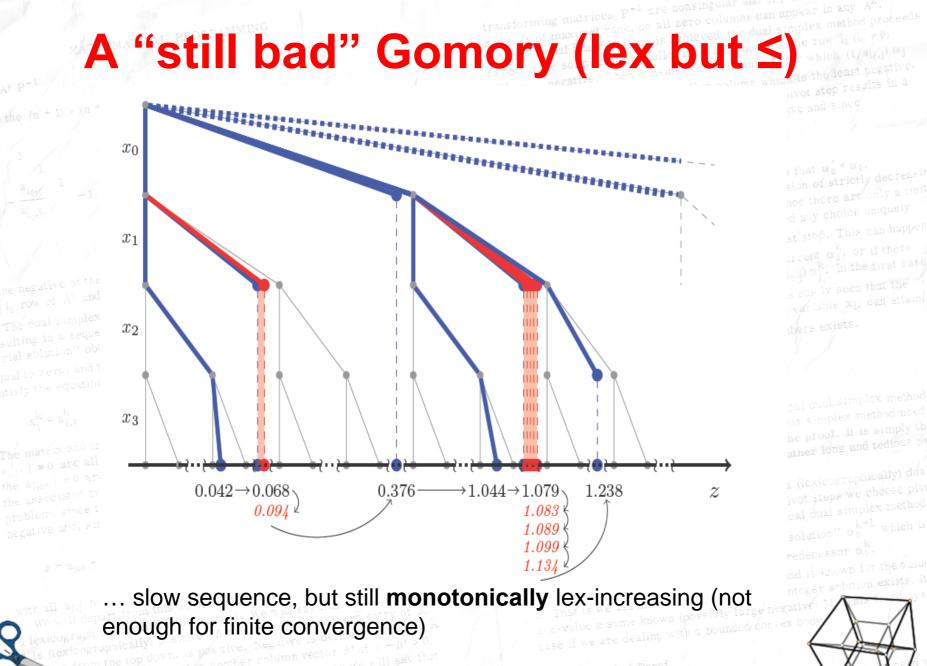
 $[\mathsf{BRANCH}] x_j = 0 \text{ for all } j \in J^+ \rightarrow \qquad x_h \ge \lceil \overline{a}_{i0} \rceil - \sum_{j \in J^-} \lceil \overline{a}_{ij} \rceil x_j \ge \lceil x_h^* \rceil$

[BACKTRACK] otherwise, a "previous component" increases → BIG lex-









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is given above choel

Let us assume that we adop

Lessons learned

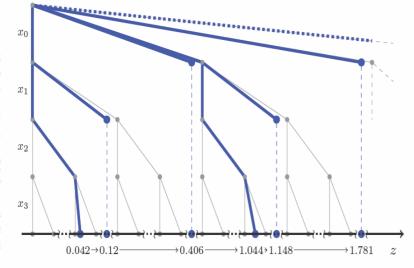
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The Gomory method is **framed** within its enumerative cast

"Good" FGCs may allow for large backtracking steps, but they cannot modify the underlying tree

Inefficient depth-first branching on an unnatural variable order → branching even on integer-valued variables!!

X_i = C_{1,0} The matrix and trial a_{1,0} j =0 are all no the a_{1,0} i =0 are n the associated trial problem, since the negative and, since



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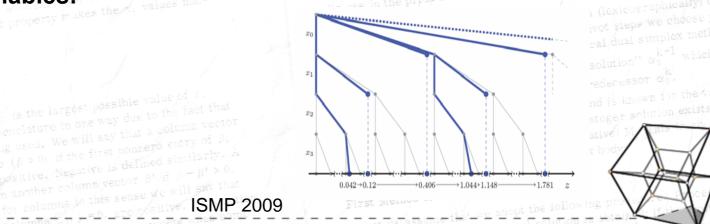
which is

the Aalu 101100 exists, 1

Good news #2: lex on the fl

Facts:

- If x^{*}_h is the **first** fractional var. of the current lex-optimal LP sol., there is no harm in changing the lex sequence from position *h*
- Our lex-reoptimization method allows one to do this "**natively**", in an effective way
- \bullet The first fractional var. $x^*_{\ h}$ plays the role of the **branching** var. in enumerative method
- One can borrow from enumerative methods any clever selection policy for the branching variable x_b^* (b for branching), and move this var. in the h-th position of the current lex-order \rightarrow (hopefully) **no more branchings on integer variables!**



Variants: get rid of the obj. function

The first branching variable x_0 is the objective function \rightarrow a very unnatural choice for an enumerative method!

In some cases, this choice forces Gomory's method to visit a same subtree several times (see e.g. the Cook-Kannan-Schrijver example below)

→ Try to get rid of the obj. function: use of invalid cuts (L-CP), binary search, etc. **BUT: are these still** <u>pure</u> cutting plane methods ??

Let z := 1000 y

z integer

 $\begin{array}{c} -z \\ x_1 \\ x_2 \\ x_2 \\ x_1 = \begin{bmatrix} -666, 667\\ 0.667\\ 0.667\\ 0.667\\ 0.667\\ 0.666\\ \end{bmatrix} \begin{bmatrix} -664, 671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.671\\ 0.750\\ 0.750\\ 0.750\\ 0.750\\ 0.750\\ 0.750\\ 0.750\\ 0.750\\ 0.750\\ 0.751\\ 0.751\\ 0.751\\ 0.751\\ 0.751\\ 0.099\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.999\\ 0.990\\ 0.999\\ 0.999\\ 0.990\\ 0.999\\ 0.990\\ 0.990\\ 0.990\\ 0.990\\ 0.999\\ 0.990\\$

 $\max\{y : x_1 + x_2 + y \le 2, y \le x_1, y \le x_2, x_1, x_2 \in \{0, 1\}, y \ge 0\}$

graphical dual simplex metho that this simplex method use ry to the proof. It is simply t intal rather long and tedious a

ained a (lexicographically) du ding pivot steps we choose pigraphical dual simplex metho "trial solution" α_0^{k+1} which "trial predecessor α_0^k .

er bound is known for the val if an integer solution exists, ge negative) is a solution of the solution of the



Role of cuts & dynamic lex-order

| | L- | CP | L | -BB | L-CI | P.dyn | L-BB.dyn | | |
|------------|---------|---------|---------|-----------|---------|------------|----------|----------|--|
| Instance | Time | Nodes | Time | Nodes | Time | Nodes | Time | Nodes | |
| bm23 | 2.65 | 2205 | 0.29 | 3372 | 0.96 | 1013 | 0.13 | 1358 | |
| hard_ks100 | 16.67 | 41303 | 7200 | 122828201 | 15.18 | 24077 | 4084.83 | 56872670 | |
| hard_ks9 | 0.07 | 164 | 0.01 | 274 | 0.05 | 133 | 0.01 | 228 | |
| l152lav | 705.22 | 1111733 | 1876.69 | 3335512 | 7201.47 | 634042 | 1406.11 | 1354164 | |
| lseu | 199.6 | 191256 | 1153.73 | 15089374 | 46.59 | 23327 | 12.38 | 109578 | |
| manna81 | 2991.28 | 142013 | 7200 | 3321404 | 7202.88 | 8276 | 7200 | 2692153 | |
| p0033 | 0.16 | 380 | 0.62 | 11538 | 0.17 | 307 | 0.83 | 12192 | |
| p0201 | 117.79 | 58101 | 59.07 | 357664 | 38.56 | 11127 | 6.51 | 26942 | |
| pipex | 9.01 | 6474 | 1.31 | 13904 | 2.83 | 2198 | 0.72 | 7686 | |
| sentoy | 114.56 | 103349 | 20.04 | 197510 | 2.39 | 1460 | 0.16 | 2038 | |
| stein15 | 0.1 | 123 | 0.03 | 418 | 0.11 | 97 | 0.02 | 260 | |
| stein27 | 6.58 | 4160 | 1.62 | 13260 | 6.47 | 3551 | 1.22 | 9210 | |

L-CP and **L-B&B** work on the same underlying tree (L-CP exploiting FGCs)

*.dyn versions modify the lex-order on the fly (no branching on integer var.s)

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THEMATICAL PROGRAMMING THEMATICAL PROGRAMMING Computational is of maximal rank, no all zero columns can appear in any A^A. Computational test is of maximal rank, no all zero columns can appear in any A^A. Lex-FGC Lex-FGC

| | | | Lex-FG | \mathbf{C} | | | | Lex-FGC. | lyn | |
|------------|---|--------|---------|--------------|--------|---|---------|-----------|----------|--------|
| Instance | | #Itr.s | Cuts | Time | Cl.Gap | | #Itr.s | Cuts | Time | Cl.Gap |
| air4 | Т | 434 | 140191 | 7207.51 | 25.19 | Т | 559 | 178404 | 7218.26 | 36.17 |
| air5 | Т | 833 | 240266 | 7209.73 | 25.26 | Т | 1119 | 321186 | 7209.53 | 30.7 |
| bm23 | 0 | 660 | 8294 | 2 | 100 | 0 | 652 | 8047 | 0.99 | 100 |
| cap6000 | Т | 16043 | 107239 | 7201.43 | 32.64 | Т | 76649 | 493654 | 7201.18 | 83.16 |
| hard_ks100 | 0 | 99 | 483 | 0.43 | 100 | 0 | 809 | 4100 | 2.73 | 100 |
| hard_ks9 | 0 | 141 | 609 | 0.22 | 100 | 0 | 130 | 542 | 0.08 | 100 |
| krob200 | 0 | 41 | 1643 | 93.94 | 100 | 0 | 23 | 375 | 18.64 | 100 |
| l152lav | 0 | 744 | 25109 | 118.2 | 100 | Т | 42184 | 1911790 | 7201.22 | 86.29 |
| lin318 | 0 | 28 | 1001 | 200.08 | 100 | 0 | 27 | 848 | 67.79 | 100 |
| lseu | 0 | 9591 | 133589 | 53.24 | 100 | 0 | 4261 | 53522 | 11.75 | 100 |
| manna81 | 0 | 12 | 280 | 19.05 | 100 | 0 | 12 | 280 | 12.63 | 100 |
| mitre | Т | 565 | 116369 | 7209.36 | 90.83 | 0 | 786 | 132399 | 6125.11 | 100 |
| mzzv11 | Т | 16 | 14516 | 8533.94 | 30.86 | Т | 28 | 23496 | 7409.5 | 37.18 |
| mzzv42z | Т | 19 | 15264 | 7519.35 | 17.45 | Т | 31 | 24081 | 7550.08 | 23.14 |
| p0033 | 0 | 501 | 4421 | 1.2 | 100 | 0 | 1214 | 10622 | 2.12 | 100 |
| p0201 | Т | 190383 | 5471004 | 7201.06 | 87.3 | Т | 259143 | 8984687 | 7201.05 | 96.49 |
| p0548 | Т | 129823 | 4832247 | 7201.06 | 76.86 | С | 12412 | 450566 | 110781.7 | 53.21 |
| p2756 | Т | 51454 | 673642 | 7201.06 | 79.78 | Ε | 134 | 7550 | 7.35 | 79.78 |
| pipex | 0 | 473607 | 4583701 | 1591.97 | 100 | С | 1081606 | ¿10000000 | 2219.38 | 97.51 |
| protfold | Т | 144 | 57617 | 7253.09 | 45.26 | Т | 253 | 94771 | 7203.1 | 36.13 |
| sentoy | 0 | 5338 | 68991 | 24.36 | 100 | 0 | 3170 | 40765 | 6.52 | 100 |
| seymour | Т | 117 | 67931 | 7224.93 | 26.89 | Т | 149 | 84533 | 7217.92 | 32.11 |
| stein15 | 0 | 68 | 708 | 0.15 | 100 | 0 | 59 | 641 | 0.1 | 100 |
| stein 27 | 0 | 3134 | 35861 | 13.96 | 100 | 0 | 2250 | 27462 | 5.63 | 100 |
| timtab | Т | 5193 | 1675111 | 7344.59 | 50.44 | Т | 4090 | 1320383 | 7550.76 | 46.37 |

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Wirst Method of Pl

MATHEMATICAL PROGRAM /Aº P-1 where the $(n + 1) \rightarrow (n + 1)$ multip P^{-1}

> ain.n -1/310.1

(7-4

is the negative of the inverse of the $(n + 1) \times (n + 1)$ matrix the 10 row of A6 and 211 the unit rows except the one int The dual simplex method gonsists of a succession "trial solution" obtained by sotting the express consist va resulting in a sequence of matrices A equal to zero, and then choosing the current besie variables, the the satisfy the equations, i.e.,

 $\mathbf{x}_{i}^{\mathbf{k}} = \mathbf{a}_{i,0}^{\mathbf{k}}$

The matrix and trial solution are usually said to be "dual feas a(1, j = 0 are all nonnegative. They are is the (or primal tensible) if all the al,o, i =0 are nonnegative. If At is both primal and dual least the associated trial solution is the spinisting to the l problem, since the primal feasible property makes the xi values negative and, since

 $z = a_{0,0} + \sum_{i=1}^{i=n} a_{0,i} (-L_{j}^{k_{ij}})$ with all $a_{5,j}$ nonnegative, $z \neq a_{5,5}$ is the largest possible value of z. We will depart from this nomenclature in one way due to the fact that a lexicographical method is being used. We will say that a column vector β is (lexicographically) positive ($\beta > 0$) if the first ponzero entry of β , counting from the top down, is positive. Negative is defined similarly. A column vector β is greater than another column vector $\beta^*/\text{if } \beta - \beta^* > 0$. this sense we will set that Controlumns of i =0, are positive. This Action rows, is of maxing to obtain the solution in the following way: choose some row 10 (10 PW) with alo, negative. Then consider the columns wi for which (1/alo) al is negative, and select from these that column which is the least pegative. It is easily verified that this pivot step results in a new matrix A' in which the columns are still positive and since

(a negative tec**G**r) has been atter (a negative tec**G**) has been atter (b no find to α_0 it follows that $\alpha'_0 < \alpha_0$. On of strictly decreasing mber of these since there are only a imit per of possible sets of nonpusse terrisber and any choice uniquely workers a stop. This can happen Lex. dual simplex to make current ag, or if there tog has been obtained, in the second it is easily seen that the is the largest value that the veriable xit can attain. and consequently no polution in nonnegative numbers exists.

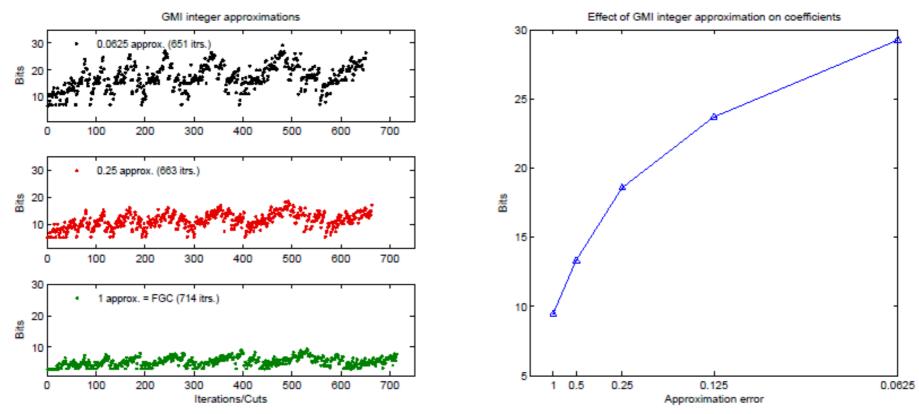
8. FINITENESS PROOFS

In these proois we will use the lexicographical dual simplex method described in Section 7. It is not implied that this simplex method need be used in practice or that it is necessary to the proof. It is simply th -its use in the proof has reduced the original rather long and tedious pr

et us assume then that we have obtained a (lexicographically) dua to relatively simple ones. ea in color and that in all succeeding pivot steps we choose pivelements in according to the succeeding pivot steps we choose pivelements in according to the succeeding pivot steps we choose pivelements in according to the succeeding pivot steps we choose pivelements in according to the succeeding pivot steps we choose pivelements in according to the succeeding pivot steps we choose pivelements in a succeeding pivelement steps we choose pivelements in a succeeding pivelement steps we choose pive After each pivot then we obtain a new "trial solution" α_0^{k+1} which is strictly lexicographically smaller than its predecessor α_0^K . We will also assume that some lower bound is known for the value 2. Inst is we assume it is known that if an integer solution exists, it a z-value > some known (possibly large negative) M. This is always case if we are dealing with a bounded convex body

Let us assume that we adopt the following procedure. Proceed a First Method of Proof

Question: what about GMI cuts?



Bits required to represent the integer cut coeff.s when approximating GMI cuts (approx. error =1 for FGCs, approx. error = 0 for GMIs)

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 \rightarrow GMI cuts appear numerically much more difficult to handle (at least, in a pure cutting plane context ...)