Benders cuts for the integrated layout and cable routing problem

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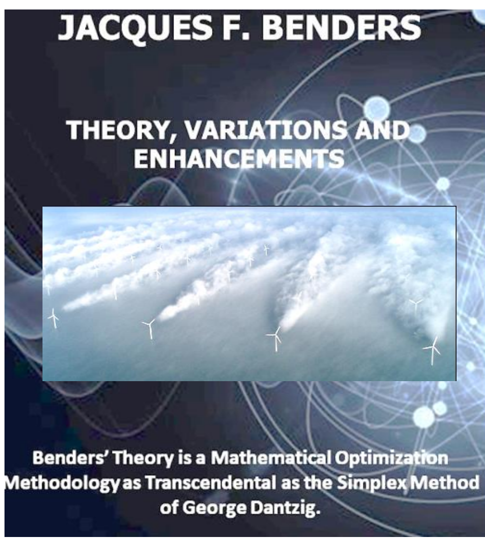


Wind farm optimal layout

Given

- a site (**offshore** or onshore)
- characteristics of the turbines to build
- measurements of the wind in the site

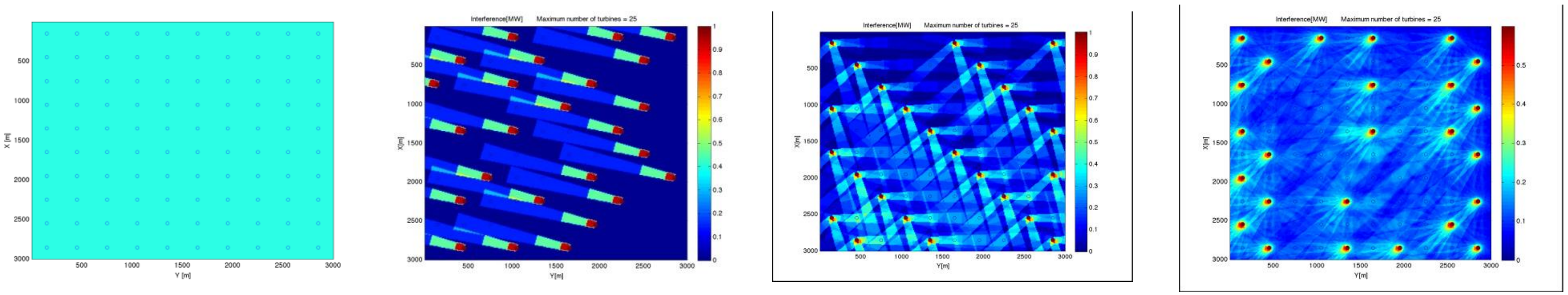
Determine a turbine allocation that **maximizes power production**

Taking into account:

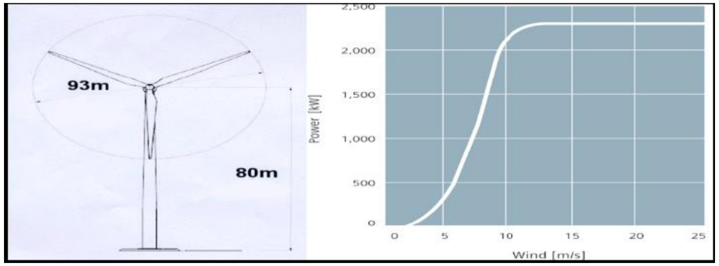
- proximity constraints (no collisions)
- minimum/maximum number of turbines
- wake effects

The layout problem

- Define a grid of **sites** (candidate points for turbine allocation)



- For each site pair *(i,j),* let I_{ij} denote the average **interference** (power loss) experienced at point *j* if a turbine is built on site *i* → it depends on average wind speed and direction, nonlinear turbine power curve, etc.



- Assume overall interference is **cumulative** (sum of pairwise interf.s)

Layout (nonconvex) quadratic model

- Let ***V*** be the site set, ***power$_i$*** be the max. power production at point *i*, ***E_I*** denote incompatible site pairs, ***N_{min}*** and ***N_{max}*** be input limits on the n. of built turbines, and let ***I_{ij}*** be the interference from *i* to *j*

$$\max \sum_{i \in V} \text{power}_i z_i - \sum_{i \in V} \sum_{j \in V} I_{ij} z_i z_j$$

$$N_{\min} \leq \sum_{i \in V} z_i \leq N_{\max}$$

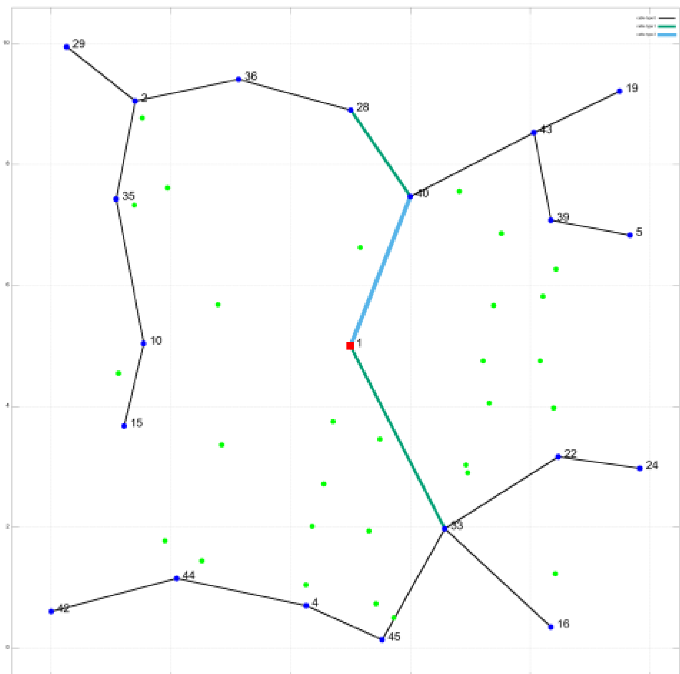
$$z_i + z_j \leq 1 \qquad \forall [i,j] \in E_I$$

$$z_i \in \{0,1\} \qquad \forall i \in V$$

The cable routing problem

- The built **turbines** (blue circles) need to be connected through different types of **cable** to bring the electrical current to a **substation** (red square) and eventually to shore through a single export cable (not shown in the figure)

- MILP model from Fischetti M, Pisinger D (2018) Optimizing wind farm cable routing considering power losses. European Journal of Operational Research 270(3):917–930



$$
\begin{aligned}
&\min \sum_{ij}\sum_{t} \mathrm{cost}_{ij}^{t} x_{ij}^{t} \\
&\sum_{j} f_{ij} = \sum_{j} f_{ji} + \mathrm{power}_i \quad \forall\, i \neq s \\
&f_{ij} \leq \sum_{t} \mathrm{cap}_t\, x_{ij}^{t}, \quad \forall\, i,j \\
&y_{ij} = \sum_{t} x_{ij}^{t}, \quad \forall\, i,j \\
&\sum_{j} y_{ij} = 1, \quad \forall\, i \neq s \\
&\sum_{j} y_{sj} = 0, \\
&\sum_{i} y_{is} \leq C_{\max}, \\
&y_{ij} + y_{ji} + \sum_{k:\ [h,k] \text{ overlaps } [i,j]} y_{hk} \leq 1, \quad \forall\, i<j,\ \forall h \notin \{i,j\} \\
&x_{ij}^{t} \in \{0,1\}, \quad \forall\, i,j,t \\
&y_{ij} \in \{0,1\}, \quad \forall\, i,j \\
&f_{ij} \geq 0, \quad \forall\, i,j \\
&f_{jj} = x_{jj}^{t} = y_{jj} = f_{sj} = x_{sj}^{t} = y_{sj} = 0, \quad \forall\, j,t \\
&f_{ij} = x_{ij}^{t} = y_{ij} = 0, \quad \forall\, i,j,t,\ s \notin \{i,j\} : \mathrm{dist}_{ij} < D_{\min}
\end{aligned}
$$

Combined layout + cable problem

$$\max\ \alpha(\sum_i \mathrm{profit}_i z_i - w_{tot}) - \sum_{ij}\sum_t \mathrm{cost}^t_{ij} x^t_{ij} \tag{1}$$

$$\sum_j f_{ij} = \sum_j f_{ji} + \mathrm{power}_i\, z_i, \quad \forall\, i \neq s \tag{2}$$

$$f_{ij} \le \sum_t \mathrm{cap}_t\, x^t_{ij}, \quad \forall\, i,j \tag{3}$$

$$y_{ij} = \sum_t x^t_{ij}, \quad \forall\, i,j \tag{4}$$

$$\sum_j y_{ij} = z_i, \quad \forall\, i \neq s \tag{5}$$

$$\sum_j y_{sj} = 0, \tag{6}$$

$$\sum_i y_{is} \le C_{\max}, \tag{7}$$

$$N_{\min} \le \sum_{i \neq s} z_i \le N_{\max} \tag{8}$$

$$z_i + z_j \le 1, \quad \forall\, i < j : s \notin \{i,j\},\ \mathrm{dist}_{ij} < D_{min} \tag{9}$$

$$y_{ij} + y_{ji} + \sum_{k:\ [h,k]\ \text{overlaps}\ [i,j]} y_{hk} \le 1, \quad \forall\, i < j,\ \forall h \notin \{i,j\} \tag{10}$$

$$w_{tot} \ge \sum_i w_i, \tag{11}$$

$$\sum_{j:\,\mathrm{dist}_{ij} \ge D_{\min}} I_{ij} z_j \le w_i + M_i(1 - z_i), \quad \forall\, i \neq s \tag{12}$$

$$x^t_{ij} \in \{0,1\}, \quad \forall\, i,j,t \tag{13}$$

$$y_{ij} \in \{0,1\}, \quad \forall\, i,j \tag{14}$$

$$z_i \in \{0,1\}, \quad \forall\, i \tag{15}$$

$$f_{ij} \ge 0, \quad \forall\, i,j \tag{16}$$

$$w_i \ge 0, \quad \forall\, i \tag{17}$$

$$f_{jj} = x^t_{jj} = y_{jj} = f_{sj} = x^t_{sj} = y_{sj} = 0, \quad \forall\, j,t \tag{18}$$

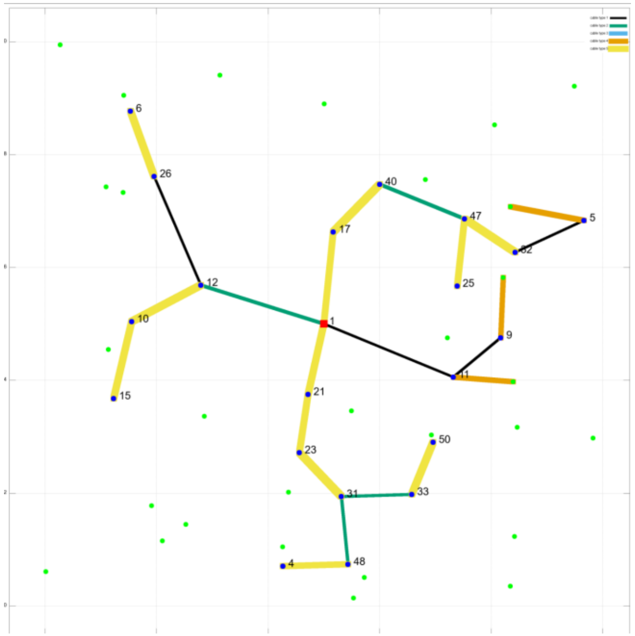
$$f_{ij} = x^t_{ij} = y_{ij} = 0, \quad \forall\, i,j,t,\ s \notin \{i,j\} : \mathrm{dist}_{ij} < D_{\min} \tag{19}$$

$$w_s = 0 \text{ and } z_s = 1 \tag{20}$$

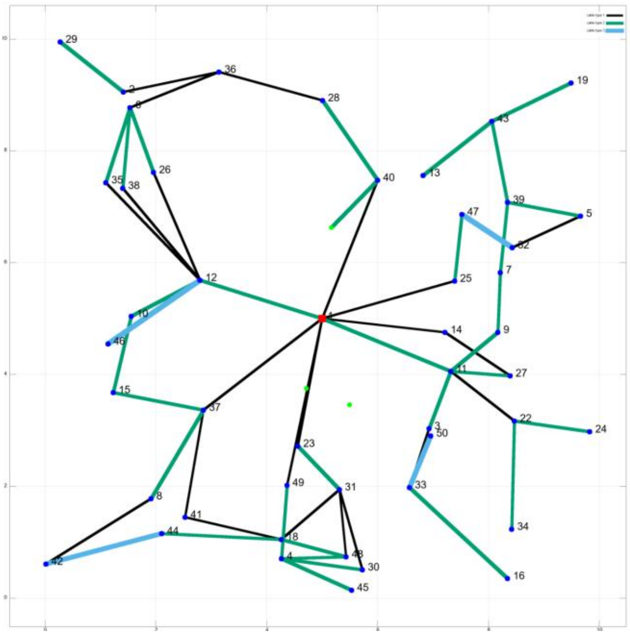
Solving combined model to optimality

- The two sub-models pull optimization from opposite directions (close to substation for cables, dispersed far from the substation for layout)

- The combination of the two sub-models favors **wild fractional LP solutions** tricking the big-M constraints in the MILP model …



No interference (almost integer)



With interference (a mess!)

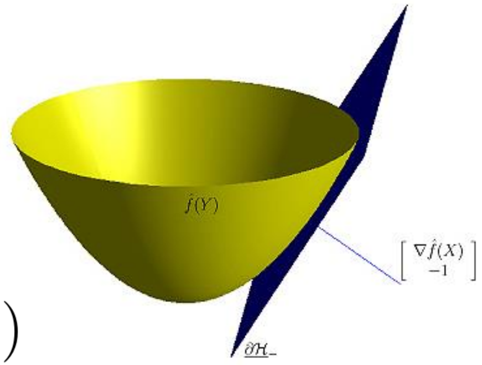
Improving the model through Benders-like cuts

- Rewrite

$$\min \; linear(x, w, z) + w_{tot}$$
$$w_{tot} \geq w^{ideal}(z) := \sum_{i \in V} \sum_{j \in V} I_{ij} z_i z_j \quad \text{(a nonconvex function)}$$
$$< \text{other constraints not involving } w_{tot} >$$
$$w_{tot} \geq 0$$

- … and replace $w^{ideal}(z)$ with a convex function $w^{relax}(z)$ that assumes the same values for binary *z*

- … and finally lower-approximate $w^{relax}(z)$ by subgradient (linear) cuts *"à la Benders"*



$$w_{tot} \geq w^{relax}(z^*) + subgrad(z^*)\,(z - z^*)$$

Cuts from McCormick linearization

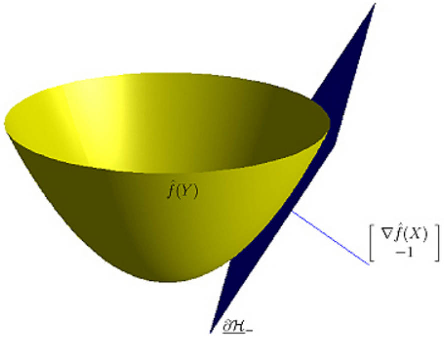
- Textbook approach to linearize quadratic terms: introduce new continuous var.s $m_{ij} = z_i\, z_j$ and define the convex function $w^{relax}(z)$ as (recall that $I_{ij} \geq 0$ for all i, j):

$$
\begin{aligned}
w^{quadr}(z) = \min_{m} \sum_{i<j} (I_{ij} + I_{ji})\, m_{ij} \\
m_{ij} \geq z_i + z_j - 1, \quad \forall\, i < j \\
m_{ij} \geq 0, \quad \forall\, i < j
\end{aligned}
$$

- For any given $z \in [0,1]^n$, the opt. sol. m is easily computed as

$m_{ij} = max\{z_i + z_j - 1, 0\}, \quad \forall\, i < j$ → exponential family of cuts

$$
\begin{aligned}
w_{tot} &\geq w^{quadr}(z) \\
&= \sum_{i<j} (I_{ij} + I_{ji}) \max\{\, z_i + z_j - 1,\ 0\,\} \\
&\geq \sum_{ij \in S} (I_{ij} + I_{ji})\, (z_i + z_j - 1)
\end{aligned}
$$



tight at $z = z^*$ by choosing $S = S(z^*) = \{i < j :\ z_i^* + z_j^* - 1 > 0\}$

Cuts from triangle inequalities

- In our model, we also have binary "cable" var.s. *y* associated with the arcs, which allows us to write (as before, $m_{ij} = z_i\, z_j$)

$$
\begin{aligned}
w_{tot} \geq w^{clique}(y) = \min_{m} \sum_{i<j} (I_{ij} + I_{ji})\ m_{ij} \\
m_{ij} \geq m_{[i,k]} + m_{[k,j]} - 1, \quad \forall\ i, j, k,\ i < j,\ k \notin \{i, j\} \\
m_{ij} \geq y_{ij} + y_{ji}, \quad \forall\ i < j \\
0 \leq m_{ij} \leq 1, \quad \forall\ i < j
\end{aligned}
$$

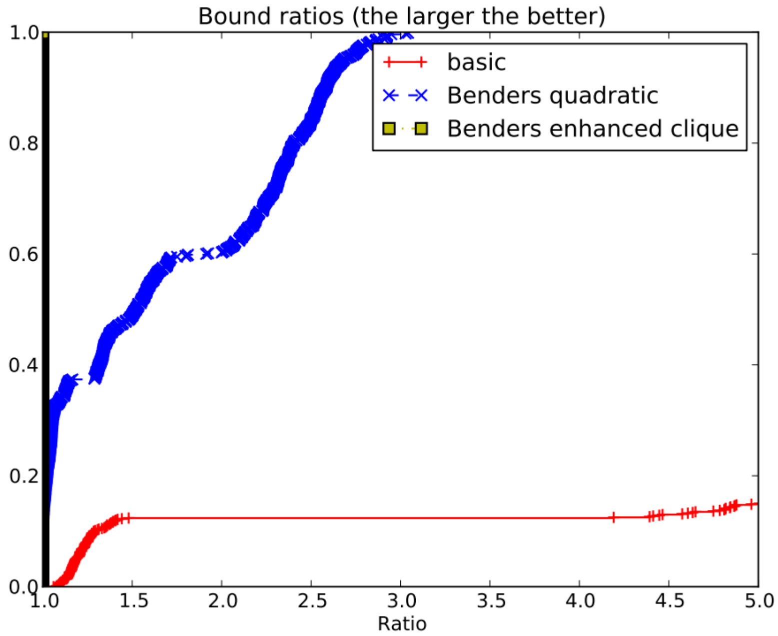
- By complementing the *m* var.s we get the all-pair shortest path model:

$$
\begin{aligned}
w^{clique}(y) = \ldots = \sum_{i<j} (I_{ij} + I_{ji})\ - \max_{\overline{m}} \sum_{i<j} (I_{ij} + I_{ji})\, \overline{m}_{ij} \\
\overline{m}_{ij} \leq \overline{m}_{[i,k]} + \overline{m}_{[k,j]}, \quad \forall\ \ i, j, k,\ i < j,\ k \notin \{i, j\} \\
\overline{m}_{ij} \leq 1 - y_{ij} - y_{ji}, \quad \forall\ \ i < j \\
0 \leq \overline{m}_{ij} \leq 1, \quad \forall\ \ i < j
\end{aligned}
$$

- … and the associated subgradient cuts (computable in $O(n^3)$ time) …

Computational results (root node)

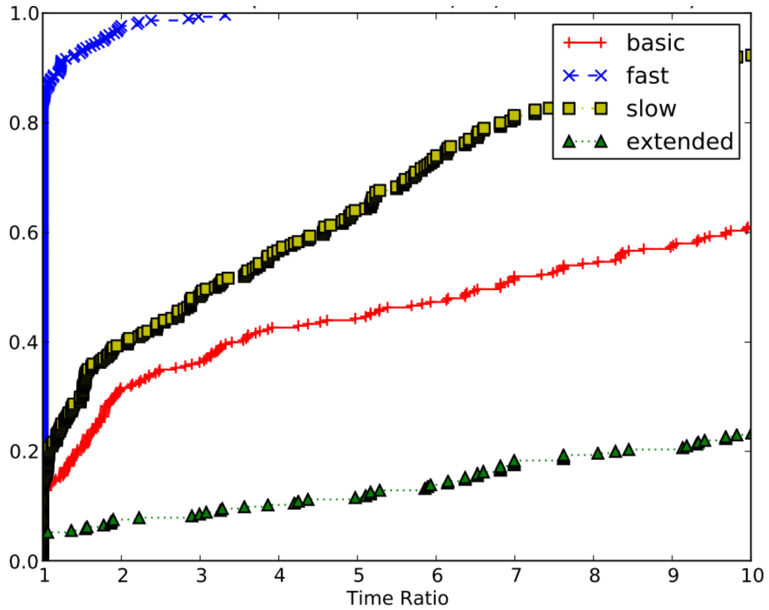
- Cut separation implemented in C and embedded into IBM CPLEX solver
- Root node bound comparison (800 random instances of various types); three main versions compared through performance profile plots:



Basic → no additional cuts
Benders quadratic → cuts from w^{quadr}
Benders enhanced cliques → cuts from w^{clique}

Computational results (exact solver)

- Branch-and-cut comparison (300 random instances of various types); four main versions compared through performance profile plots:



Basic → no additional cuts

Slow → all cuts at every B&C node

Fast→ all cuts at the B&C root node only

Extended → extended formulation with explicit m_{ij} variables + triangle inequalities

Thanks for your attention

Slides available at http://www.dei.unipd.it/~fisch/papers/slides/

Reference papers:

• Fischetti M, Monaci M (2016) Proximity search heuristics for wind farm optimal layout. Journal of Heuristics 22(4):459–474.

• Fischetti M, Pisinger D (2018) Optimizing wind farm cable routing considering power losses. European Journal of Operational Research 270(3):917–930

• Fischetti M, Kristoffersen JR, Hjort T, Monaci M, Pisinger D (2020) Vattenfall optimizes offshore wind farm design. INFORMS Journal On Applied Analytics 50(1):80–94 (finalist of the 2019 Edelman Prize)

• Fischetti M, Fischetti M (2021) Integrated layout and cable routing in wind farm optimal design, Technical Report DEI, University of Padua (submitted)