OR 1 18 - OCT - 2021

TABLEAU FORM of the Simplex Method

$B^{-1} \rightarrow ?$ $\quad O(m^3)$

$$\begin{cases} x_B = B^{-1}b - B^{-1}F x_F \\ z = c_B^T B^{-1} b + (c_F^T - c_B^T B^{-1} F) x_F \end{cases}$$

canonical form w.r.t. current basis B

$$\boxed{Ax = b} \qquad c^T x - z = 0$$

$$\begin{array}{c|c|ccccc|c}
 & \text{col.}0 & x_1 & x_2 & \cdots & & x_n & z \\
\hline
\text{row } 0 & 0 & c_1 & c_2 & \cdots & & c_n & -1 \\
\hline
1 & & & & & & & 0 \\
 & b & & & A & & & 0 \\
 & & & & & & & \vdots \\
m & & & & & & & 0
\end{array}$$

" ORIGINAL TABLEAU "

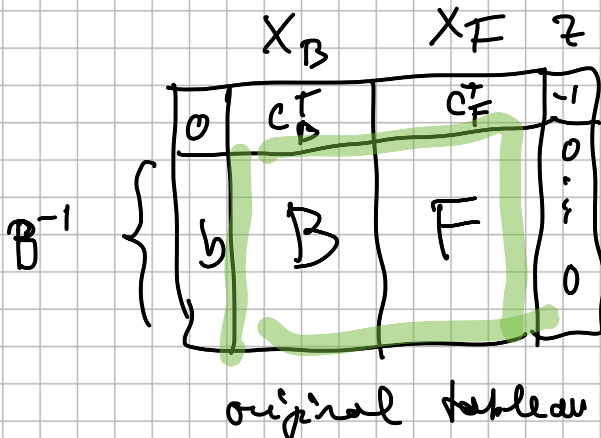
$$\begin{cases} x_B = B^{-1}b - B^{-1}F x_F \\ z = c_B^T B^{-1} b + \underbrace{(c_F^T - c_B^T B^{-1} F)}_{\bar{c}_F^T} x_F \end{cases}$$

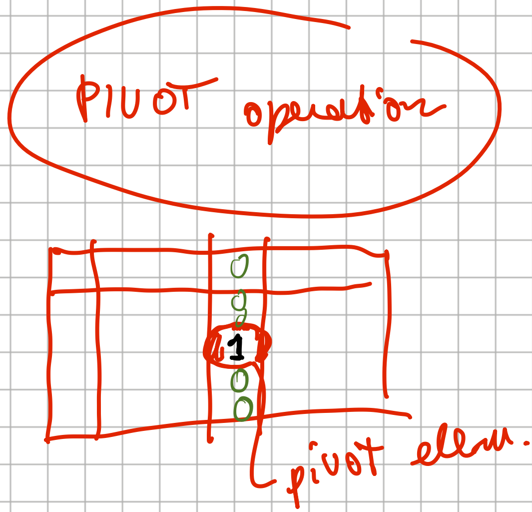
$$\begin{cases} B^{-1} b = I x_B + B^{-1} F x_F \\ \underbrace{-c_B^T B^{-1} b}_{\bar{c}_0} = 0 x_B + \bar{c}_F x_F - z \end{cases}$$

$\bar{b} := B^{-1} b$

$\bar{F} := B^{-1} F$

$$\begin{array}{c|c|c|c}
-c_B^T B^{-1} b & \leftarrow x_B \rightarrow & \leftarrow x_F \rightarrow & z \\
\hline
\bar{c}_0 & 0\;0\;0\;0\;0 & \bar{c}_F^T & -1 \\
\hline
 & & & 0 \\
\bar{b} & I & \bar{F} & 0 \\
 & & & \vdots \\
 & & & 0
\end{array}$$





Example

$$\begin{cases} \min \ -x_1 - x_2 \\ 6x_1 + 4x_2 + x_3 = 24 \\ 3x_1 - 2x_2 + x_4 = 6 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

		x_1	x_2	x_3	x_4	z
$-z=$	●	-1	-1	0	0	1
$x_3=$	24	6	4	1	0	0
$x_4=$	6	3	-2	0	1	0
		x_F		x_B		

$24/6 = 4$

$6/3 = 2$

● $= -x_1 - x_2 - z$

$\bar{c}_1 = -1$, "x_1 enters the basis"

$$\theta = \min\left\{ \frac{\bar{b}_1}{\bar{a}_{11}}, \frac{\bar{b}_2}{\bar{a}_{21}} \right\} = 6/3$$

$24/6$ $\quad$ $6/3$

"$x_{B[t]} = x_4$ leaves the basis"

		x_1	x_2	x_3	x_4	z
$-z=$	0	-1	-1	0	0	-1
$x_3=$	24	6	4	1	0	0
$x_4=$	6	3	-2	0	1	0

$\longleftrightarrow x_F \qquad \longleftrightarrow x_B$

		x_1	x_2	x_3	x_4
$-z=$	2	0	-5/3	0	1/3
$x_3=$	12	0	8	1	-2
$x_1=$	2	1	-2/3	0	1/3

pivot 3

	x_1	x_2	x_3	x_4	
0	-1	-1	0	0	+ ← row 0
2	1	$-\frac{2}{3}$	0	$\frac{1}{3}$	= ← new pivot row
2	0	-5/3	0	1/3	← new row 0

	24	6	4	1	0	+ ← row 1
$-6 \times$	2	1	-2/3	0	1/3	← new pivot row
	12	0	8	1	-2	

NEW TABLEAU

		x_1	x_2	x_3	x_4	z	
$-z=$	2	0	-5/3	0	1/3	-1	
$x_3=$	12	0	8	1	-2	0	← PIVOT ROW
$x_1=$	2	1	-2/3	0	1/3	0	

$$-z = 2$$
$$x_3 = 12$$
$$x_1 = 2$$

current basic solution
(FEASIBLE : $\bar{b} \geq 0$)

$\bar{c}_2 = -5/3 < 0$

" x_2 enters the basis "

" x_3 leaves the basis "

		x_1	x_2	x_3	x_4
$-z=$	$9/2$	0	0	$5/24$	$-1/12$
$x_2=$	$3/2$	0	1	$1/8$	$-1/4$
$x_1=$	3	1	0	$1/12$	$1/6$

← new pivot row

$z = -9/2 \quad x_1 = 3, \; x_2 = 3/2, \; (x_3 = x_4 = 0)$

		x_1	x_2	x_3	x_4
$-z=$	6	$1/2$	0	$1/4$	0
$x_2=$	6	$3/2$	1	$1/4$	0
$x_4=$	18	6	0	$1/2$	1

$z = -6 \quad x_1 = 0, \; x_2 = 6, \; \ldots$

$\Longrightarrow$ TABLEAU OPTIMAL !

- How to select the negative reduc. cost?

ONLY negative $\bar{c}_h$ are candidate

A $\rightarrow$ $\boxed{|\bar{c}_h| \ \max.}$ "VERY FAST"

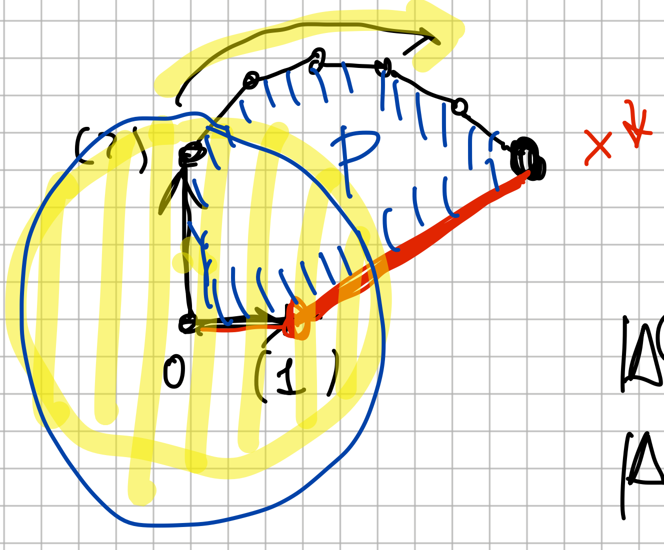
$$\underbrace{|\Delta z|}_{\max} = \underbrace{|\bar{c}_h|}_{\max} \cdot \underbrace{\vartheta_h}_{?}$$

B $\rightarrow$ $\boxed{|\bar{c}_h|\vartheta_h \rightarrow \max}$ $O(m)$ "MORE ACCURATE BUT SLOWER"

$\rightarrow$ take the first h : $\bar{c}_h < 0$

("min h")

$\rightarrow$ RANDOM choice



$|\Delta_z^{(1)}| = 10$

$|\Delta_z^{(2)}| = 1000$