Self-Splitting Tree Search in a Parallel Environment

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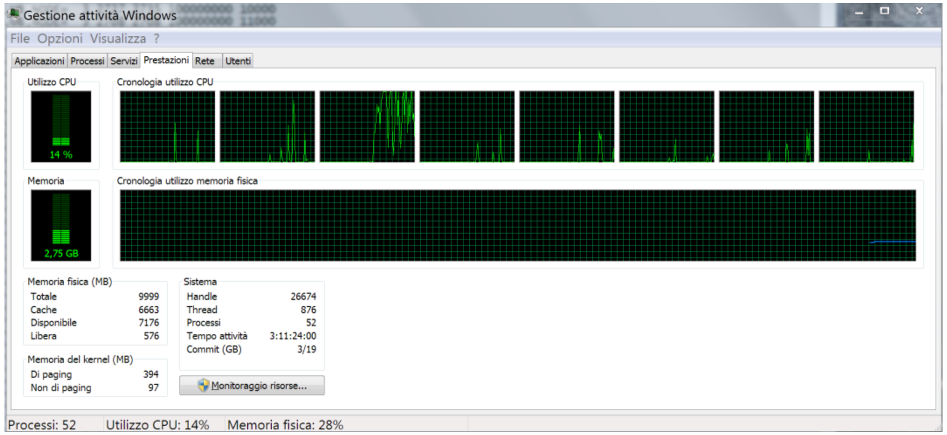
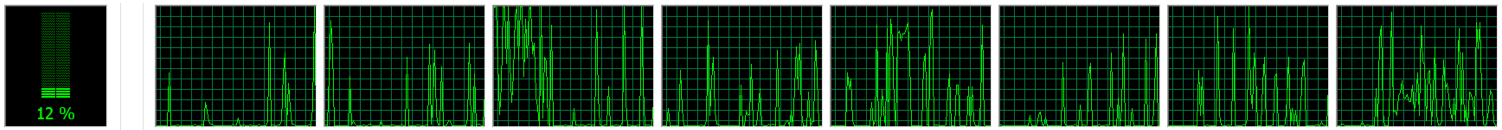
Parallel computation

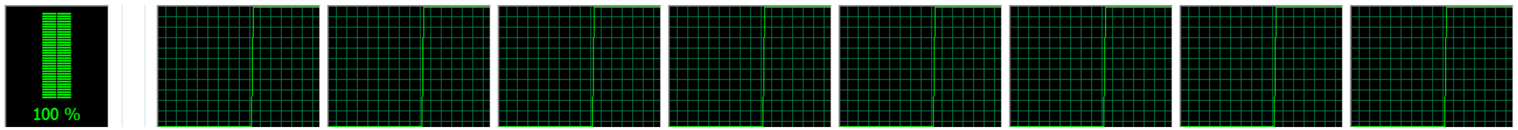
- Modern PCs / notebooks have several processing units (cores) available



- Running a sequential code on 8 cores only uses 12% of the available power…

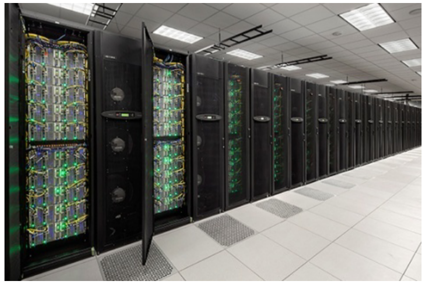


- … whereas one would of course aim at using 100% of it



Distributed computation

- Affordable **servers** offer 24+ quadcore units (blades)



- **Grids** of 1000+ computers are available worldwide

- No doubt that **parallel computing** is becoming **a must** for CPU intensive applications, including optimization

Parallelization of a sequential code

- We are given a **deterministic sequential** source code based on a divide-and-conquer algorithm (e.g., **tree search**)
- We want to **slightly modify** it to exploit a given set of *K* (say) processors called **workers**
- **IDEA:** just run *K* times the **same sequential code** on the *K* workers
- … but modify the source code so as to **just skip some nodes** (that will be processed instead by one of the other workers…)

"Workload automatically splits itself among the workers"

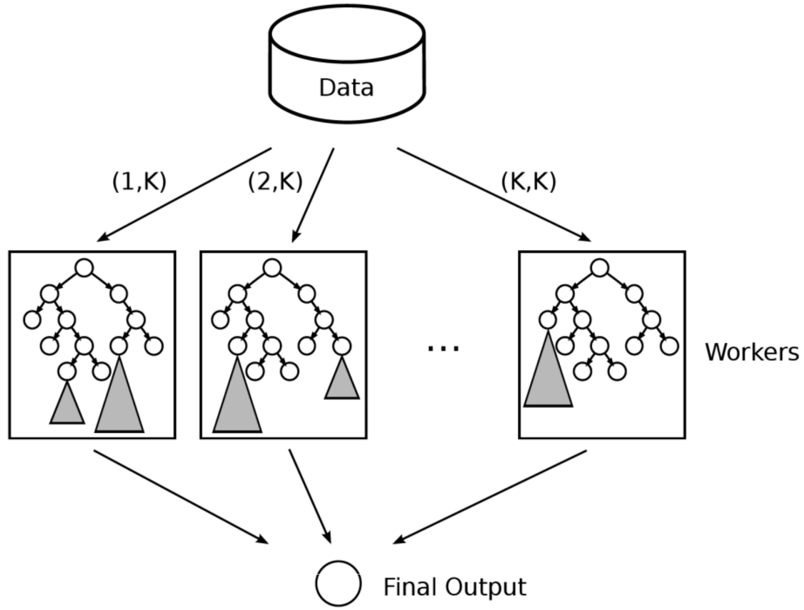
The basic idea

- Assume you have ***K* workers available** and a sequential tree-search code
- In the source code, locate the place where tree nodes are popped-out from the node queue
- Add the following statements to your sequential code:

  When a sufficient n. of nodes has been generated, **just kill some nodes** according to a rule that depends on an additional integer input parameter **k**

- Run the resulting <u>sequential code</u> on the K workers, with input **k=1,2,…,K**
- **Naïve Rule:** kill nodes with a certain probability (using k as random seed) → heuristic as a same node can be killed by all K workers
- **SelfSplit**: use a rule that guarantees a node be **killed in all but one** of the K runs

SelfSplit

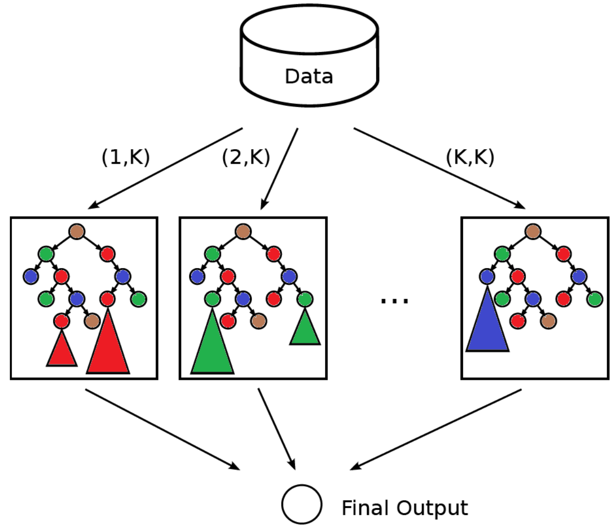


- Each worker reads the original input data and receives an **additional input** pair *(k,K),* where *K* is the total number of workers and *k=1,…,K* identifies the current worker

- The same deterministic computation is initially performed, in parallel, by all workers (**sampling phase**), without any communication

- When enough open nodes have been generated, each worker applies a **deterministic rule** to **skip the nodes that belong to other workers**. No communication required at this stage

Vanilla implementation

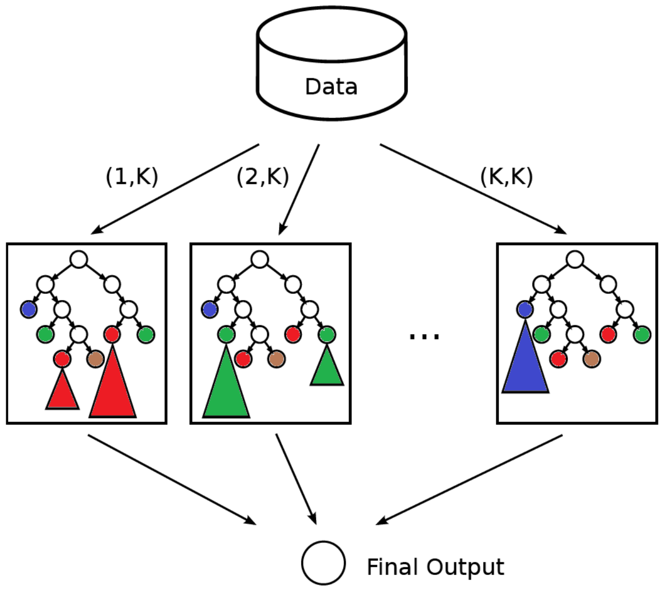


1. Two integer parameters (k, K) are added to the original input
2. A global flag **ON_SAMPLING** is introduced and initialized to true. The flag becomes false when there are enough open nodes in the branch-and-bound tree.
3. Each time a node n is created, it is deterministically assigned a *color* $c(n)$ which is a pseudo-random integer in $\{1, \cdots, K\}$ during the sampling phase, and $c(n) = k$ otherwise.
4. Whenever the modified algorithm is about to process a node n, condition

$$\textbf{(not ON\_SAMPLING)} \quad \textbf{and} \quad (c(n) \neq k)$$

is evaluated. If the condition evaluates to true, node n is just **discarded**, as it corresponds to a subproblem assigned to a different worker; otherwise, the processing of node n continues as usual and no modified action takes place.

Paused-node implementation



1. As before, input parameters (k, K) are added.

2. Whenever the modified algorithm is about to process a node n, a boolean function **NODE_PAUSE**(n) is called: if true is returned, node n is just **paused** and the next node is considered.

3. When there are no nodes left to process, the *sampling phase* ends. All paused nodes, if any, are assigned a color $c(n)$ between 1 and K, according to a deterministic rule.

4. All nodes n with color $c(n) \neq k$ are just discarded. The remaining nodes are processed (in any order and possibly in a nondeterministic way) till completion.

Extensions

- SelfSplit can be run with just *K' << K* workers, with input pairs *(1,K), (2,K), …,(K',K)* → kind of **multistart heuristic** that guarantees non-overlapping explorations

- It can be used to obtain a quick **estimate of the sequential computing time**, e.g. by running SelfSplit with (1,1000), …(8,1000) and taking

  sampling_time +

  1000 * (average_computing_time – sampling_time)

- Allows for a **pause-and-resume** exploration of the tree (useful e.g. in case of computer failures)

- Applications to **High Performance Computing** and **Cloud Computing?**

Related approaches

- The idea of parallelizing without communication is not new…

  Laursen, Per S. 1994. Can parallel branch and bound without communication be effective? SIAM Journal on Optimization 4(2) 288-296.

  … but is was apparently ignored by the **Mathematical Programming** community

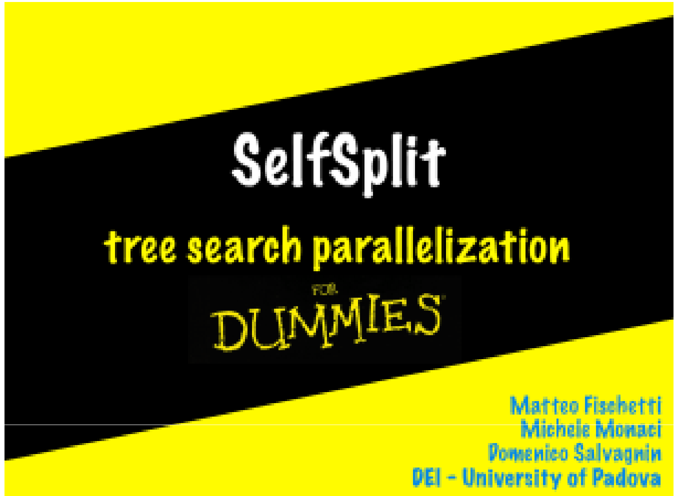
- Recent work for **Constraint Programming** (CP)

  Regin, Jean-Charles, Mohamed Rezgui, Arnaud Malapert. 2013. Embarrassingly parallel search. Christian Schulte, ed., Principles and Practice of Constraint Programming, Lecture Notes in Computer Science, vol. 8124. Springer Berlin Heidelberg, 596-610.

  Moisan, Thierry, Jonathan Gaudreault, Claude-Guy Quimper. 2013. Parallel discrepancy-based search. Christian Schulte, ed., Principles and Practice of Constraint Programming, Lecture Notes in Computer Science, vol. 8124. Springer Berlin Heidelberg, 30-46.

Our hashtags

- SelfSplit is **#easy** to implement
- SelfSplit can be the **#firstoption** to try
- SelfSplit can in fact be the **#onlyoption** when complicated (industrial) codes need to be parallelized→ **#justforget** to modifying the sources heavily
- SelfSplit can be rather effective indeed **#itworks**



Self-Split

SelfSplit for CP #itworks

	time (s)	speedup		
instance	$K = 1$	$K = 4$	$K = 16$	$K = 64$
golomb_12	41.5	3.84	14.31	41.50
golomb_13	1,195.8	4.00	15.67	57.49
golomb_14	19,051.9	3.97	15.71	61.34
partition_16	30.0	3.75	13.64	46.15
partition_18	354.8	3.90	14.78	54.58
partition_20	4,116.4	3.86	15.64	59.40
ortholatin_5	29.3	3.89	13.95	36.63
sports_10	98.7	3.91	14.51	44.86
hamming_7_4_10	32.3	3.85	14.04	40.38
hamming_7_3_6	2,402.4	3.91	15.44	59.76

Table 1 Speedups for the Constraint Programming solver *Gecode*.

Pure B&B codes #stillworkswell

Sequential code to parallelize: an old FORTRAN code of 3000+ lines from

> M. Fischetti, P. Toth, "An Additive Bounding Procedure for the Asymmetric Travelling Salesman Problem", Mathematical Programming A 53, 173-197, 1992.

- Parametrized AP relaxation (no LP)
- Branching on subtours
- Best-bound first

Vanilla SelfSplit: just **8 new lines** added to the sequential original code

time (sec)	time speedup			
$K = 1$	$K = 4$	$K = 16$	$K = 32$	$K = 64$
1,504	7.76	13.37	21.56	30.55

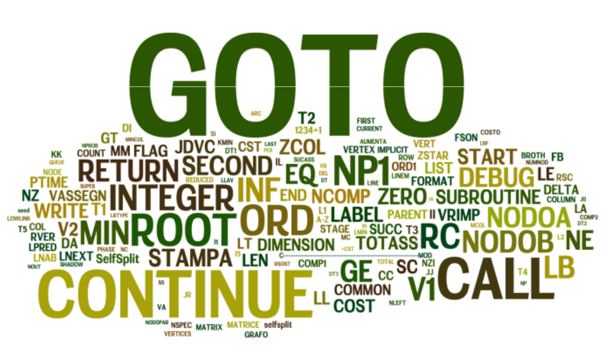
Table 3 Parallelization of a sequential ATSP branch-and-bound code.

B&Cut codes #fair

Sequential code to parallelize: B&C FORTRAN code (10K lines) from

- M. Fischetti, P. Toth, “A Polyhedral Approach to the Asymmetric Traveling Salesman Problem” Management Science 43, 11, 1520-1536, 1997.

- M. Fischetti, A. Lodi, P. Toth, “Exact Methods for the Asymmetric Traveling Salesman Problem”, in The Traveling Salesman Problem and its Variations, G. Gutin and A. Punnen ed.s, Kluwer, 169-206, 2002.



Main Features

- LP solver: CPLEX 12.5.1
- Cuts: SEC, SD, DK, RANK (and pool) separated along the tree
- Dynamic (Lagrangian) pricing of var.s
- Variable fixing, primal heuristics, etc.

time (sec)	time speedup			
$K = 1$	$K = 4$	$K = 16$	$K = 32$	$K = 64$
2,465	6.74	10.89	14.54	17.91

MIP application (CPLEX)

We performed the following experiments

1. We implemented **SelfSplit** in its paused-node version using CPLEX callbacks.

2. We selected the instances from **MIPLIB 2010** on which CPLEX consistently needs a large n. of nodes, even when the incumbent is given on input, and still can be solved within 10,000 sec.s (single-thread default). **This produced a testbed of 32 instances**.

3. All experiments have been performed in **single thread**, by giving the incumbent on input and disabling all heuristics → approximation of a production implementation involving **some limited amount of communication** in which the incumbent is shared among workers.

MIP application (CPLEX)

Experiment n. 1

We compared CPLEX default (with empty callbacks) with **SelfSplit_1**, i.e. **SelfSplit with input pair (1,1),** using 5 random seeds. The slowdown incurred was just 10-20%, hence Self_Split_1 is comparable with CPLEX on our testbed

Experiment n. 2

We considered the availability of **16 single-thread machines** and compared two ways to exploit them without communication:

(a) running **Rand_16**, i.e. SelfSplit_1 with 16 random seeds and taking the best run for each instance **(concurrent mode)**

(b) running **SelfSplit_16,** i.e. SelfSplit with input pairs (1,16), (2,16),…,(16,16)

MIP application (CPLEX) #notbad

	SelfSplit_1		SelfSplit_16			Rand_16		
	time	opt	Time	opt	speedup	Time	opt	speedup
beasleyC3	10,000.01	0	10,000.00	0	1.00	1,601.26	1	6.25
csched007	10,000.01	0	1,445.56	1	6.92	2,166.22	1	4.62
csched010	5,471.81	1	475.61	1	11.50	1,183.64	1	4.62
danoint	2,579.58	1	234.82	1	10.99	1,767.17	1	1.46
enlight16	272.44	1	10.35	1	26.32	154.53	1	1.76
iis-bupa-cov	10,000.01	0	1,762.88	1	5.67	10,000.01	0	1.00
k16x240	3,526.51	1	365.03	1	9.66	3,526.51	1	1.00
mcsched	4,744.56	1	371.82	1	12.76	3,735.39	1	1.27
mik-250-1-100-1	1,131.14	1	1,543.85	1	0.73	1,129.12	1	1.00
momentum1	8,730.25	1	2,224.69	1	3.92	3,476.15	1	2.51
neos-1426662	5,591.18	1	2,980.37	1	1.88	590.31	1	9.47
neos-1442657	639.15	1	83.95	1	7.61	180.05	1	3.55
neos-1616732	2,792.79	1	549.99	1	5.08	1,410.80	1	1.98
neos-1620770	10,000.01	0	208.04	1	48.07	1,356.77	1	7.37
neos-942830	1,626.58	1	3,662.14	1	0.44	258.65	1	6.29
neos15	5,096.19	1	3,081.10	1	1.65	5,094.56	1	1.00
neos16	10,000.01	0	127.86	1	78.21	2,041.42	1	4.90
neos858960	2,043.96	1	173.69	1	11.77	794.92	1	2.57
newdano	10,000.01	0	1,406.58	1	7.11	9,820.67	1	1.02
nobel-eu-DBE	10,000.01	0	7,577.27	1	1.32	5,641.82	1	1.77
noswot	147.14	1	17.39	1	8.46	23.99	1	6.13
ns1766074	89.45	1	10.30	1	8.68	85.88	1	1.04
ns2081729	6,588.38	1	10,000.00	0	0.66	6,588.38	1	1.00
nu60-pr9	10,000.01	0	3,248.67	1	3.08	5,088.17	1	1.97
pg5_34	9,416.21	1	826.22	1	11.40	9,407.77	1	1.00
pigeon-10	421.67	1	65.16	1	6.47	395.49	1	1.07
ran14x18	3,163.16	1	236.98	1	13.35	2,438.44	1	1.30
ran14x18-disj-8	1,709.97	1	201.25	1	8.50	1,709.75	1	1.00
reblock166	10,000.02	0	10,000.00	0	1.00	10,000.01	0	1.00
rmine6	10,000.01	0	1,716.65	1	5.83	4,763.89	1	2.10
rococoB10-011000	10,000.01	0	5,092.32	1	1.96	10,000.01	0	1.00
timtab1	1,675.12	1	171.30	1	9.78	868.85	1	1.93
sum		**21**		**29**			**29**	
avg					**10.37**			**2.69**
geomean					**5.29**			**2.01**

Why speedups change so much?

- Empirical rule: the more **sophisticated** the code, the smaller the **speedup** **#curseofbeingtoosmart**

- Typically explained by the fact that the solver **"learns during the run"** important information (cuts, conflicts, etc.) that cannot be shared by the workers in a no-communication framework

- However SelfSplit learns a lot during its sampling phase: is loss of communication the only issue? We believe that **performance variability** plays a role here

- Sophisticated tree-search codes behave like **chaotic systems** (marginal changes modify the search path and may heavily affect performance)

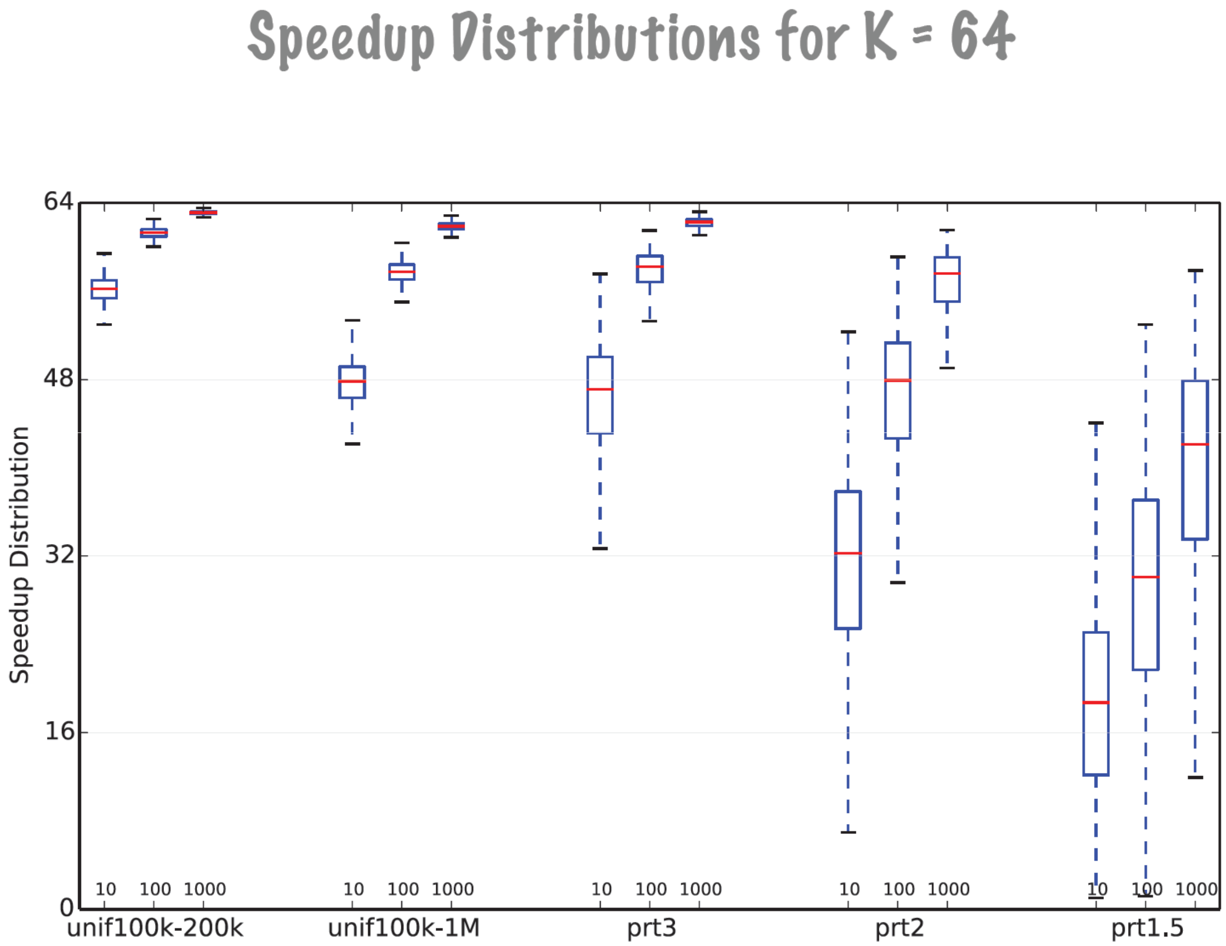
- Maybe simpler B&B codes preferable when **#millioncores** will be available?

Role of variability in workload split

- Synthetic experiments with 10, 100, 1000 random subtrees per worker (subtree size as a random variable)

unif = uniform

prt = Pareto heavy t.



Thank you for your attention

SelfSplit paper available at **www.dei.unipd.it/~fisch/papers**

Slides (also of this talk) available at **www.dei.unipd.it/~fisch/papers/slides**