A Lagrangian Heuristic for Robust Train Timetabling

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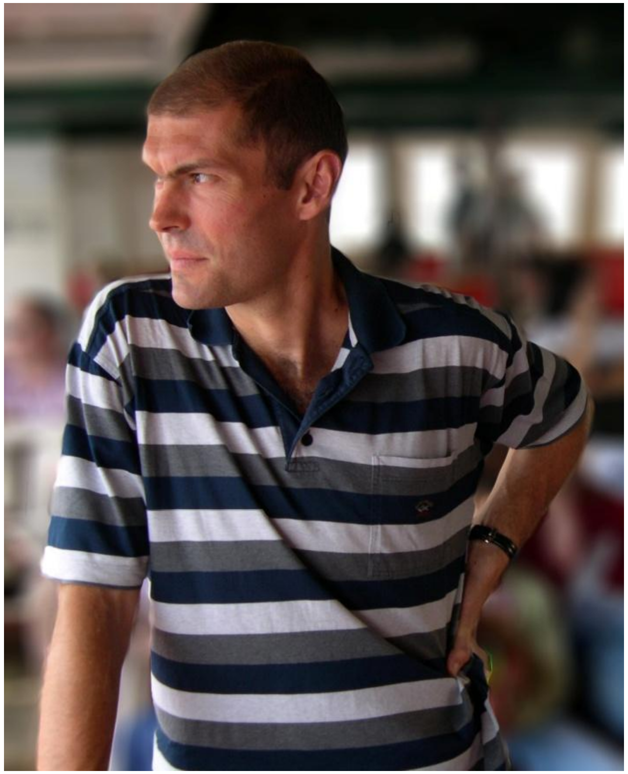


Lagrangian + subgradient opt. in a (dual) heuristic context

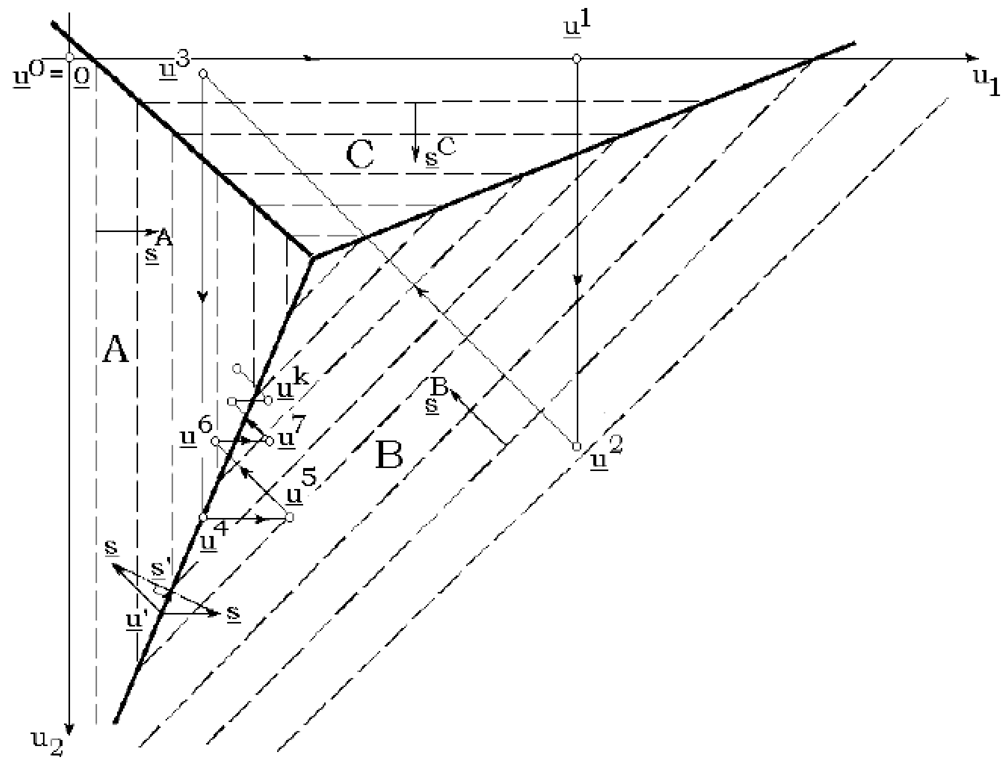
- Lagrangian vs LP

→ Lagrangian costs often **more informative** than LP reduced costs when used to drive heuristics

- Subgradient vs bundle

→subgradient as a quick-and-dirty way to **sample** near-optimal dual vectors

- Lagrangian+subgradient attractive for heuristics



Light Robustness

(F. and Monaci, 2009)

- Robust optimization avoids risky sol.s that **overfit** the nominal model
- **Light Robustness**: constraint slacks used as a proxy for robustness

Let x^* be an optimal solution of nominal problem

$$\min \sum_{j \in N} c_j \, x_j$$

$$\sum_{j \in N} a_{ij} \, x_j \leq b_i \quad i \in M$$

$$x_j \geq 0 \quad j \in N$$

$$\max \ \sigma$$

$$\sum_{j \in N} a_{ij} \, x_j + s_i = b_i \quad i \in M$$

$$\sigma \leq \frac{s_i}{L_i^*} \quad i \in U$$

$$\sum_{j \in N} c_j \, x_j \leq (1+\delta) \, z^*$$

$$x_j \geq 0 \quad j \in N$$

$$s_i \geq 0 \quad i \in M$$

Lagrangian for (Light) Robustness

- **Idea**

1. Add a **parameter** to the objective function as a dial between cost and (total) slacks

2. Use a **Lagrangian heuristic** as a black-box, and collect good (undominated) solutions for both objectives (cost and slack)

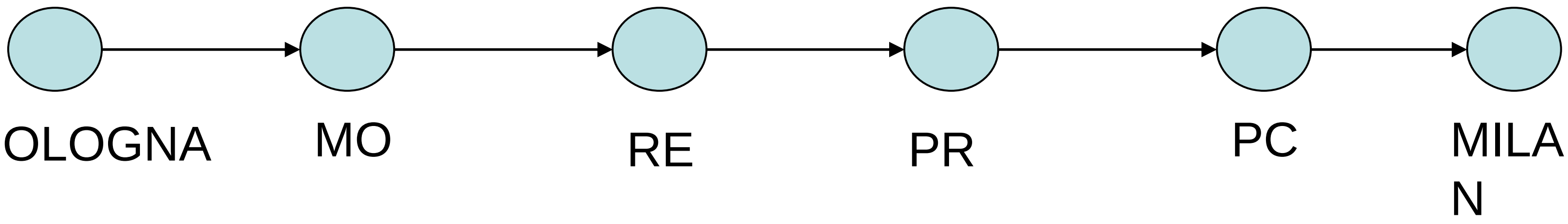
3. To favor **diversification**, smoothly change the parameter during Lagrangian optimiztion (only cost → only slacks)

4. Select and validate the final solution(s) by running a (possibly time consuming) **simulation tool**

Nominal Train Timetabling Problem

INPUT :

- **Single Line with a one-way track** (approach easy to extend to railway network)



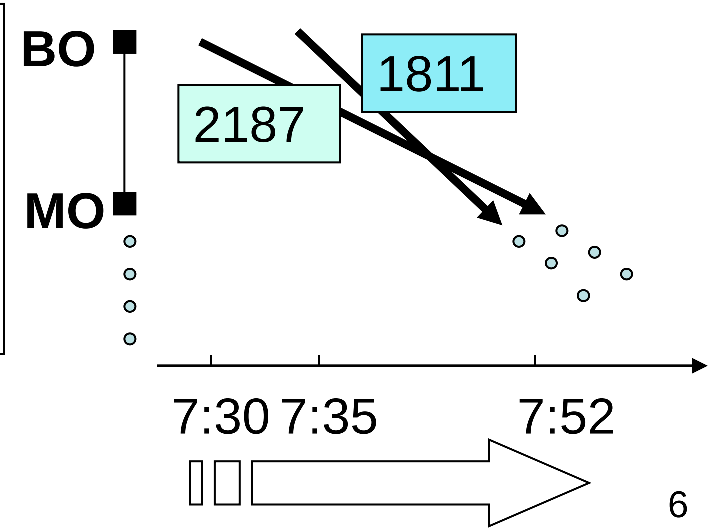
- **List T of Trains with "ideal timetables"**

Ideal Timetables are CONFLICTING!!!!

EUROSTAR 1811: BO 7:35 - MI 9:10

REGIONAL 2187:

BO	7:30	-	MO	7:52
MO	7:54	-	RE	8:12
RE	8:14	-	PR	8:26
PR	8:28	-	PC	8:55



Track Capacity Constraints:

- **no overtaking** between stations (allowed only within stations)
- **min time** between consecutive **departures** from each station
- **min time** between consecutive **arrivals** at each station

OUTPUT :

- **“Adjusted” non-conflicting timetables with maximum total profit:**

Train Adjustments:

- **shift** departure time from initial station
- **stretch** stopping time at intermediate stations

Train Profit:

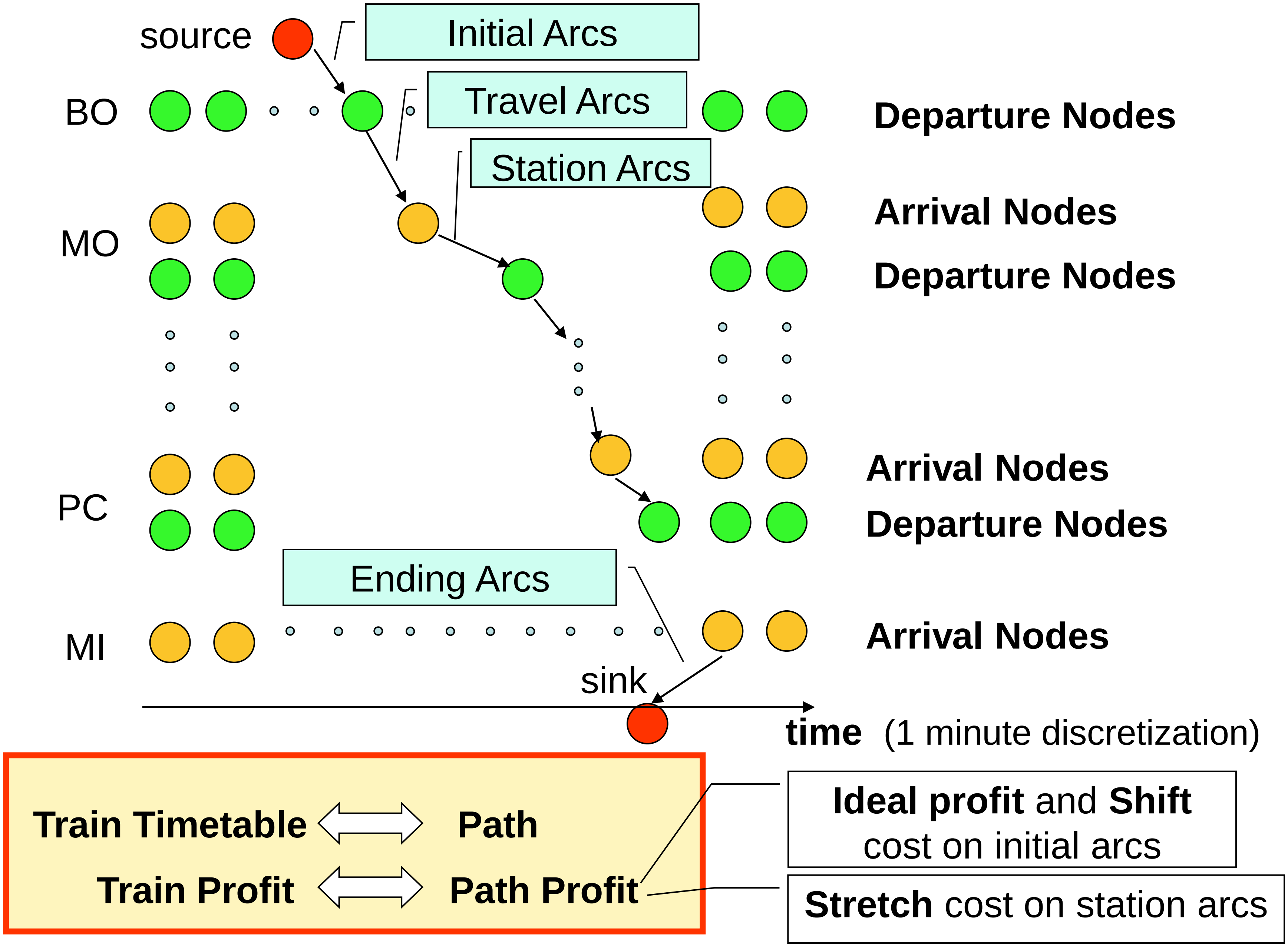
$$\pi_t - \phi_t(shift_t) - \sum_i \varphi_{it}(stretch_{it})$$

Ideal profit

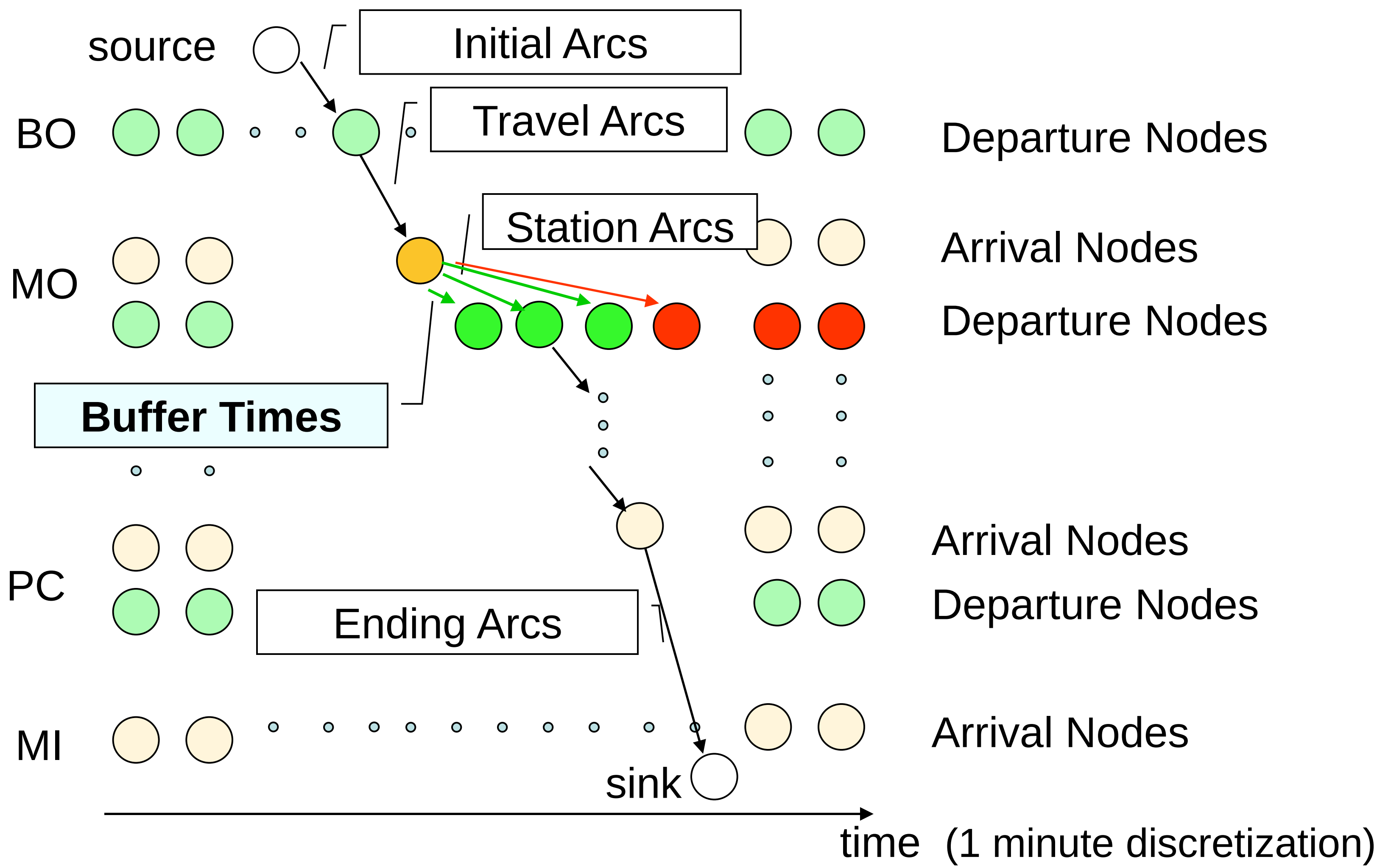
Arbitrary monotone functions

If profit is null or negative cancel the train

Representation on Time-Space Graph



Robust Train Timetabling Problem



Robust Train Timetabling Problem

$$\max \sum_{t \in T} \sum_{r \in R^t} p_r x_r + F_K \sum_{t \in T} \sum_{r \in R^t} b_r^t x_r$$

Efficiency

Buffer Times

Track capacity constraints are relaxed in a Lagrangian way.

The Lagrangian relaxed problem calls for a set of paths for the trains, each having maximum Lagrangian profit (given by the sum of the original profits for the arcs in the path including the weights for the buffer times, minus the sum of the penalties assigned to the nodes visited by the path).

Robust Train Timetabling Problem

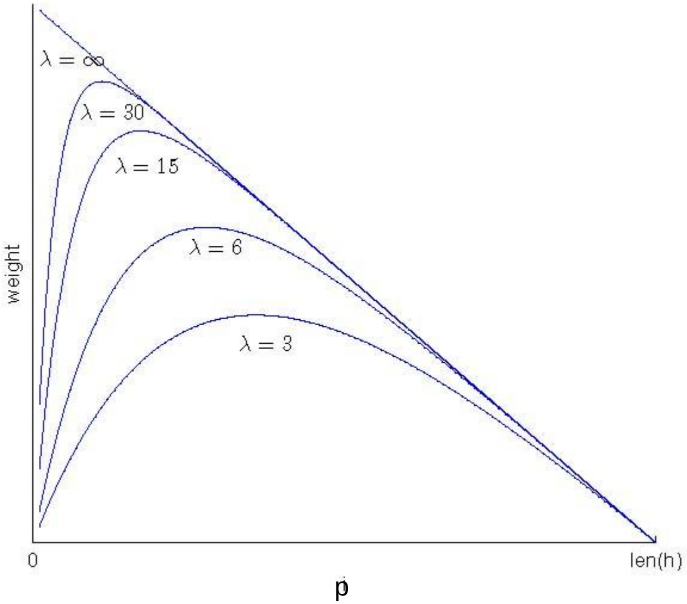
$$\max \sum_{t\in T}\sum_{r\in R^t} p_r x_r + F_K \sum_{t\in T}\sum_{r\in R^t} b_r^t x_r$$

F_K changes along the subgradient iterations (first push efficiency, later robustness)

$$b_r^t = \min\{q_r^t, M\}(1-e^{-\lambda p})(len(t)-p)$$

Minutes of buffer

Max buffer



It is observed by Kroon et al. (2007) that buffers that are placed too early are not very useful (since the probability to face any delay at this early position is very small).

Robust Train Timetabling Problem

Heuristic Algorithm

A Lagrangian-based heuristic algorithm is developed, in a subgradient framework

Local search procedures are used to improve the solution found

Validation Method

A simulation tool is used to test the robustness of the heuristic solution

Given a solution, it considers different realistic external delay scenarios and, assuming that all the trains in the solution have to be scheduled and all train precedences are fixed, adapts the solution to make it feasible with the given external delays, evaluating the resulting cumulative delay.

Computational Experiments

Instance	Number of trains	Nominal solution (efficiency)	Nominal Cumulative delay	Time (sec)	Robust solution (efficiency)	Robust Cumulative delay	Time (sec)
Ch-Ro	41	5567	39181	462	5515 5418 5418	35241 33658 33658	1753
Md-Mi	100	9316	17027	566	9240 8935 8621	14657 12902 12743	2299
Md-Mi	200	18542	37365	1830	18357 17668 16723	35094 26889 24419	5136
Md-Mi	300	24638	45145	3479	24412 23410 22181	44738 37733 32094	9365
Md-Mi	400	27259	53059	5227	26989 25958 24540	49798 44432 38095	12150
Ch-Mi	194	20816	3068	519	20618 20252 20252	2906 2739 2739	1471

Comparison with F., Salvagnin, Zanette (Transp. Sci. 2009)

Instance	Number of trains	Robust solution (efficiency)	Robust Cumulative delay	Time (sec)	Robust solution FSZ	Robust Cumulative delay FSZ	Time (sec)
Ch-Ro	41	5515 5418 5418	35241 33658 33658	1753	5512 5289 5011	37332 35081 31849	14862
Md-Mi	100	9240 8935 8621	14657 12902 12743	2299	9209 8837 8372	16683 14070 12675	14966
Md-Mi	200	18465 17737 16818	37202 30538 23319	5136	18437 17692 16761	36376 32355 29716	16230
Md-Mi	300	23410 22493 21398	37733 33485 29630	9365	23313 22371 21193	45465 40433 37673	17879
Md-Mi	400	27230 26081 24708	51691 44573 37287	12150	27170 26072 24700	52202 47527 44258	19627
Ch-Mi	194	20252 20252 20252	2739 2739 2739	1471	20041 19231 18219	3328 2675 2703	14919

Comparison with FSZ, same timelimit (1200 sec.s)

Instance	Number of trains	Robust solution (efficiency)	Robust Cumulative delay	Time (sec)	Robust solution FSZ	Robust Cumulative delay FSZ	Time (sec)
Ch-Ro	41	5488 5398 5398	35265 33333 33333	1200	5442 5222 4947	51788 42319 46857	1200
Md-Mi	100	9249 8991 8672	14724 13748 12955	1200	9229 8856 8390	16652 14088 12621	1200
Md-Mi	200	18307 17674 16833	38017 31120 25857	1200	18412 17668 16738	36652 32544 29766	1200
Md-Mi	300	20425 20425 20425	26154 26154 26154	1200	16712 16037 15193	44223 40745 40403	1200
Md-Mi	400	21552 21552 21552	29063 29063 29063	1200	20530 19700 18663	53917 51021 48508	1200
Ch-Mi	194	20080 20080 20080	2826 2826 2826	1200	20011 19190 18192	3263 2729 2656	1200

Thanks for your attention

For more information, see

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