

OR 1

2-10V-2021

STANDARD FORM

$\bar{x} \in \mathbb{R}^n, \bar{u} \in \mathbb{R}^m$

min $\{ c^T x : Ax \geq b, x \geq 0 \}$
 max $\{ u^T b : c^T \geq u^T A, u \geq 0 \}$

min $\{ c^T x : Ax = b, x \geq 0 \}$
 max $\{ u^T b : c^T \geq u^T A \}$

(\bar{x}, \bar{u}) optimal iff:

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(1) $\bar{x} \geq 0 : A\bar{x} \geq b$

(1') $\bar{x} \geq 0 : A\bar{x} = b$

(2) $\bar{u} \geq 0 : c^T \geq \bar{u}^T A$

(2') $\bar{u} : c^T - \bar{u}^T A \geq 0$

(3.a) $\bar{u}^T (A\bar{x} - b) = 0$

~~ALWAYS OK~~

(3.β) $(c^T - \bar{u}^T A)\bar{x} = 0$

(3') $(c^T - \bar{u}^T A)\bar{x} = 0$

SIMPLEX METHOD:

$\bar{x} = \begin{bmatrix} B^{-1}b \geq 0 \\ 0 \end{bmatrix} \Rightarrow (1') \text{ OK}$

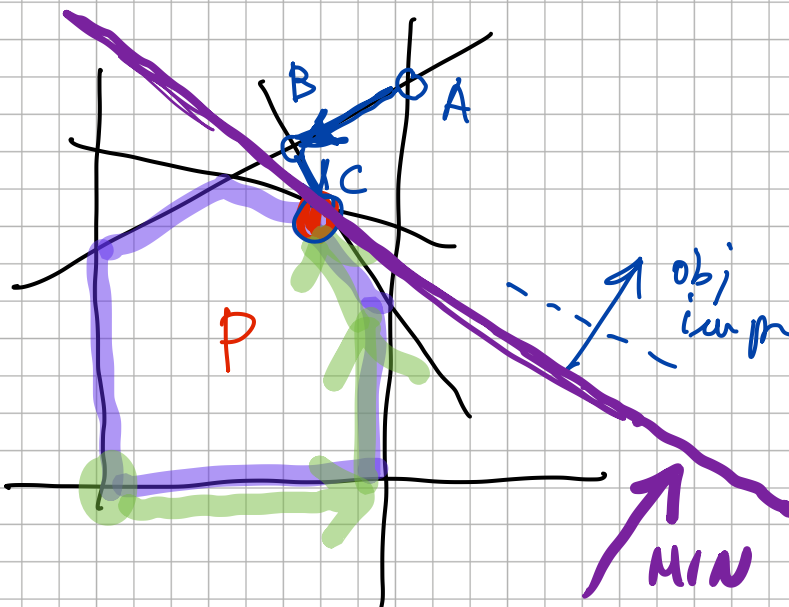
$\bar{u}^T := c_B^T B^{-1} \Rightarrow (3') \text{ OK, because:}$

$\begin{bmatrix} c_B^T - \bar{u}^T B & c_F^T - \bar{u}^T F \end{bmatrix} \begin{bmatrix} \bar{x}_B = B^{-1}b \\ \bar{x}_F = 0 \end{bmatrix} = 0$

DUAL SIMPLEX METHOD

(2') - (3') : OK by design

$\Rightarrow (1')$ at the end



MAX
 $c^T x = z^*$ opt. value
 MIN

	x_1	x_2	...	x_m	x_{m+1}	...	x_h	...	x_n
$-z =$	\bar{c}_0	0	0	0	\bar{c}_{m+1}		\bar{c}_h		\bar{c}_n
$x_1 =$	\bar{b}_1	1			$\bar{a}_{1,m+1}$		\bar{a}_{1h}		.
$x_2 =$	\bar{b}_2		1	0	.		\vdots		.
	\bar{b}_t			1	X	X	\bar{a}_{th}	X	X
		0		1	.		\vdots		.
$x_m =$	\bar{b}_m			1	.		\bar{a}_{mh}		$\bar{a}_{m,n}$

!!!
← row t

BASIC var.
NON BASIC var.s

$\bar{b} \geq 0 \Rightarrow \text{STOP}$ "OPTIMAL TABLEAU"

$\exists \bar{b}_t < 0 :$

PIVOT elem: $\bar{a}_{th} < 0$

AFTER THE PIVOT OPERATION

$$\tilde{c}_j = \bar{c}_j - \frac{\bar{c}_h \bar{a}_{tj}}{\bar{a}_{th}} \geq 0, \forall j$$

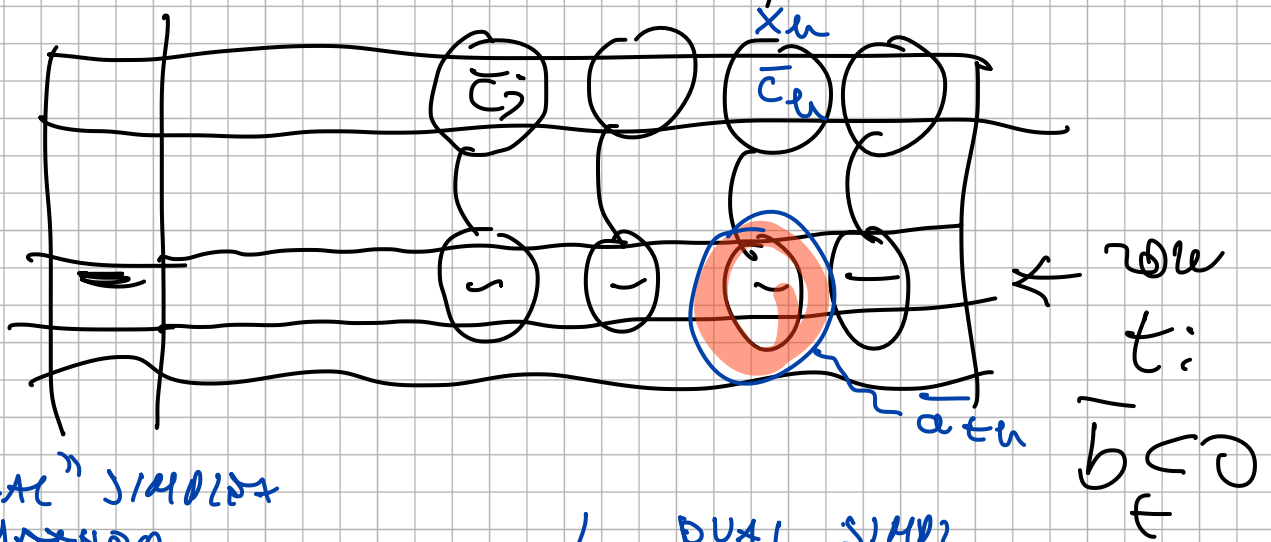
$$|\bar{c}_j| \geq \frac{|\bar{c}_h|}{|\bar{a}_{th}|} \cdot \bar{a}_{tj}$$

$\bar{a}_{tj} \geq 0 \rightarrow \text{OK}$
 $\bar{a}_{tj} < 0 \rightarrow$

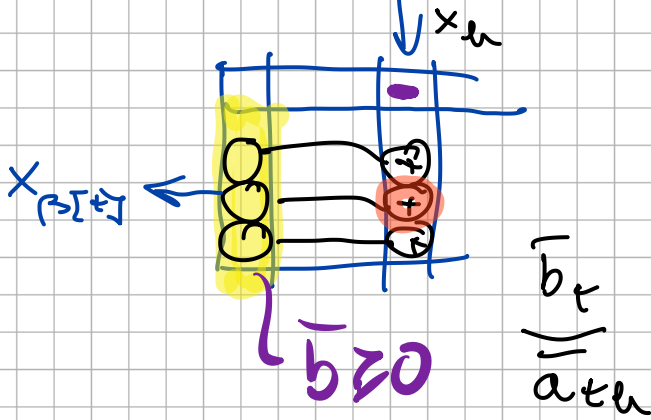
$$\frac{|\bar{c}_j|}{|\bar{a}_{tj}|} \geq \frac{|\bar{c}_h|}{|\bar{a}_{th}|}$$

⇒ JUST DEFINE

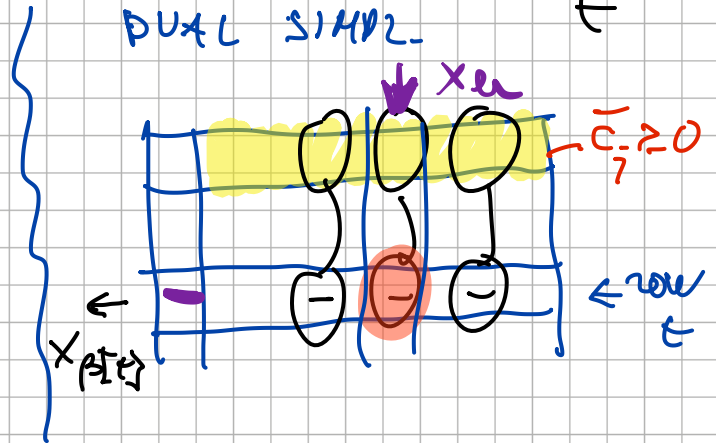
$$h = \arg \min \left\{ \left| \frac{\bar{c}_j}{\bar{a}_{tj}} \right| : \bar{a}_{tj} < 0 \right\}$$



« PRIMAL » SIMPLEX METHOD



DUAL SIMPL.



EX :

$$\begin{cases} \min & 3x_1 + 4x_2 + 5x_3 \\ & 2x_1 + 2x_2 + x_3 \geq 6 \quad -x_4 \\ & x_1 + 2x_2 + 3x_3 \geq 5 \quad -x_5 \\ & x_1, x_2, x_3 \geq 0 \end{cases}$$

	x_1	x_2	x_3	x_4	x_5
$-z =$	0	3	4	5	0
$x_4 =$	-6	-2	-2	+1	0
$x_5 =$	-5	-1	-2	0	+1

Annotations: $\bar{c}_j \geq 0$ (pointing to the top row), row 4 (pointing to the second row), row 5 (pointing to the third row).

"infeasible SUPER-OPTIMAL basis"

	x_1	x_2	x_3	x_4	x_5
$-z =$	-9	0	1	7/2	3/2
$x_1 =$	+3	1	1	1/2	-1/2
$x_5 =$	-2	0	-1	-5/2	-1/2

Annotations: row 5 (pointing to the third row), row 4 (pointing to the third row).

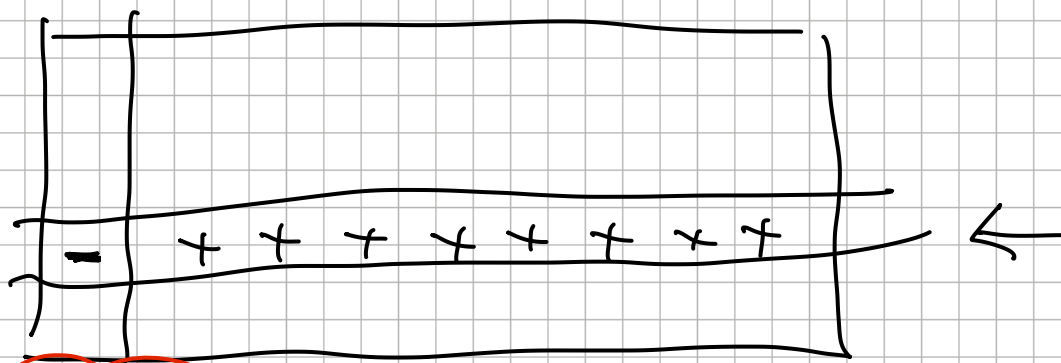
	x_1	x_2	x_3	x_4	x_5
$-z =$	-11	0	0	1	1
$x_1 =$	1	1	0	-2	-1
$x_2 =$	2	0	1	5/2	1/2

Annotations: ≥ 0 (pointing to the bottom row), ≥ 0 (pointing to the right side).

$$(x_1 = 1, x_2 = 2, x_3 = x_4 = 0)$$

\Rightarrow FEASIBLE BASIC SOL.

\Rightarrow OPTIMAL \Rightarrow STOP



" $\bar{b}_t < 0$ & $\bar{a}_{tj} \geq 0 \quad \forall j \Rightarrow ?$ "

Write down eq. corresp. to row t

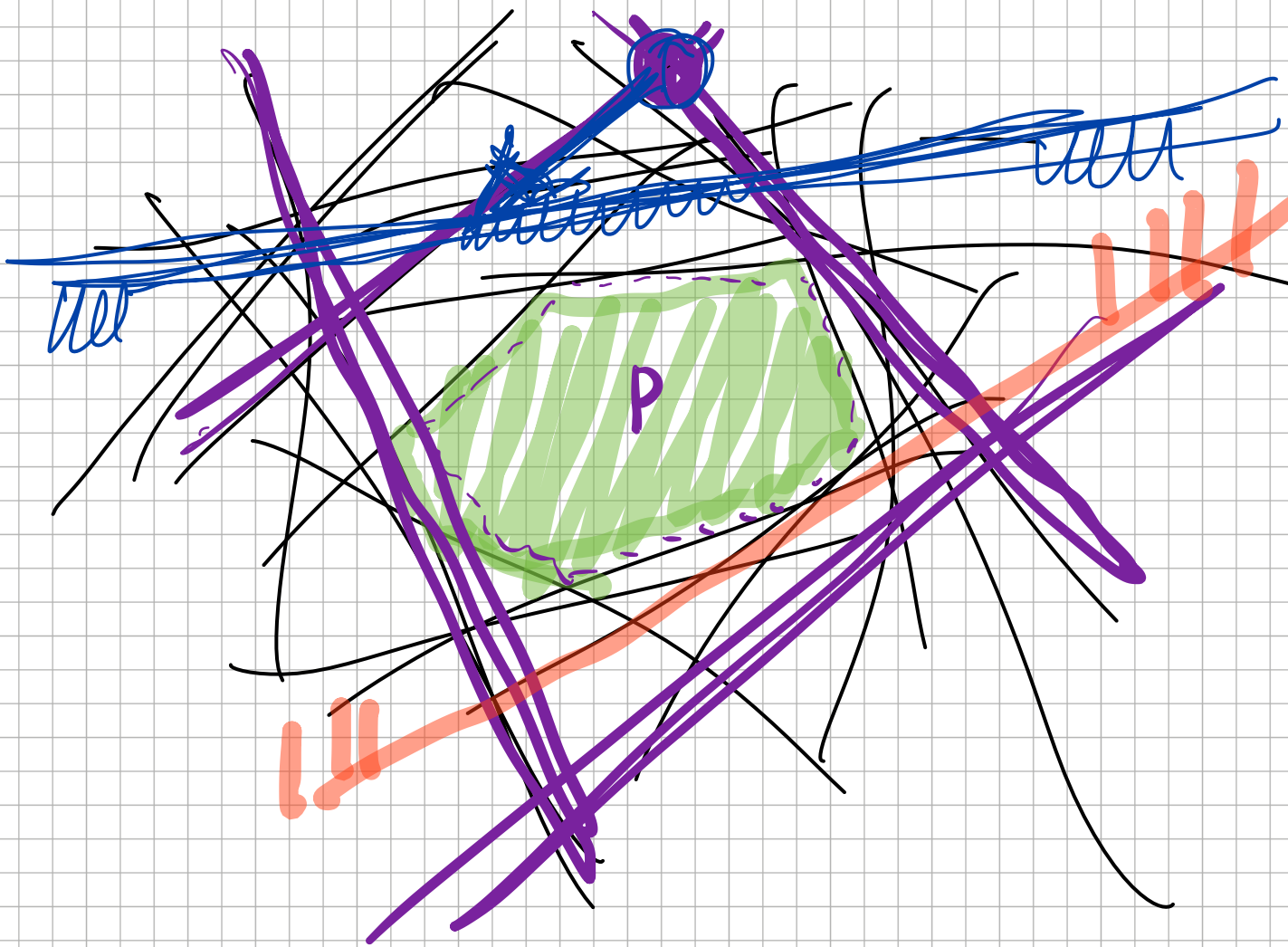
$$\sum_j \underbrace{\bar{a}_{tj}}_{\geq 0} \underbrace{x_j}_{\geq 0} = \underbrace{\bar{b}_t}_{< 0} \quad (*)$$

\Rightarrow IMPOSSIBLE!

\Rightarrow STOP " PRIMAL PR. IS INFEASIBLE "

$\nexists x \geq 0$ that satisfies (*)

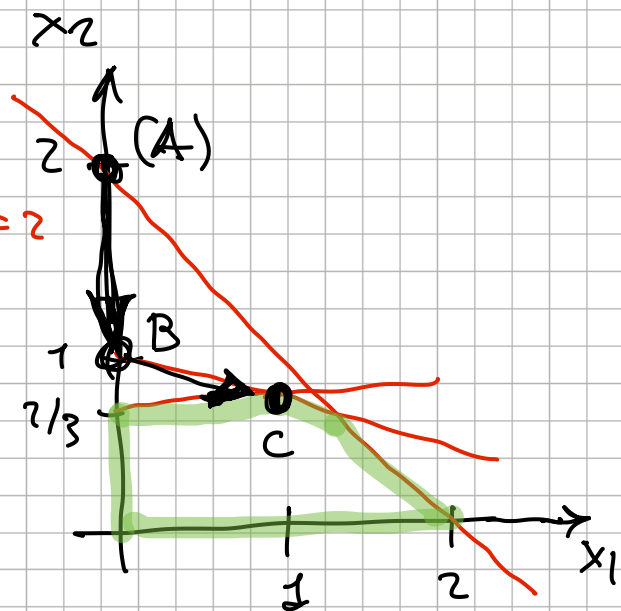
DUAL METHOD very useful:



" ADDING violated constr. s
on the fly "

EX.

$$\begin{cases}
 \text{min} & -x_1 - 4x_2 \\
 & x_1 + x_2 \leq 2 \\
 & x_1 + 3x_2 \leq 3 \\
 & x_2 \leq 2/3 \\
 & x_1, x_2 \geq 0
 \end{cases}$$



	x_1	x_2	x_3
$-z =$	0	-1	-4
$x_3 =$	2	1	1

	x_1	x_2	x_3
$-z =$	8	3	0
$x_2 =$	2	1	1

$\rightarrow \text{opt} \Leftrightarrow \bar{c}_j \geq 0$

$x^* = (0, 2) \rightarrow \text{point (A)}$

	x_1	x_2	x_3	x_4	x_5
$-z =$	8	3	0	4	0
$x_2 =$	2	1	1	1	0
	3	1	3	0	1
	$\frac{2}{3}$	0	1	0	1

$\leftarrow \geq 0$

	x_1	x_2	x_3	x_4	x_5
$-z =$	8	3	0	4	0
$x_2 =$	2	1	1	0	0
$x_4 =$	-3	-2	0	-3	1
$x_5 =$	$-\frac{4}{3}$	-1	0	-1	0

$\bar{c}_j \geq 0 \forall j$

\Rightarrow PERFECT for DUAL SIMPLEX!

\rightarrow DUAL PROOF \rightarrow OPTIMAL TABLEAU!