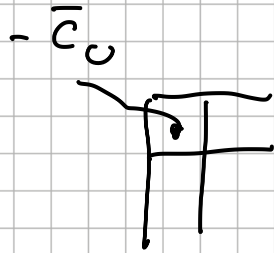
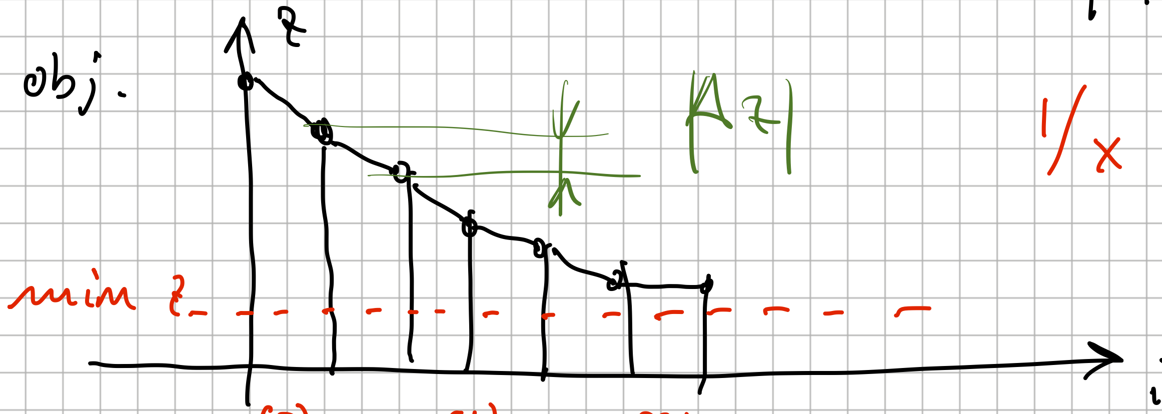
OR 1 20-OCT-2021

CONVERGENCE





$1/x$

$B^{(0)} \to B^{(1)} \to B^{(2)} \to \cdots$

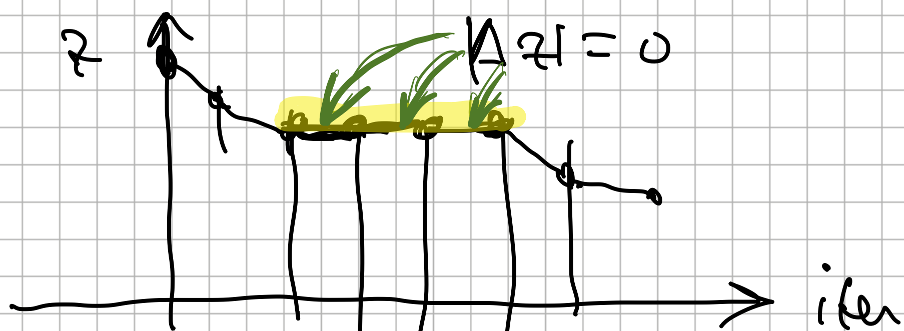
$\boxed{|\Delta z| \gneqq 0}$ ⇒ all the $B^{(i)}$'s are DIFFERENT

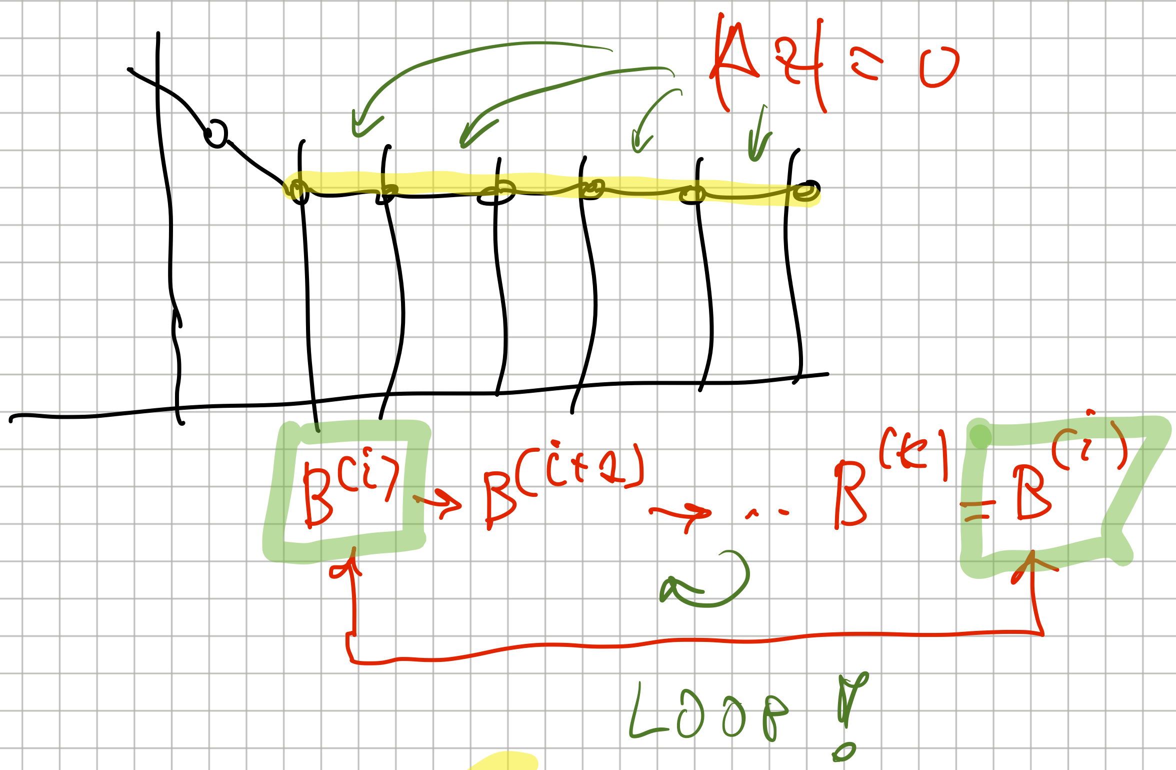
⇒ In the worst case, I'll enumerate ALL possible bases

⇒ after, at most, $\binom{n}{m} = \dfrac{n!}{m!(n-m)!}$ iterations

the alg. MUST stop!

- What if $|\Delta z| = 0$ for some iterations?





$$|\Delta z| := \underbrace{|\bar{c}_h|}_{\neq 0} \cdot \vartheta_h$$

where $$\boxed{\vartheta_h := \min\left\{ \frac{\bar{b}_i}{\bar{a}_{ih}} : \bar{a}_{ih} > 0 \right\}}$$

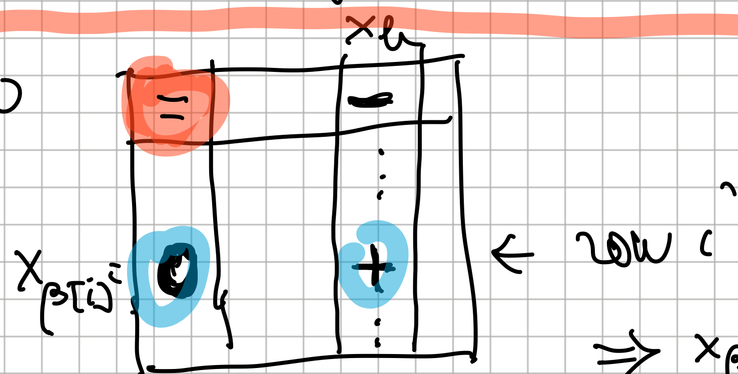
where $\bar{c}_h \lneq 0 \Rightarrow$

$$|\Delta z| = 0 \Rightarrow \vartheta_h = 0$$

$$\Rightarrow \boxed{\bar{b}_i = 0 \text{ for some } i \text{ (s.t. } \bar{a}_{ih} > 0)}$$

$\Delta z = 0$

$\bar{b}_i = 0$

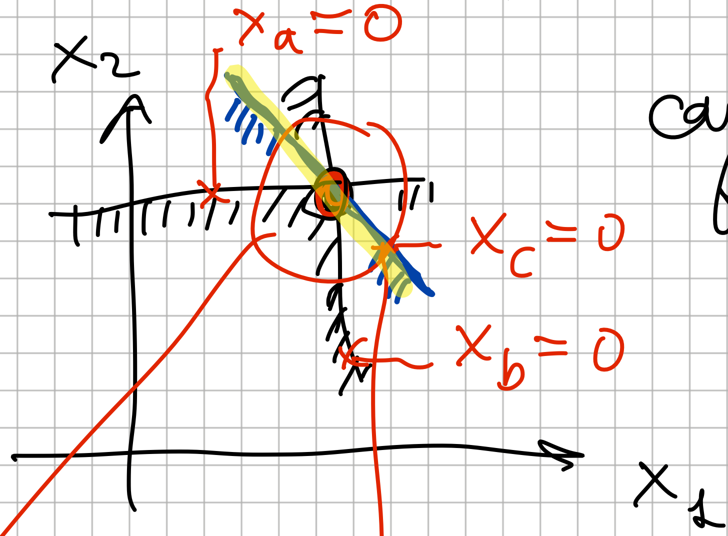


$\Rightarrow x_{\beta[i]} = 0$

$\Rightarrow$ DEGENERACY

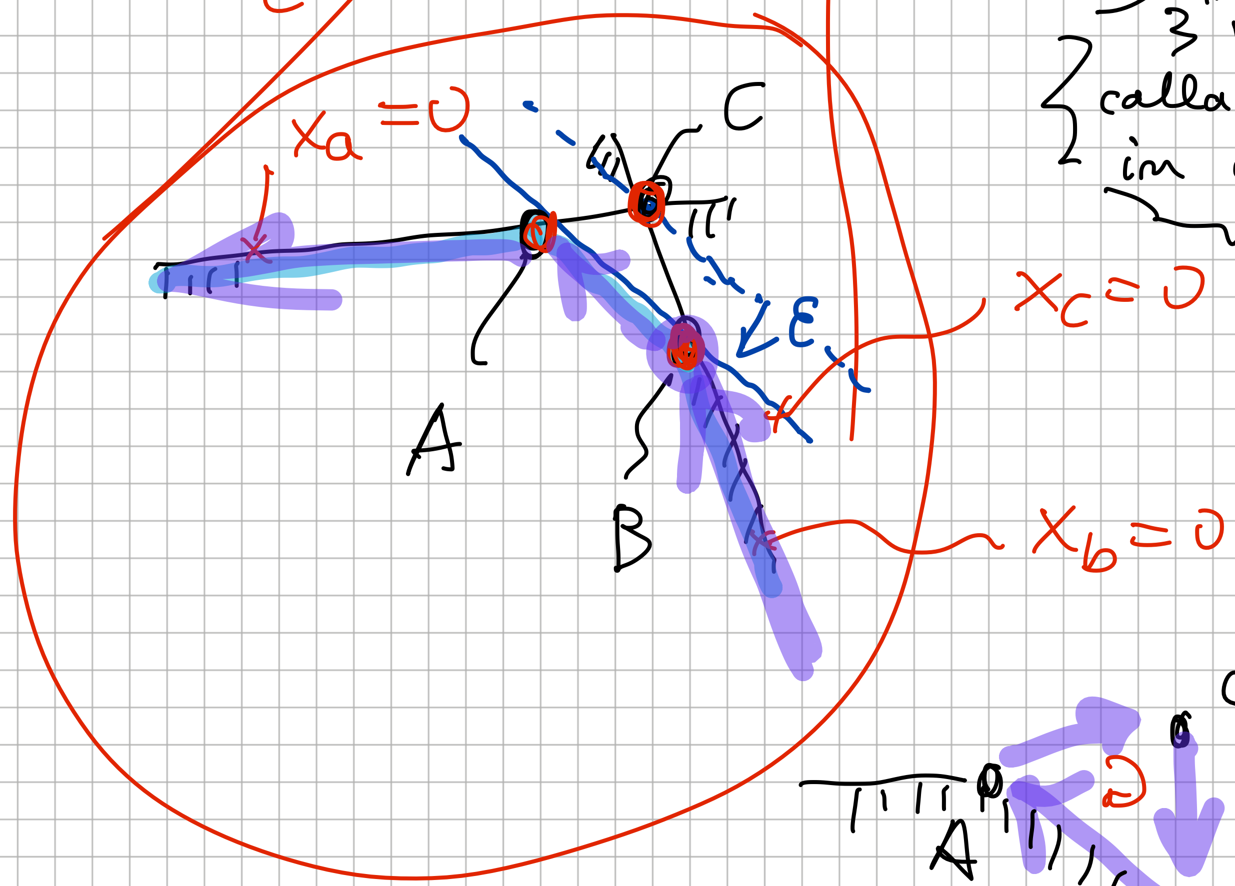
∴ Risk of LOOP ⟺ occurrence of DEGENERACY

Ex. $\mathbb{R}^2$

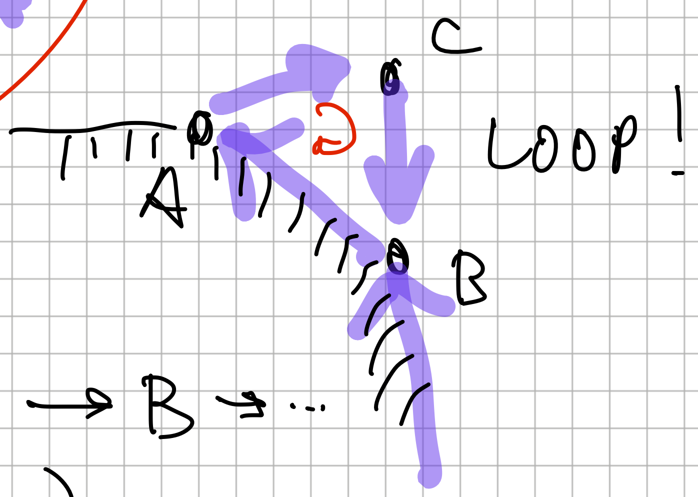


canonical form ($\leq$)

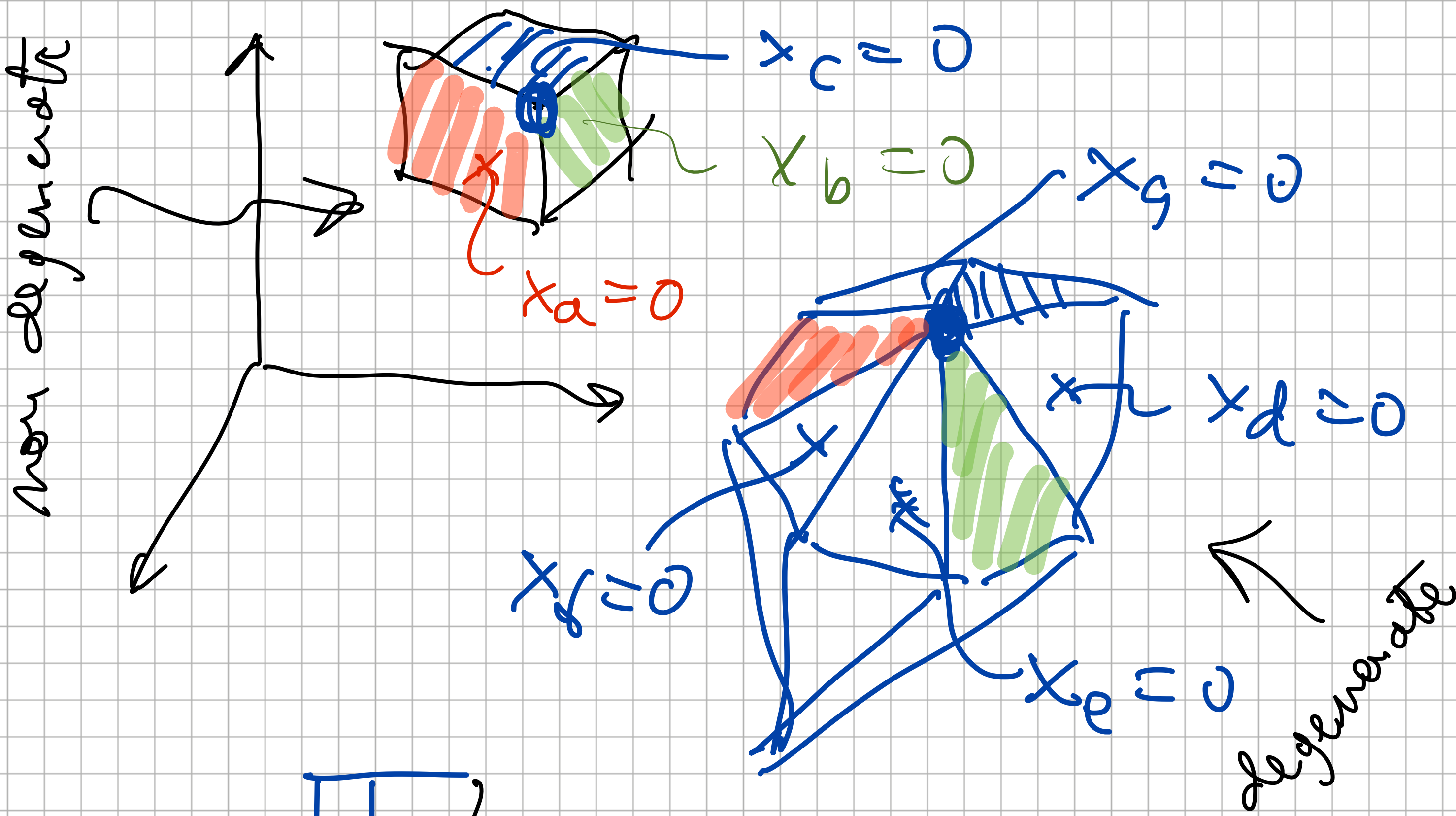
« x_c is basic but $= 0$ »

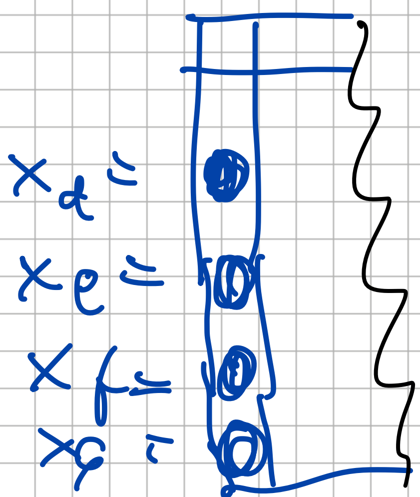


3 "vertices" collapsed in a single point ...



$B \to A \to C \to B \to \dots$
(loop)





very degenerate!

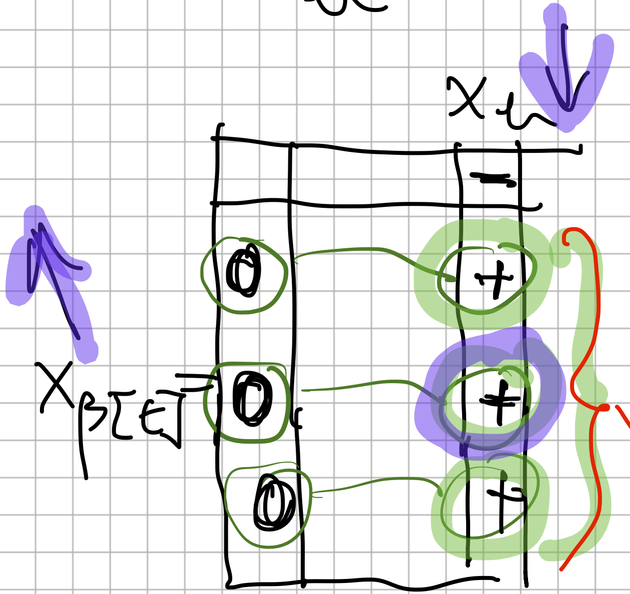
NO CONSTR. IS REDUNDANT!

HOW TO COPE WITH DEGEN.?

(1) DETERMINISTIC RULE for the choice of the PIVOT element can lead to a loop

⇒ RANDOMIZE the choice!

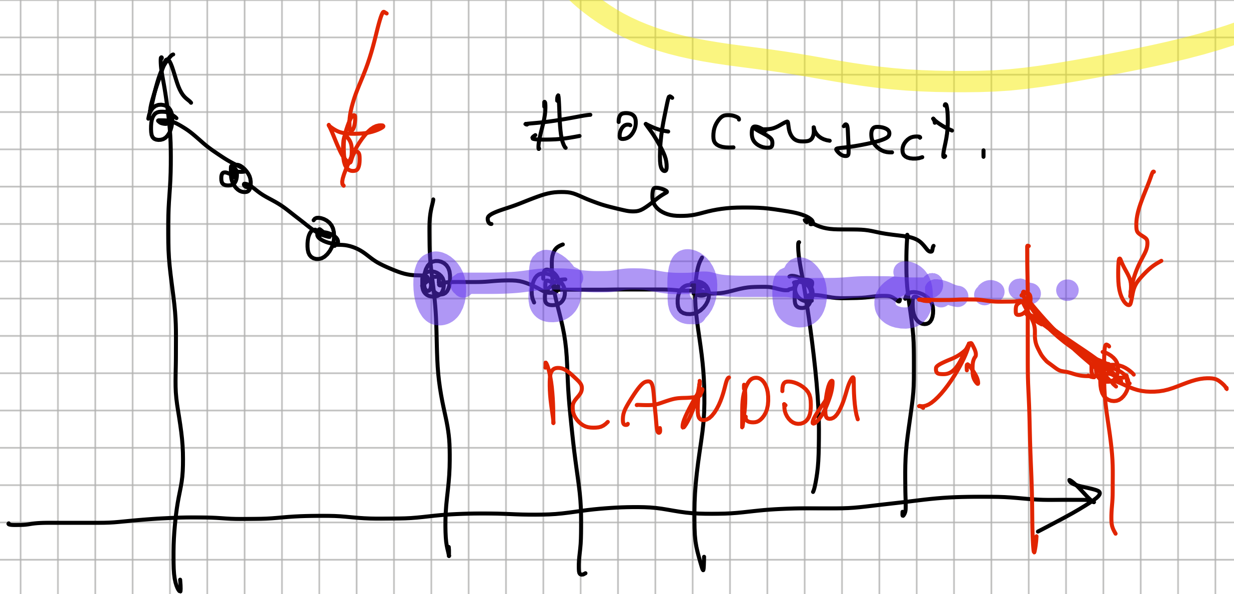
$\bar{c}_h < 0$ ⇝ RANDOM CHOICE



$\bar{b}_i / \bar{a}_{ih} \to$ MIN

$\theta = \min = 0$

RANDOM CHOICE!



• BLAND's rule:

very simple DETERMINISTIC rule that avoid the LOOP!

" whenever one can make a choice, prefer the entering/leaving variable x_j with MINIMUM index j "

EX :

x_2 $\quad$ x_4

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	
$-z=$	-10	5	-1	0	-10	0	0	0	
$x_5=$	8		4					0	$8/4$
$x_3=$	6		3					0	$6/3$
$x_6=$	1		-2					0	$-$
$x_7=$	2		1					1	$2/1$

shall I select x_5 or $\boxed{x_3}$ or x_7 (to leave the basis)?

TH: Using BLAND's rule, the simplex alg. converges after, at most, $\binom{n}{m}$ iterations.

Proof By contradiction.

Smallest case in which we have a loop (even using Bland's rule)

_obj

$x_n = x_{\beta[t]}$

$T =$

		$\dots x_{\beta[i]} \dots$	$x_h \dots$	x_n	
$-z =$	$=$	0	$-$	0	
	0	0	$\ominus$	0	
$x_{\beta[i]} =$	0	1	$\ominus$	0	← row i
	0	0	$\ominus$	0	
$x_{\beta[t]} =$	0	0	$+$	1	← row t
	0	0	$\ominus$	0	

$\ominus \leq 0$

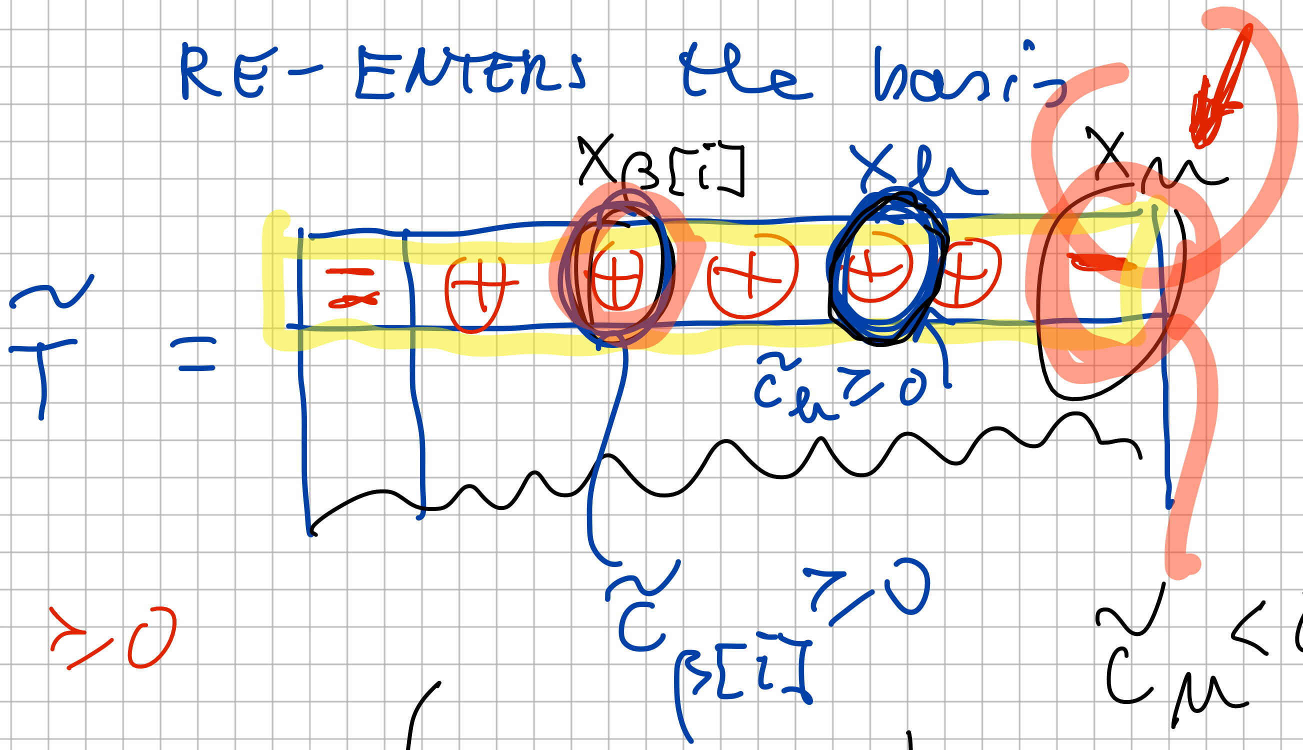
$+ > 0$

"During the loop, I'm making PIVOT op.s on EVERY row/col."

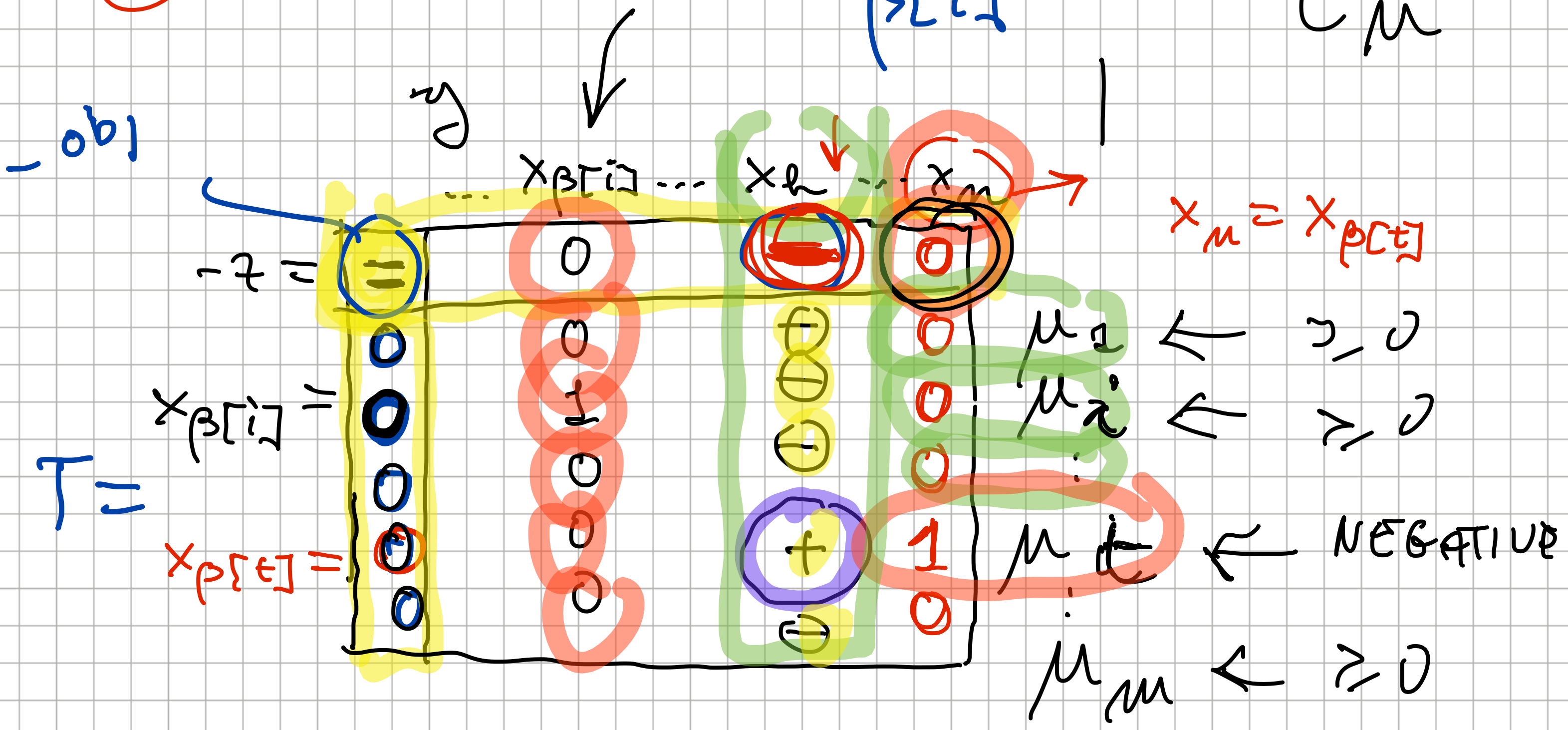
- tableau T in x_n LEAVES the basis
  - $\bar{c}_h < 0$ because x_h enters the basis
  - $\bar{a}_{th} > 0$ " if is the PIVOT elem.

- $\bar{a}_{ih} \le 0 \quad \forall\, i \ne t$ ($\Leftarrow$ BLAND)
- $\tilde{c}_{\beta[i]} = 0 \quad \forall\, i$ (reduced cost basic. var.)
- $\bar{b}_i = 0 \quad \forall\, i$ ($\Leftarrow$ MINIMALITY of this case)

- tableau $\tilde{T}$ when x_n RE-ENTERS the basis



$\oplus \ge 0$



$$[\text{row } 0 \text{ in } \tilde{T}] = [\text{row } 0 \text{ in } T] + \sum_{i=1}^{m} \mu_i \begin{bmatrix} \text{row } i \\ \text{of } T \end{bmatrix}$$

$$\underbrace{\tilde{c}_n}_{<0} = \underbrace{\bar{c}_n}_{0} + \mu_t \cdot 1 \Rightarrow \mu_t < 0$$

$$\underbrace{\tilde{c}_{\beta[i]}}_{\geq 0} = \underbrace{\bar{c}_{\beta[i]}}_{=0} + \mu_i \cdot 1 \Rightarrow \mu_i \geq 0 \quad \forall i \neq t$$

$x_{\beta[i]}$ is basic in T

$$\tilde{c}_h = \bar{c}_h + \sum_{i=1}^{m} \mu_i \bar{a}_{ih}$$

$$= \underbrace{\bar{c}_h}_{<0} + \sum_{i \neq t} \underbrace{\mu_i}_{\geq 0} \underbrace{\bar{a}_{ih}}_{\leq 0} + \underbrace{\mu_t}_{<0} \cdot \underbrace{\bar{a}_{th}}_{>0}$$

< 0 impossible!