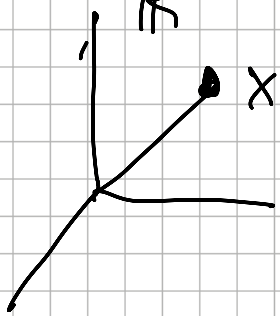
OR1 6-oct-2021

$\mathbb{R}^n$



$x = (x_1, \dots, x_n)$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

"column vector"

$$u \in \mathbb{R}^m \longrightarrow u^T = [u_1, u_2, \dots, u_m]$$

"row vector"

$\boxed{\text{Matrix}}$

$$A = \begin{array}{c} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ m \end{array} \begin{bmatrix} & & \vdots & & \\ & & \vdots & & \\ \dots & \dots & a_{ij} & - & - \\ & & \vdots & & \\ & & \vdots & & \end{bmatrix} \quad \begin{array}{ccccc} 1 & 2 & j & \dots & n \end{array}$$

$\cancel{a_{i,j}}$

$\cancel{a_{i;k}}$

$$A = \begin{bmatrix} A_1 & A_2 & \cdots & A_n \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ \vdots \\ m \end{array}$$

$1 \quad 2 \quad \dots \quad n$

$A_i \in \mathbb{R}^m$

$$A = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & \cdot & \dots & n \end{array} \\ \begin{bmatrix} \quad a_1^T \quad \\ \hline [\; \dots \; a_i^T \; \dots \;] \\ \hline a_m^T \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ \vdots \\ m \end{array}$$

$a_i \in \mathbb{R}^n$

$$A\,x = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & & x_n \end{array} \\ \begin{bmatrix} A_1 & A_2 & \cdots & A_n \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} =$$

$$= A_1 x_1 + A_2 x_2 + \cdots + A_n x_n$$

$$u^T A = [u_1, u_2, \dots, u_m] \begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_m^T \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{matrix}$$

$$= u_1 a_1^T + u_2 a_2^T + \dots + u_m a_m^T$$

$$\boxed{Ax = b} \in \mathbb{R}^m$$ system of m equations

$$\begin{bmatrix} A_1 \end{bmatrix} x_1 + \begin{bmatrix} A_2 \end{bmatrix} x_2 + \cdots + \begin{bmatrix} A_n \end{bmatrix} x_n = \begin{bmatrix} b \end{bmatrix}$$

$$\begin{cases} a_1^T x = b_1 \\ \vdots \\ a_m^T x = b_m \end{cases} \qquad \begin{cases} a_{11} x_1 + a_{12} x_2 + \dots = b_1 \\ \ddots \\ a_{m1} x_1 + a_{m2} x_2 + \dots = b_m \end{cases}$$

Scalar product

$c, x \in \mathbb{R}^n$ $\quad \langle c, x \rangle \quad \underbrace{[c_1, \dots, c_n]}_{c^T} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x}$

$$\sum_{j=1}^{n} c_j x_j = c^T x$$

LINEAR OPTIMIZATION / PROGRAMMING

$$\begin{cases} \min \ c^T x \\ Ax \geq b \\ \boxed{x \geq 0} \end{cases}$$

explicit constr.s

implicit nonnegativity constr.s

CANONICAL FORM

$$\begin{cases} \min \ c^T x \\ Ax = b \\ \boxed{x \geq 0} \end{cases}$$

STANDARD FORM

Conversion "tricks"

- $\boxed{a_i^T x \geq b_i} \iff \begin{cases} a_i^T x - s_i = b_i \\ s_i \geq 0 \end{cases}$

  surplus var.

- $\boxed{a_i^T x \leq b_i} \iff \begin{cases} a_i^T x + s_i = b_i \\ s_i \geq 0 \end{cases}$

  slack var.

- $\boxed{a_i^T x = b_i} \iff \begin{cases} a_i^T x \geq b_i \\ -a_i^T x \geq -b_i \end{cases}$

- $x_i \gtrless 0$ "x_i free" $\iff \begin{cases} x_i = x_i^+ - x_i^- \\ x_i^+, x_i^- \geq 0 \end{cases}$

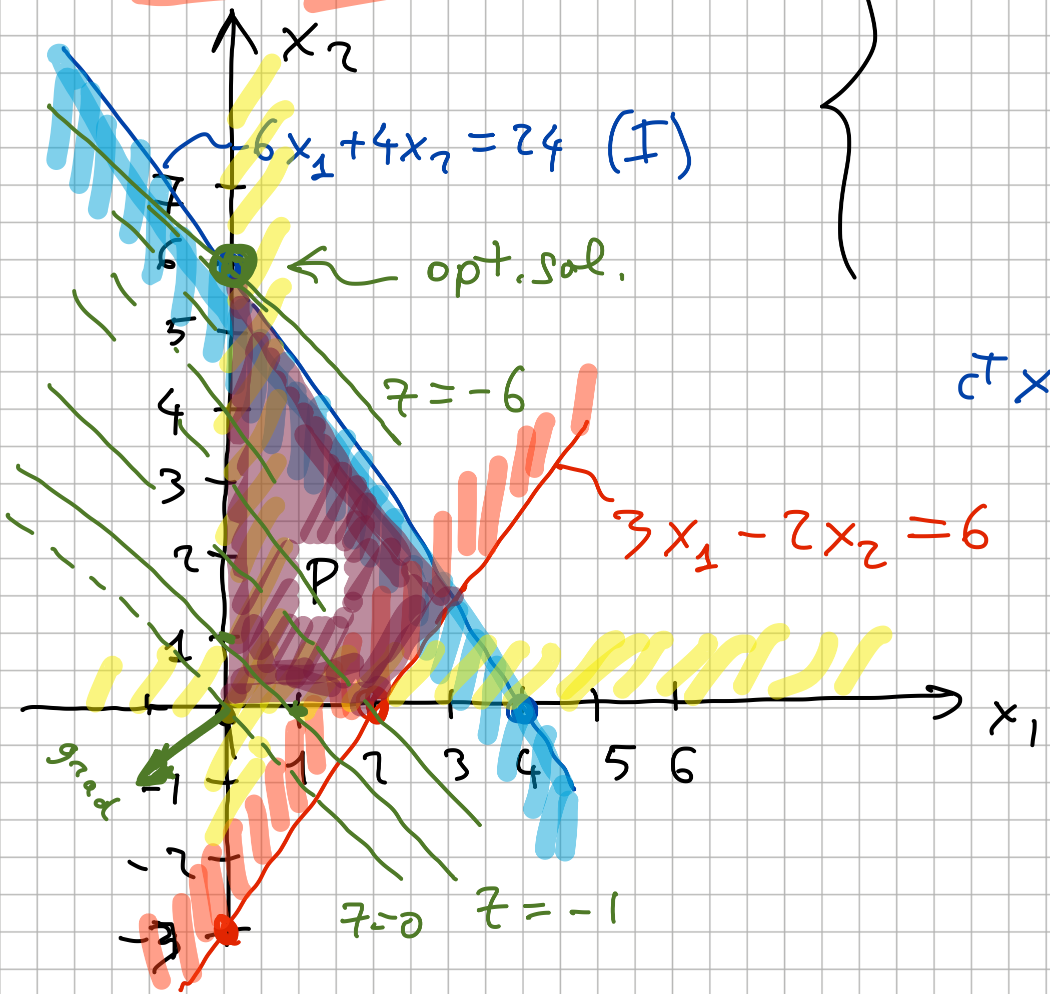
$$+3\,x_i + \dots \iff \dots + 3(x_i^+ - x_i^-) + \dots$$

$$x_i = +5 \;\longrightarrow\; x_i^+ = 5 + \delta, \quad x_i^- = 0 + \delta$$

$$x_i = -3 \;\longrightarrow\; x_i^+ = 0 + \delta, \quad x_i^- = 3 + \delta$$

$$\boxed{\max\; w^T x \;\longrightarrow\; -\min\; \underbrace{c^T x}_{c^T = -w^T}}$$

$$\boxed{\text{EXAMPLE}}$$

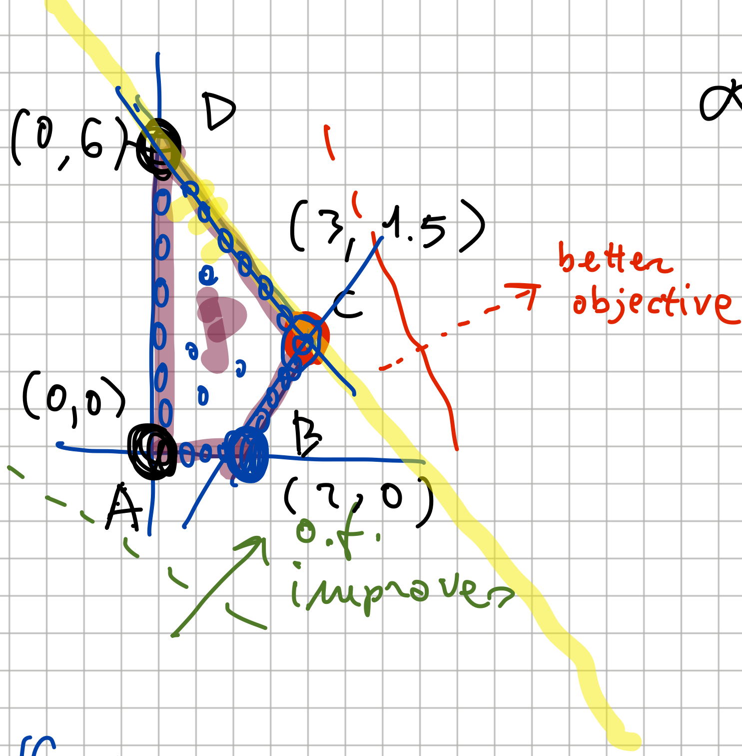


$$\begin{cases} \min\; -x_1 - x_2 \\ 6x_1 + 4x_2 \le 24 & \text{(I)} \\ 3x_1 - 2x_2 \le 6 & \text{(II)} \\ x_1 \ge 0 \\ x_2 \ge 0 \end{cases}$$

$$c^T x = \boxed{-x_1 - x_2 = z}$$

$$-x_1 - x_2 = -1$$

$$\text{grad}(c^T x) = c = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$



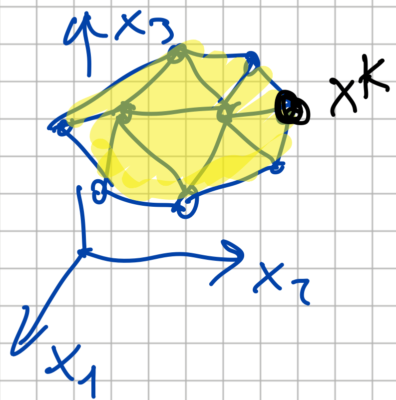
∞^2 solutions

→ only 4 "corner" points are really candidate for optimality

"A, B, C, D are the VERTICES of P"

- Definition of VERTEX
- How to compute them

\# vertices is HUGE



$\boxed{\text{NAIVE ALG.}}$

for each vertex x^k of P do
 evaluate $c^t x^k$ → MIN

⇒ FINITE ALG. for L.P.

BETTER METHOD ⤳