# Proximity-driven MIP heuristics

## with an application to wind farm layout optimization

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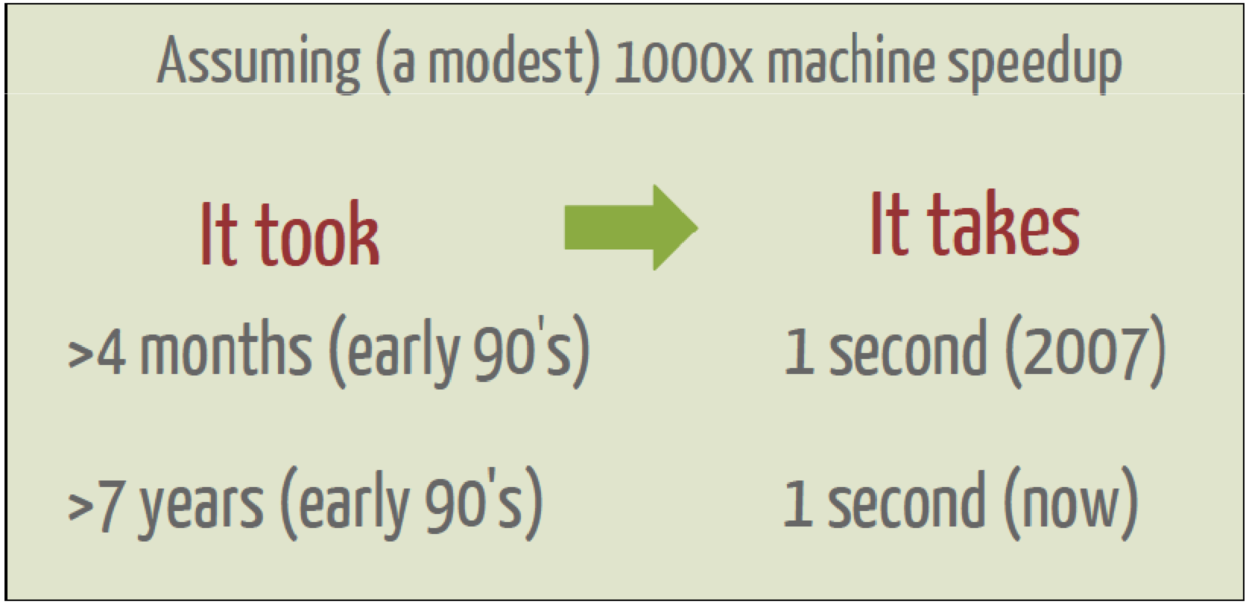
# MIP technology

- **Mixed-Integer (Linear) Programming is a powerful technique …**
  … that recently became a feasible and appealing tool to solve complex/huge real problems



# Advantages of the MIP approach

- Many **industrial problems** can be modeled as MIPs
- Different constraints can easily be added → **what if analysis**
- In many cases, off-the-shelf MIP software is able to produce a **proven-optimal solution**
- In the hardest cases, **MIP-based heuristics** yield very good solutions within acceptable computing times
- **MIP-based heuristics can be easier to design and implement than ad-hoc heuristics**

# A case study: wind farm layout

Given

- a site (**offshore** or onshore)
- characteristics of the turbines to build
- measurements of the wind in the site

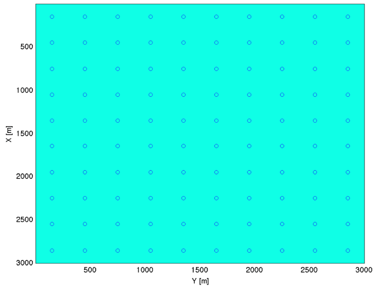
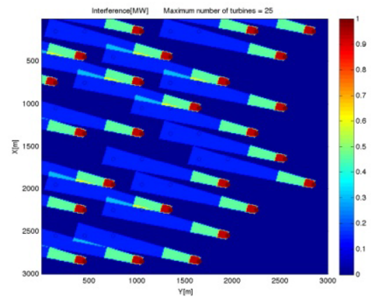
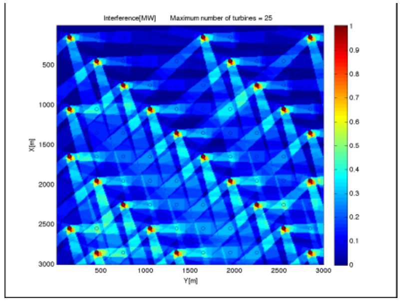
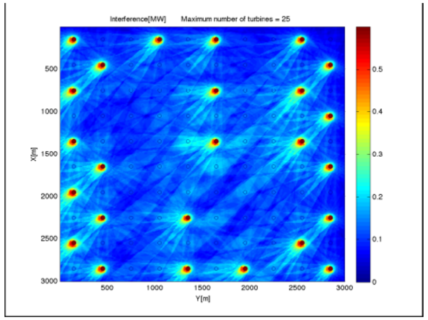
Determine a turbine allocation that **maximizes power production**

Taking into account:

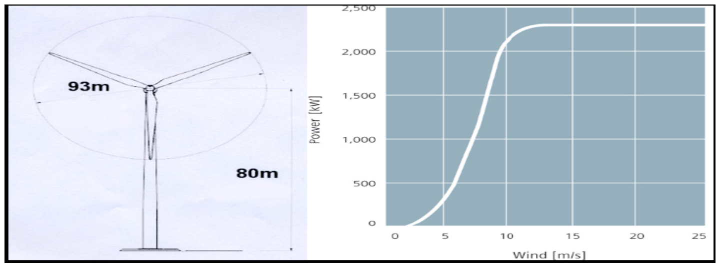
- proximity constraints (no collisions)
- minimum/maximum number of turbines
- wake effects

# The problem

- Define a grid of **sites** (candidate points for turbine allocation)

- For each site pair *(i,j),* let $I_{ij}$ denote the average **interference** (power loss) experienced at point *j* if a turbine is built on site *i* → it depends on average wind speed and direction, nonlinear turbine power curve, etc.


- Assume overall interference is **cumulative** (sum of pairwise interf.s)

# Basic (quadratic) model

- Let ***V*** be the site set, ***$P_i$*** be the max. power production at point *i*, ***$E_I$*** denote incompatible site pairs, and ***$N_{MIN}$*** and ***$N_{MAX}$*** be input limits on the n. of built turbines

$$
\begin{aligned}
& \max \sum_{i \in V} P_i x_i - \sum_{i \in V} \sum_{j \in V} I_{ij} x_i x_j && && (1) \\
\text{s.t.} \quad & N_{MIN} \le \sum_{i \in V} x_i \le N_{MAX} && && (2) \\
& x_i + x_j \le 1 && \forall [i,j] \in E_I && (3) \\
& x_i \in \{0,1\} && \forall i \in V && (4) \\
& && && (5)
\end{aligned}
$$

# Designing a simple wind-farm heuristic

- Our first heuristic is **not** based on the MIP model → hopefully easy to implement…

- **Basic move:** Given a feasible solution ***x***, we want to see if we can improve it by flipping a single variable $x_j$ → **1-opt exchange**

- **Simple heuristic**: for each ***j***, compute the objective improvement $\delta_j$ when flipping $x_j$ (alone), and find **max { $\delta_i$ }**

$$\sum_{i \in V} P_i x_i - \sum_{i \in V} \sum_{j \in V} I_{ij} x_i x_j$$

where $I_{ij} = +\infty$ for incompatible pairs $[i,j] \in E_I$

- **Complexity:** $O(|V|^2)$ for each max computation

# Improving the basic heuristic

- Complexity can be reduced from $O(|V|^2)$ to $O(|V|)$ by using parametric techniques:

1. Initialize in $O(|V|^2)$
$$\delta_j = \begin{cases} P_j - \sum_{i \in V : x_i = 1} (I_{ij} + I_{ji}) & \text{if } x_j = 0 \\ -P_j + \sum_{i \in V : x_i = 1} (I_{ij} + I_{ji}) & \text{if } x_j = 1 \end{cases}$$

2. When a certain $x_{j*}$ is going to be flipped, incrementally update all $\delta_j$'s in $O(|V|)$ time

$$\delta_j = \begin{cases} -\delta_j & \text{if } j = j^* \\ \delta_j - (I_{jj^*} + I_{j^*j}) & \text{if } j \neq j^* \text{ and } x_j = x_{j^*} \\ \delta_j + (I_{jj^*} + I_{j^*j}) & \text{if } j \neq j^* \text{ and } x_j \neq x_{j^*} \end{cases}$$

# … and improving

• **2-opt exchanges** can be implemented as well → time consuming, but we can apply it only from time to time, etc.

• Start with a **better initial** solution?

1. Start with the null solution $x = 0$ **#toobad**

2. Greedy heuristic **#better**

3. Randomized greedy **#grasp**

4. Smart solutions tend to put more turbines on the border of offshore area **#smart**

5. … more and more ideas pop out and require to be implemented, debugged and tested **#curseofbeingtoosmart**

# … and improving …

- Escaping **local optimal solutions** by using

    1. Random restarts
    2. Tabu Search
    3. Variable Neighborhood Search (VNS)
    4. Simulated Annealing
    5. Genetic Algorithms
    6. Evolutionary Heuristics
    7. ….

- ***… our first heuristic is not based on the MIP model → hopefully easy to implement…*** → **are we sure?**

# MIP heuristics

- Consider a generic Mixed-Integer convex 0-1 Problem (0-1 MIP)

$$\begin{aligned} \min f(x) \\ g(x) \leq 0 \\ x_j \in \{0,1\} \;\; \forall j \in J \end{aligned}$$

where *f* and *g* are convex functions and $\emptyset \subset J \subseteq N := \{1, \cdots, n\}$
→ removing integrality leads to an easy-solvable <u>continuous relaxation</u>

- A **black-box** (exact or heuristic) MIP solver is available

- **How to use the MIP solver to quickly provide a sequence of improved heuristic solutions (time vs quality tradeoff)?**

# Large Neighborhood Search

- **Large Neighborhood Search** (LNS) paradigm:
  1. introduce **invalid constraints** into the MIP model to create a nontrivial sub-MIP "centered" at a given heuristic sol. $\tilde{x}$ (say)
  2. Apply the MIP solver to the sub-MIP for a while…

- Possible implementations:
  - **Local branching**: add the following linear cut to the MIP

$$\Delta(x, \tilde{x}) = \sum_{j \in J:\, \tilde{x}_j = 0} x_j + \sum_{j \in J:\, \tilde{x}_j = 1} (1 - x_j) \leq k$$

  - **RINS**: find an optimal solution $x^*$ of the continuous relaxation, and fix all binary variables such that $x^*_j = \tilde{x}_j$
  - **Polish**: evolve a population of heuristic sol.s by using RINS to create offsprings, plus mutation etc.

# Proximity search

- We want to work with a modified objective function that hopefully allows the black-box solver to quickly improve the incumbent solution $\tilde{x}$

- **"Stay close" principle**: we bet on the fact that improved solutions live near the incumbent, hence we **attract** the search within a neighborhood of $\tilde{x}$ (**without imposing any artificial neighborhood constraints**)

- **Step 1.** Add an explicit **cutoff** constraint $f(x) \leq f(\tilde{x}) - \theta$

- **Step 2**. Replace the objective $f(x)$ by **the proximity function**

$$\Delta(x, \tilde{x}) = \sum_{j \in J:\, \tilde{x}_j = 0} x_j + \sum_{j \in J:\, \tilde{x}_j = 1} (1 - x_j) = ||x - \tilde{x}||_J^2$$

# Proximity search heuristic

1. run a black-box solver on the original 0-1 MIP, until a "reasonably good" feasible solution $\tilde{x}$ is found;

   **repeat**

2. explicitly add the *cutoff constraint* $f(x) \leq f(\tilde{x}) - \theta$ to the MIP model, where $\theta > 0$ is a given parameter;
3. replace the objective function $f(x)$ by a new "proximity" one, say $\Delta(x, \tilde{x})$;
4. run the MIP solver on the new model until a termination condition is reached, and let $x^*$ be the best feasible solution found;
5. **if** $J \subset N$ **then** refine $x^*$ by solving the convex program $x^* := \text{argmin}\{f(x) : g(x) \leq 0,\ x_j = x_j^*\ \forall j \in J\}$;
6. recenter $\Delta(x, \cdot)$ by setting $\tilde{x} := x^*$, and/or update $\theta$

   **until** *an overall termination condition is reached*;

   **return** $\tilde{x}$

# A MIP-based heuristic for wind farm

- What you need here is

1. A robust MIP solver

2. An idea of the **size** and difficulty of that practical instances that we want to solve (100 sites? or 1,000? or 10,000?)

3. A sound MIP model that **“does not die”** for the instances of interest → for heuristics, speed is sometimes more important than polyhedral tightness…

4. An idea about “**how to drive the MIP solver**” to deliver the solution you want → LNS, local branching, polish, proximity search…

# A standard MIP linearization

- Introduce a quadratic n. of var.s ***$z_{ij} = x_i\, x_j$***

$$
\begin{aligned}
&\max \sum_{i \in V} P_i x_i - \sum_{i \in V} \sum_{j \in V, i<j} (I_{ij} + I_{ji}) z_{ij} && (1)\\
&\text{s.t.} \qquad N_{MIN} \le \sum_{i \in V} x_i \le N_{MAX} && (2)\\
&x_i + x_j \le 1 \qquad \forall [i,j] \in E_I && (3)\\
&x_i + x_j - 1 \le z_{ij} \qquad \forall i,j \in V, i<j && (4)\\
&x_i \in \{0,1\} \qquad \forall i \in V && (5)\\
&z_{ij} \in \{0,1\} \qquad \forall i,j \in V, i<j && (6)
\end{aligned}
$$

# An alternative MIP linearization

**Glover's trick:** the objective function

$$\sum_{i \in V} P_i x_i - \sum_{i \in V} \Big( \sum_{j \in V} I_{ij} x_j \Big) x_i \tag{11}$$

is restated as

$$\sum_{i \in V} (P_i x_i - w_i) \tag{12}$$

where

$$w_i := \Big( \sum_{j \in V} I_{ij} x_j \Big) x_i = \begin{cases} \sum_{j \in V} I_{ij} x_j & \text{if } x_i = 1; \\ 0 & \text{if } x_i = 0. \end{cases}$$

→ the new continuous variable $w_i$ is the product between a continuous term (∑ …) and a binary variable ($x_i$) → **McCormick linearization**

# An alternative MIP linearization

A linearized model with linear n. of additional var.s ***$w_i$*** and BIGM constr.s

$$
\begin{aligned}
& \max z = \sum_{i \in V} (P_i x_i - w_i) && && (13) \\
\text{s.t.} \quad & N_{MIN} \le \sum_{i \in V} x_i \le N_{MAX} && && (14) \\
& x_i + x_j \le 1 && \forall [i,j] \in E_I && (15) \\
& \sum_{j \in V} I_{ij} x_j \le w_i + M_i (1 - x_i) && \forall i \in V && (16) \\
& x_i \in \{0,1\} && \forall i \in V && (17) \\
& w_i \ge 0 && \forall i \in V && (18)
\end{aligned}
$$

# Which linearization is better?

| | $n = 100$ | | $n = 150$ | | $n = 200$ | | $n = 300$ | | $n = 500$ | | $n = 1000$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mod 2 | Mod 4 | Mod 2 | Mod 4 | Mod 2 | Mod 4 | Mod 2 | Mod 4 | Mod 2 | Mod 4 | Mod 2 | Mod 4 |
| n. variables | 4786 | 200 | 9631 | 300 | 16403 | 400 | 32669 | 600 | 92257 | 1000 | 381983 | 2000 |
| n. constraints | 4866 | 280 | 9608 | 277 | 16376 | 373 | 32704 | 635 | 92173 | 916 | 382420 | 2437 |
| n. nonzeros | 14418 | 5246 | 28950 | 10288 | 49300 | 17294 | 99451 | 35313 | 279012 | 96498 | 1166942 | 406976 |
| root LP time | 0.05 | 0.01 | 0.02 | 0.02 | 0.04 | 0.07 | 20.05 | 0.07 | 26.37 | 0.36 | 380.91 | 4.32 |
| root time | 78.6 | 0.29 | 22.79 | 0.35 | 42.38 | 0.82 | 299.40 | 2.63 | 368.29 | 3.73 | 3600 | 25.97 |
| root bound | 61.19 | 70.12 | 66.05 | 66.96 | 87.55 | 88.69 | 73.30 | 76.88 | 105.09 | 108.26 | 112.40 | 112.78 |
| root heu.sol. | 54.38 | 53.1 | 53.46 | 53.96 | 63.50 | 63.84 | 54.67 | 56.13 | 70.24 | 70.78 | 65.64 | 65.64 |
| root cuts | 4960 | 37 | 1073 | 23 | 2173 | 11 | 1038 | 290 | 1040 | 15 | 178 | 40 |
| final n. nodes | 16K | 16M | 178K | 27M | 33K | 12M | 16K | 3M | 1778 | 2M | 0 | 129K |
| final best bound | 54.49 | 54.49 | 57.47 | 56.66 | 78.98 | 79.16 | 67.98 | 68.53 | 102.68 | 100.52 | 112.4 | 109.96 |
| final best heu.sol | 54.49 | 54.49 | 54.15 | 54.36 | 65.09 | 66.28 | 55.66 | 56.67 | 70.72 | 74.19 | 65.64 | 71.31 |
| final time | 2006.05 | 1698.70 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 | 3600 |

- Comparison between the linearizations with quadratic n. of var.s/constr.s **(Mod 2)** and with linear n. of var.s/constr.s **(Mod 4)**

  time limit of 3600 sec.s on a PC

  → Mod 4 (linear n. of var.s/constr.s) much better for heuristics

# Our overall MIP-based heuristic

- **Step 0**. read input data and compute the overall interference matrix ($I_{ij}$);
- ***Step 1***. *(optional) apply* ***ad-hoc heuristics*** *(1-opt) to get a first incumbent* $\tilde{x}$*;*
- ***Step 2.*** *(optional) apply quick* ***ad-hoc refinement heuristics*** *(few iterations of iterated 1- and 2-opt) to possibly improve* $\tilde{x}$*;*
- **Step 3.** if $n > 2000$, randomly **remove points** $i$ with $\tilde{x}_i = 0$ so as to reduce the number of candidate sites to *2000*;
- **Step 4**. build a MIP model for the resulting subproblem and apply **proximity search** to refine $\tilde{x}$ until the very first improved solution is found (or time limit is reached);
- **Step 5.** if time limit permits, repeat from Step 2.

# Computational results

**Alternative heuristics** implemented in C and run on a quad-core PC (16GB RAM)

a) **proxy**: our MIP-based proximity-search heuristic built on top of Cplex 12.5.1

b) **cpx_def**: Cplex 12.5.1 in its default setting, starting from the same heuristic solution $\tilde{x}$ used by **proxy**

c) **cpx_heu**: same as **cpx_def**, with an internal tuning intended to improve heuristic performance (aggressive RINS & Polish)

d) **loc_sea**: an ad-hoc **local-search** procedure not based on any MIP solver

**Testbed** (real offshore site: Horns Rev 1 in Denmark)

- offshore 3,000 x 3,000 (m) square with 400m minimum turbine separation
- no limit on the number of turbines to build
- *Siemens SWT-2.3-93* turbines (rotor diameter 93m)
- pairwise interference computed using Jensen's model, by averaging 250,000+ real-world wind samples from *Horns Rev 1 (Denmark)*

# Computational results

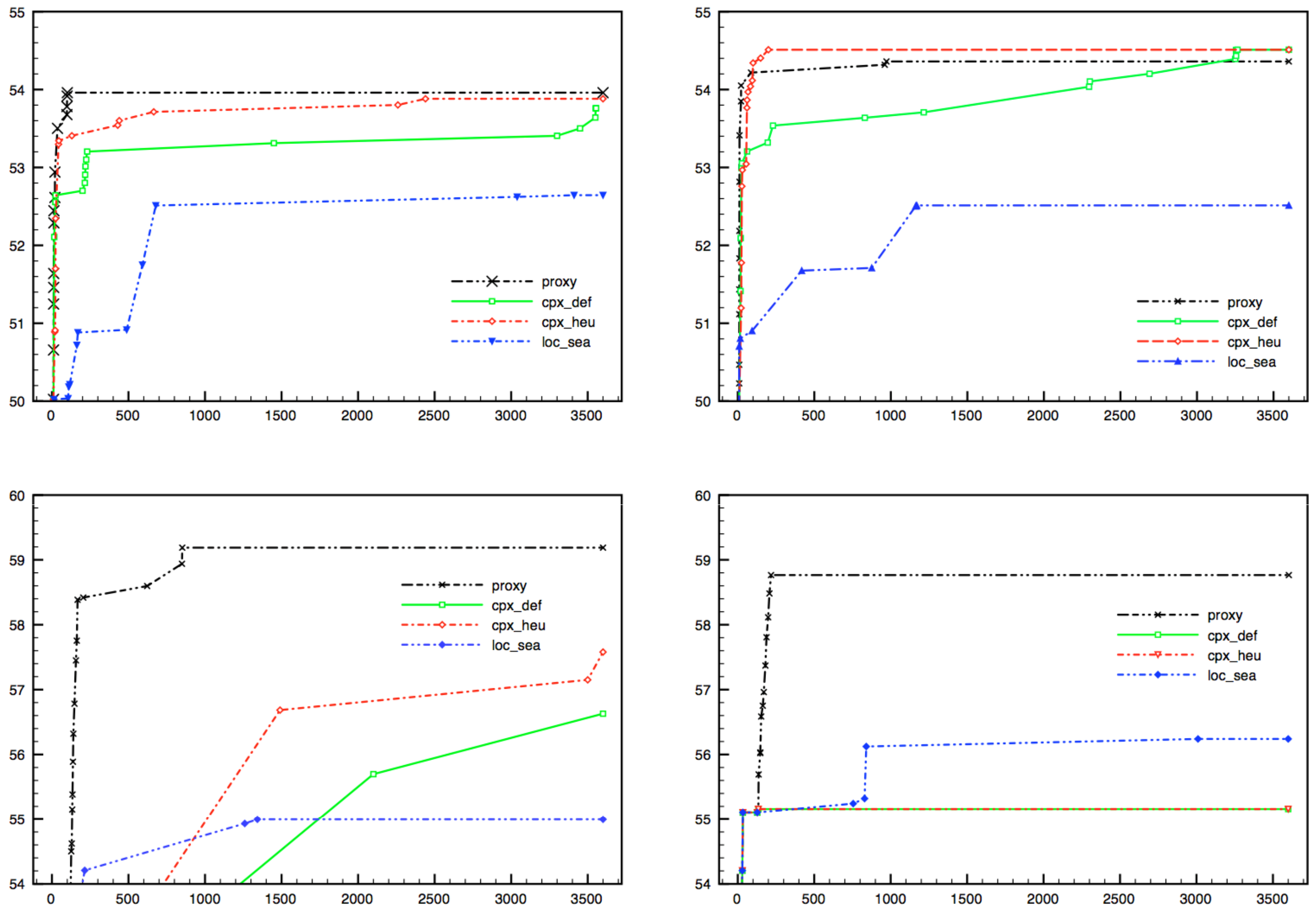



**Fig. 4** Solution profit over time for 4 sample instances with $n = 1{,}000$ (top left and top right), $n = 5000$ (bottom left), and $n = 10{,}000$ (bottom right); the higher the profit the better.

# Thanks for your attention

Papers

- M. Fischetti, M. Monaci, "Proximity Search for 0-1 Mixed-Integer Convex Programming", 2013 (accepted in *Journal of Heuristics*)
- M. Fischetti, M. Monaci, "Proximity search heuristics for wind farm optimal layout", 2013 (submitted to *Journal of Heuristics*).
- M. Fischetti, M. Fischetti, M. Monaci, "Proximity search heuristics for Mixed Integer Programs", 2014 (*RAMP 2014* proceedings)

and slides available at **[www.dei.unipd.it/~fisch](www.dei.unipd.it/~fisch)**