OR 1 — 2-nov-2021

STANDARD FORM

$\bar{x} \in \mathbb{R}^n$, $\bar{u} \in \mathbb{R}^m$

$\min \{ c^T x : Ax \geq b, x \geq 0 \}$

$\max \{ u^T b : c^T \geq u^T A, u \geq 0 \}$

$\min \{ c^T x : Ax = b, x \geq 0 \}$

$\max \{ u^T b : c^T \geq u^T A \}$

$(\bar{x}, \bar{y})$ optimal iff:

(1) $\bar{x} \geq 0 : A\bar{x} \geq b$

(2) $\bar{u} \geq 0 : c^T \geq \bar{u}^T A$

(3.a) $\bar{u}^T (A\bar{x} - b) = 0$

(3.β) $(c^T - \bar{u}^T A)\bar{x} = 0$

$(\bar{x}, \bar{y})$ optimal iff:

(1') $\bar{x} \geq 0 : A\bar{x} = b$

(2') $\bar{u} : c^T - \bar{u}^T A \geq 0^T$

ALWAYS OK

(3') $(c^T - \bar{u}^T A)\bar{x} = 0$

SIMPLEX METHOD:

$\bar{x} = \begin{bmatrix} B^{-1}b \geq 0 \\ \hline 0 \end{bmatrix} \Rightarrow$ (1') OK

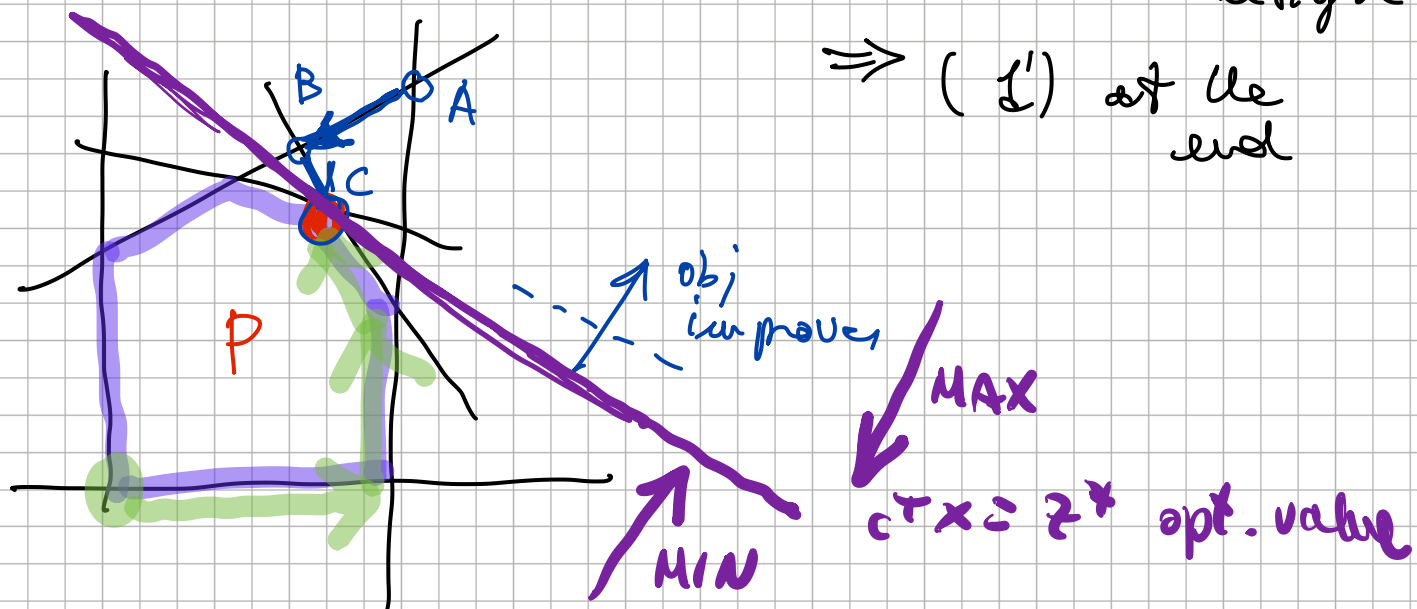
$\bar{u}^T := c_B^T B^{-1} \Rightarrow$ (3') OK, because:

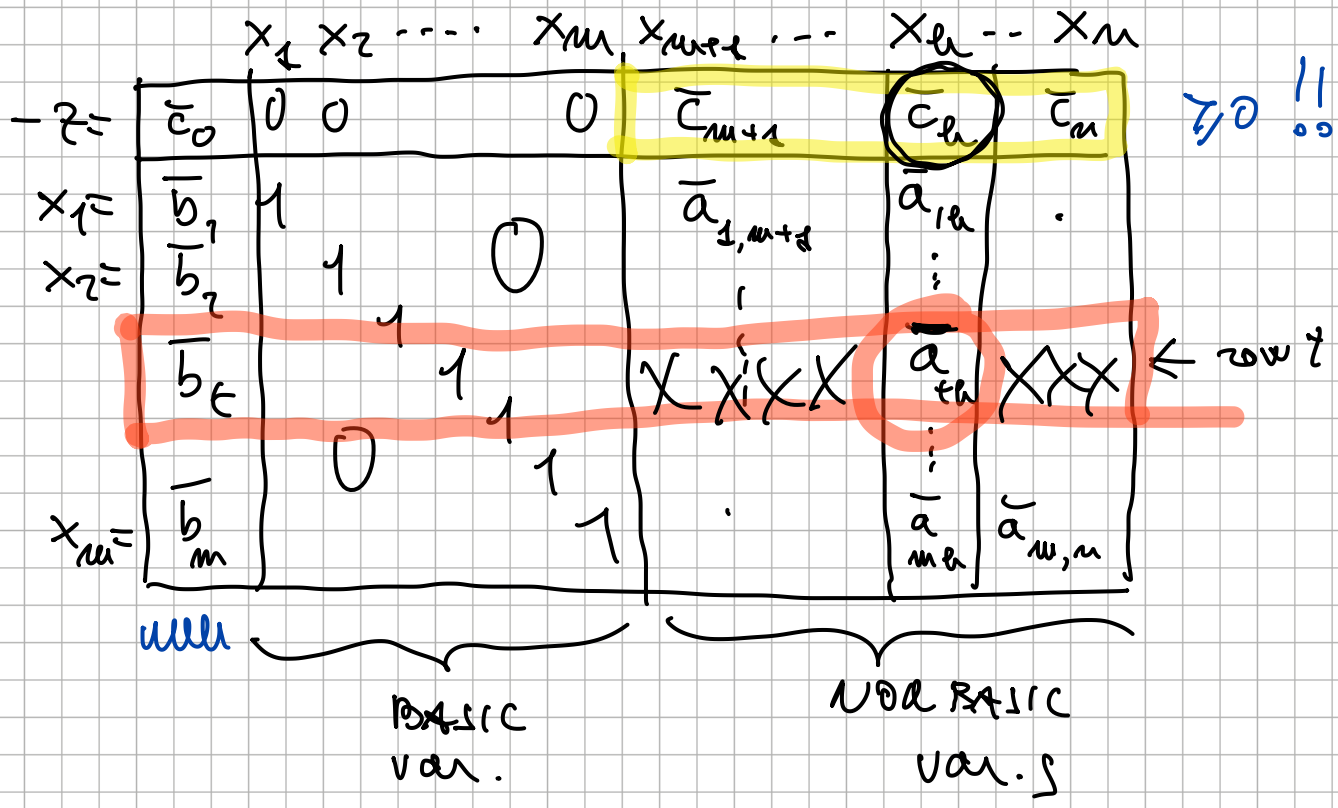
$$\underbrace{\left[c_B^T - \bar{u}^T B \right.}_{= 0^T}, \left. c_F^T - \bar{u}^T F \right] \left[\begin{array}{c} \bar{x}_B = B^{-1} b \\ \hline \bar{x}_F = 0 \end{array} \right] = 0$$

DUAL SIMPLEX METHOD

(2')-(3') : OK by design

$\Rightarrow$ (1') at the end





$\bar{b} \geq 0 \Rightarrow$ STOP "OPTIMAL TABLEAU"

$\exists\ \bar{b}_t < 0$:

PIVOT ELEM: $\bar{a}_{tk} < 0$

AFTER THE PIVOT OPERATION

$$\tilde{c}_j = \underbrace{\bar{c}_j}_{\geq 0} - \frac{\bar{c}_k}{\bar{a}_{tk}} \bar{a}_{tj} \geq 0, \quad \forall j$$

$$\underbrace{|\bar{c}_j|}_{\geq 0} \geq \frac{|\bar{c}_k|}{-|\bar{a}_{tk}|} \cdot \bar{a}_{tj}$$

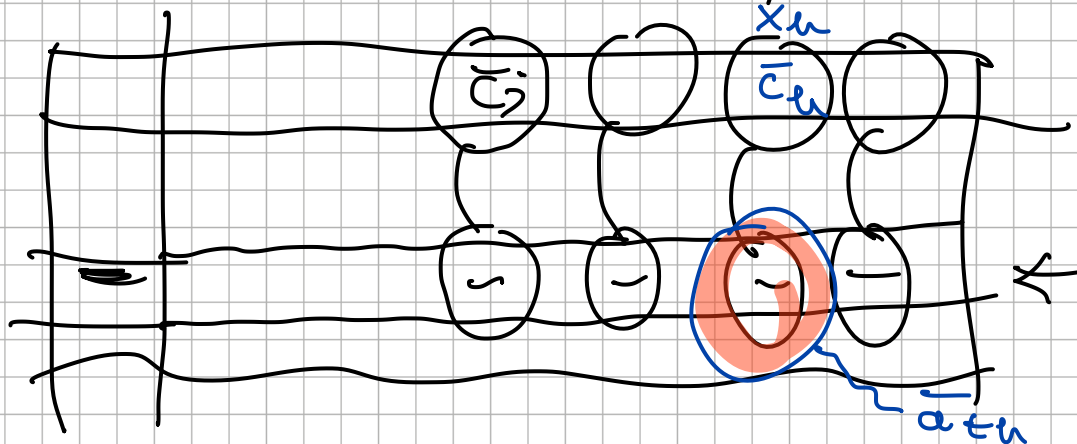
$\bar{a}_{tj} \geq 0 \rightarrow$ OK

$\bar{a}_{tj} < 0 \rightarrow$

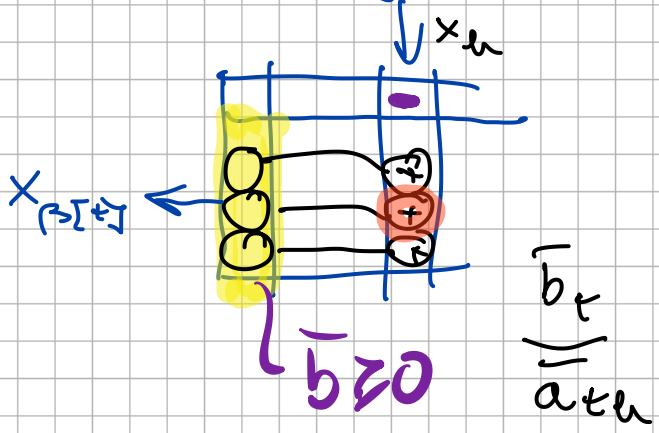
$$\frac{|\bar{c}_j|}{|\bar{a}_{tj}|} \geq \frac{|\bar{c}_k|}{+|\bar{a}_{tk}|} \quad \forall j$$

$\Rightarrow$ JUST DEFINE

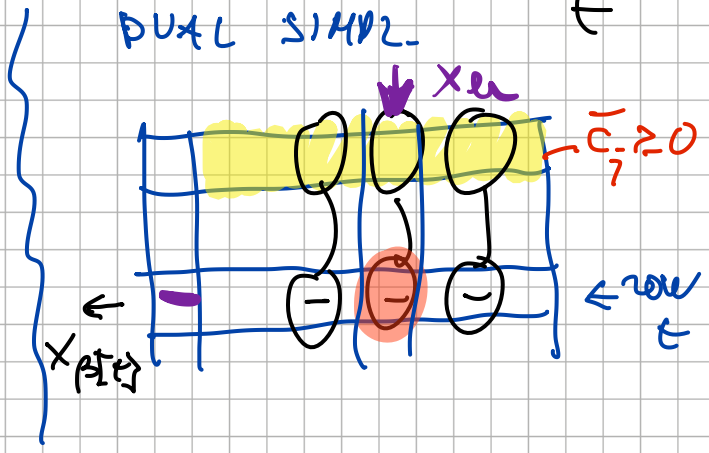
$$h = \arg\min \left\{ \left| \frac{\bar{c}_j}{\bar{a}_{tj}} \right| : \bar{a}_{tj} < 0 \right\}$$



"PRIMAL" SIMPLEX METHOD

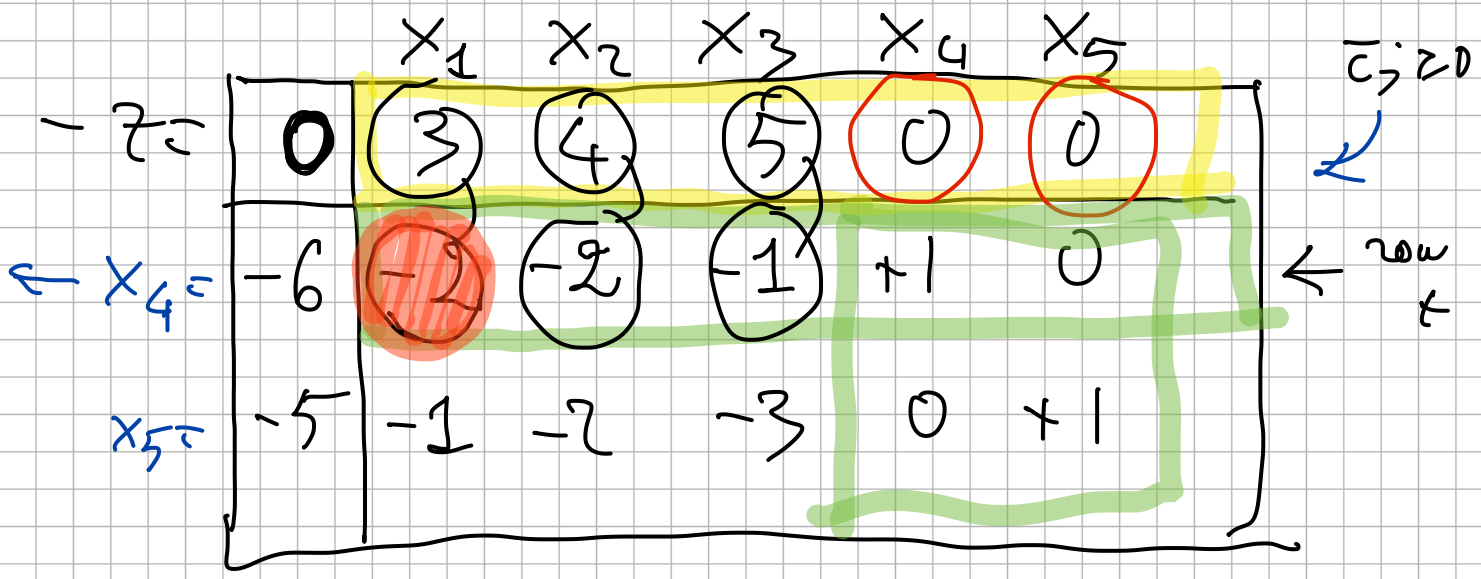


DUAL SIMPLEX

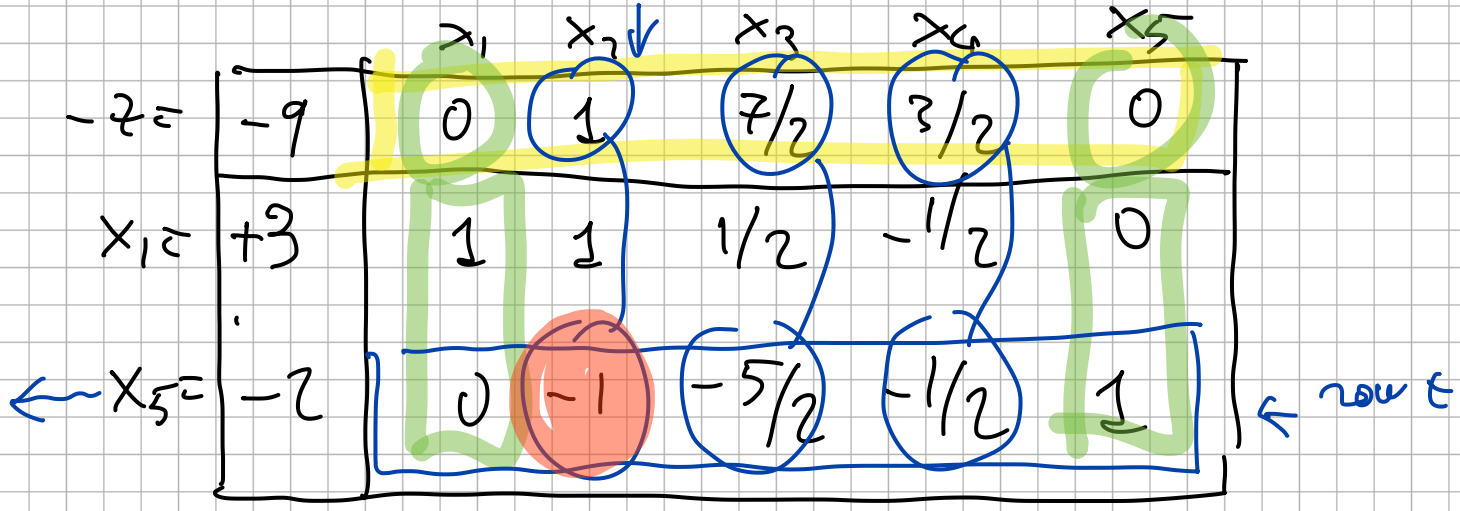


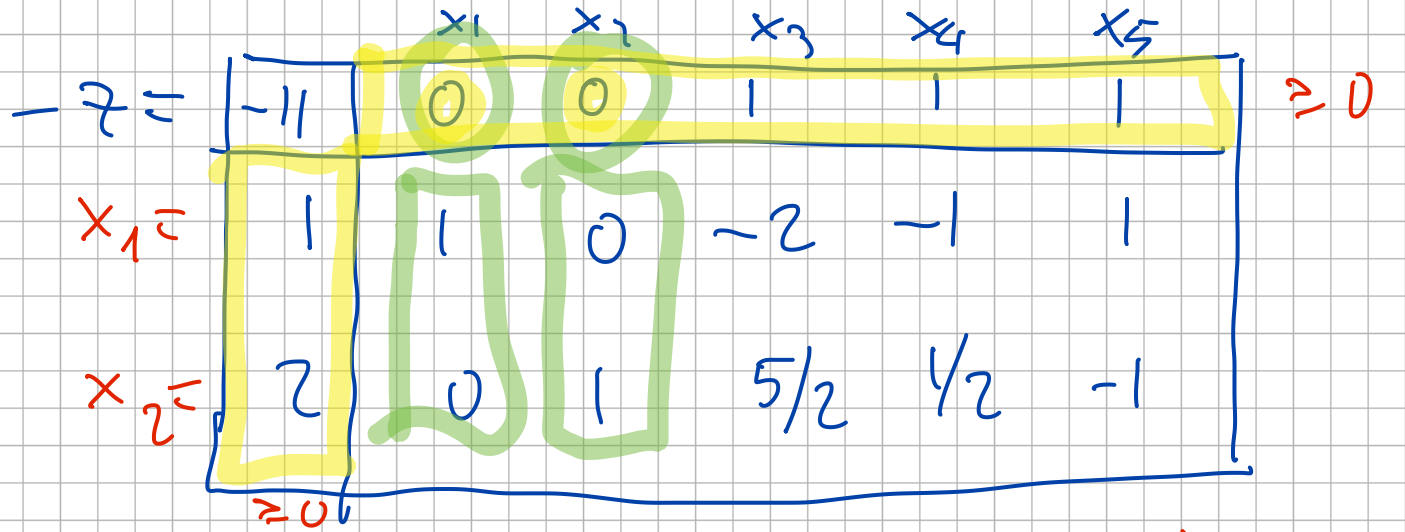
EX:

$$\begin{cases} \min \; 3x_1 + 4x_2 + 5x_3 \\ 2x_1 + 2x_2 + x_3 \geq 6 \quad (\times -1) \\ x_1 + 2x_2 + 3x_3 \geq 5 \quad (\times -1) \\ x_1, x_2, x_3 \geq 0 \end{cases}$$



"infeasible SUPER-OPTIMAL basis"

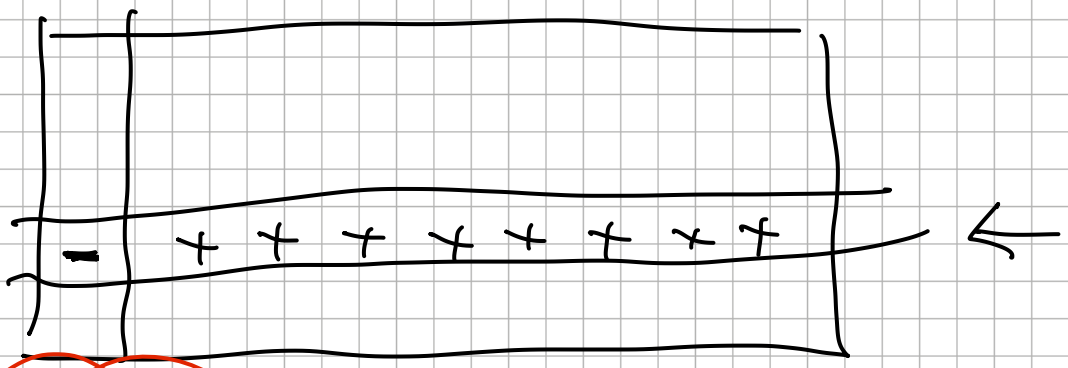




$(x_1 = 1, x_2 = 2, x_3 = x_4 = 0)$

$\Rightarrow$ FEASIBLE BASIC SOL.

$\Rightarrow$ OPTIMAL $\rightarrow$ STOP



" $\bar{b}_t < 0$ & $\bar{a}_{tj} \geq 0 \quad \forall j \Rightarrow$? "

Write down eq. corresp. to row t

$$\sum_j \bar{a}_{tj} x_j = \bar{b}_t \qquad (*)$$

$\bar{a}_{tj} \geq 0$, $x_j \geq 0$, $\bar{b}_t \neq 0$

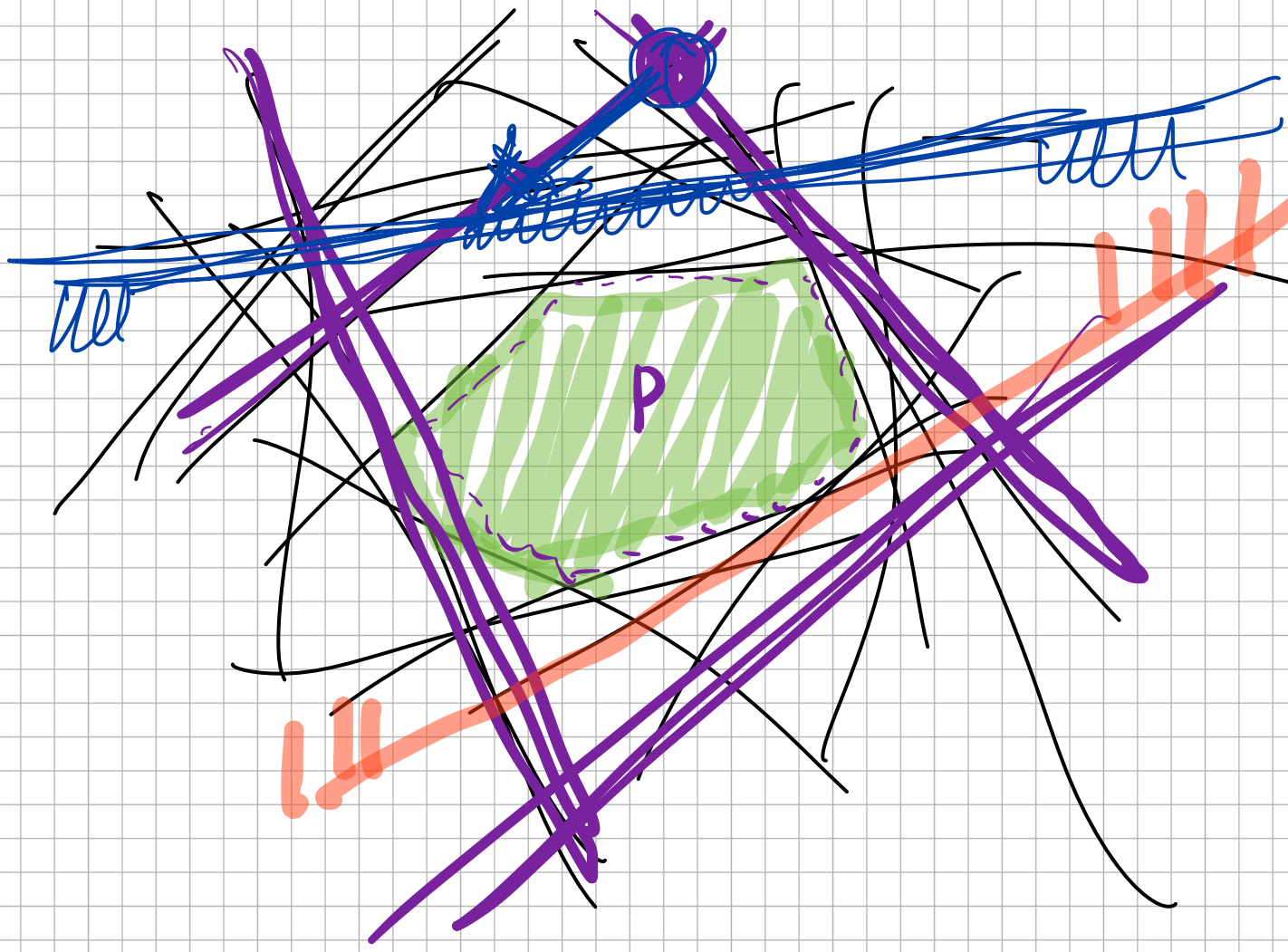
$\Rightarrow \geq 0$

$\Rightarrow$ IMPOSSIBLE!

$\Rightarrow$ STOP " PRIMAL PR. IS INFEASIBLE "

$\nexists \; x \geq 0$ that satisfies $(*)$

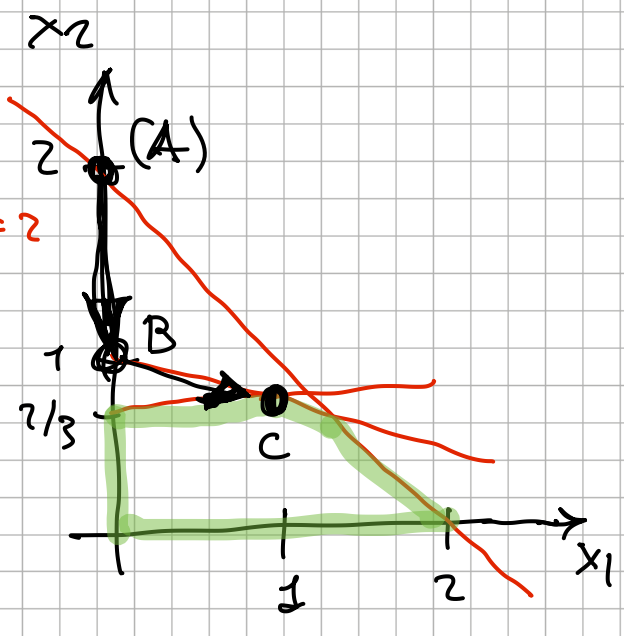
DUAL METHOD very useful:



" ADDING violated constr.s on the fly "

Ex.

$$\begin{cases} \min\ -x_1 - 4x_2 \\ x_1 + x_2 \le 2 \\ x_1 + 3x_2 \le 3 \\ x_2 \le 2/3 \\ x_1, x_2 \ge 0 \end{cases}$$



	x_1	x_2	x_3
$-z =$	0	-1	-4	0
$x_3 =$	2	1	(1)	1

	x_1	x_2	x_3
$-z =$	8	3	0	4
$x_2 =$	2	1	(1)	1

$\rightarrow$ opt $\Leftrightarrow \bar{c}_j \geq 0$

$x^T = (0, 2)$ $\rightarrow$ point (A)

		x_2		x_4	x_5
	8	3	0	4	0	0
	2	1	(1)	1	0	0
	3	1	3	0	1	0
	1/3	0	(1)	0	0	1

≥ 0

	x_1	x_2	x_3	x_4	x_5
$-z =$	8	3	0	4	0	0
$x_2 =$	2	1	1	0	0	0
$x_4 =$	-3	-2	0	-3	1	0
$x_5 =$	-4/3	-1	0	-1	0	1

$\bar{c}_j \geq 0 \; \forall j$

$\Rightarrow$ PERFECT for DUAL SIMPLEX !

$\rightarrow$ DUAL ... PIVOTS $\rightarrow$ OPTIMAL TABLEAU !