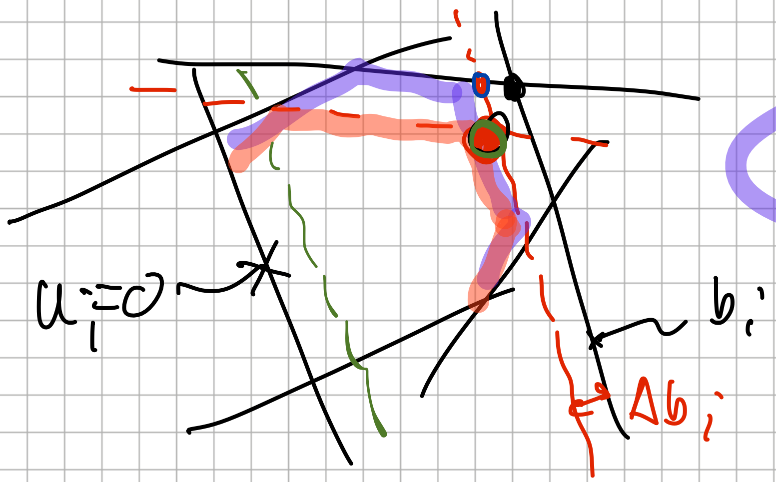
OR 1 - 3-NOV-2021

SENSITIVITY ANALYSIS



$c_i + \Delta c_i$

ROBUST OPT.

- OPTIMAL BASIS B W.R.T. "NOMINAL" DATA
- INPUT : $(A, b, c) \to (A + \cancel{\Delta A}, b + \Delta b, c + \Delta c)$

- STABILITY OF THE BASIS

(C.1) $B^{-1} b \geq 0$ primal feasibility

(C.2) $\bar{c}^T := c^T - \underbrace{c_B^T B^{-1}}_{=: u^T} A \geq 0^T$ dual feas.

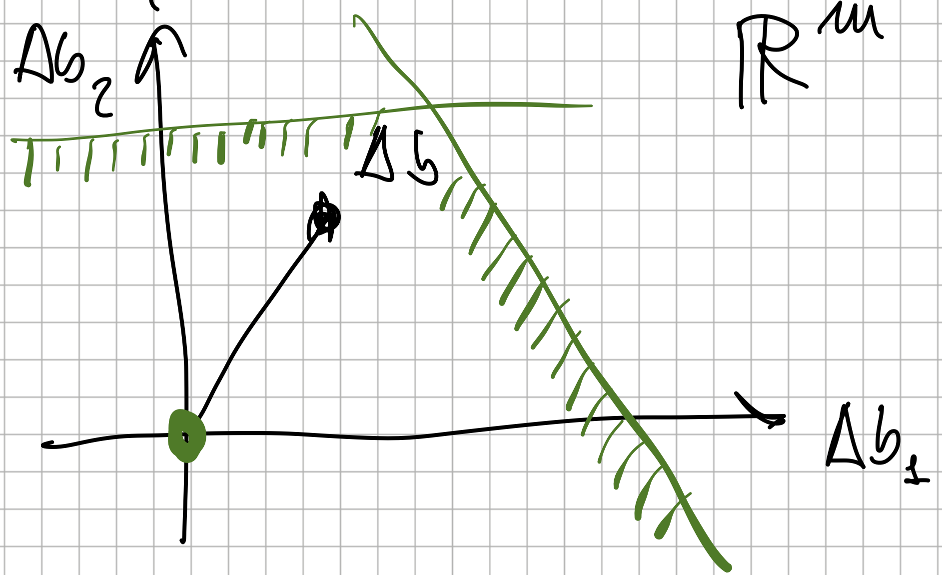
$\boxed{\text{1 RHS vector } b \to b + \Delta b}$

(C.1) $B^{-1}(b + \Delta b) \geq 0$

(C.2) OK

$$\boxed{B^{-1} b \geq - B^{-1} \Delta b}$$

m ineq. in $\mathbb{R}^m$



OPTIMAL SOL. VALUE:

- $c_B^T B^{-1} b \to$ opt. sol. value before the change

- $c_B^T B^{-1} (b + \Delta b) \Rightarrow$

$$\Delta z = \underbrace{c_B^T B^{-1}}_{=: u^T} \Delta b =$$

optimal sol. of the dual

$$= u^T \Delta b = \sum_{i=1}^{m} u_i \Delta b_i$$

$u_i = 0 \rightsquigarrow \Delta b_i \to \Delta z = 0$

$|u_i|$ very large: small Δb_i $\to$ large Δz

$$\boxed{②\qquad c_F \to c_F + \Delta c_F}$$

(C.1) $\cancel{x \; B^{-1} b \geq 0}$ X OK

(C.2) $\tilde{c}^T =$

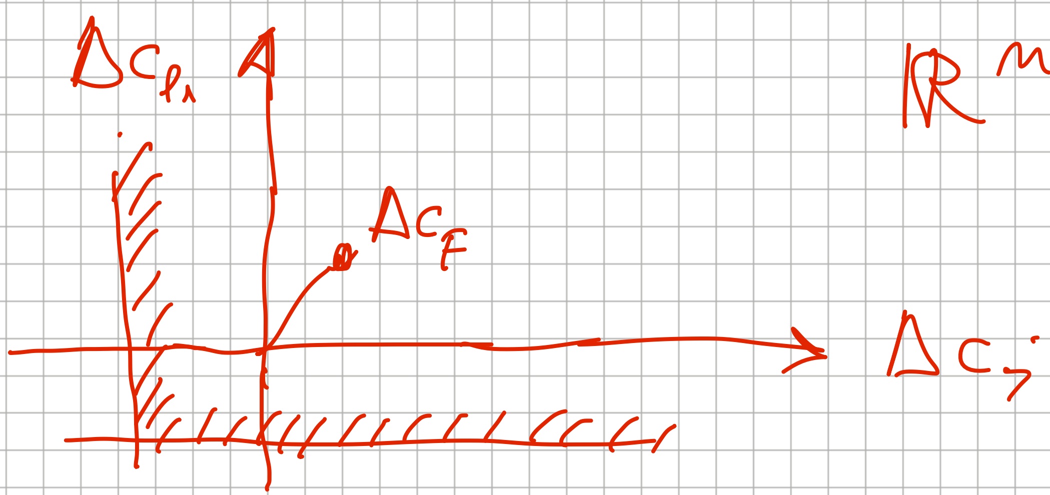
$$[\tilde{c}_B^T, \tilde{c}_F^T] = [0^T, (c_F^T + \Delta c_F^T) - c_B^T B^{-1} F] \geq 0^T$$

$$= [0^T, \underbrace{\tilde{c}_F^T + \Delta c_F^T}_{\geq 0^T}]$$

$$\iff \boxed{I \, \Delta c_F \geq - \tilde{c}_F}$$

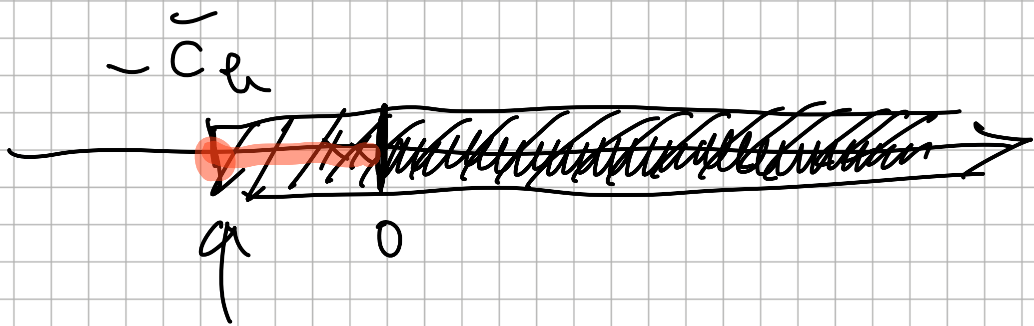
n − m ineq.s in $\mathbb{R}^{n-m}$

$\mathbb{R}^{n-m}$



For each non basic var. x_h

$$\Delta c_h \geq -\bar{c}_h \quad \forall h \ / \ \text{NONBASIC}$$



$-\bar{c}_h$ breakpoint

$\bar{c}_h \gg 0$ very large $\rightarrow$ very large changes without selecting this var.

$\bar{c}_h \simeq 0 \rightarrow$ small changes suffice ...

$$\boxed{③ \quad c_B \rightarrow c_B + \Delta c_B}$$

(C.1) OK

(C.2) $\tilde{c}^T = [\tilde{c}_B^T, \tilde{c}_F^T] =$

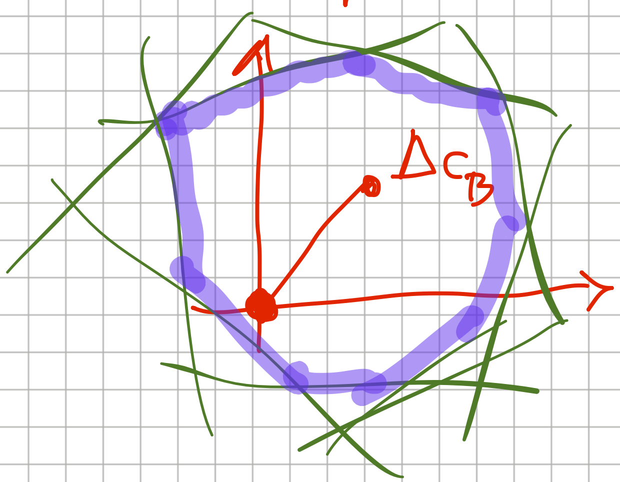
$u^T = c_B^T B^{-1} \rightarrow \tilde{u}^T = (c_B + \Delta c_B)^T B^{-1}$

$$= \left[0^T \;,\; c_F^T - \underbrace{(c_B^T + \Delta c_B^T)}_{\text{new } u^T} B^{-1} F \right] \geq 0^T$$

$$\iff \underbrace{\left(c_F^T - c_B^T B^{-1} F\right)}_{=:\, \bar{c}_F^T} - \Delta c_B^T B^{-1} F \geq 0^T$$

$$\iff \boxed{\Delta c_B^T B^{-1} F \leq \bar{c}_F^T}$$

$n-m$ ineq.s in $\mathbb{R}^m$



$\mathbb{R}^m$

$\boxed{\text{EXAMPLE}}$

INITIAL TABLEAU

		x_1	x_2	x_3	x_4	x_5	x_6
$-z =$	0	-3	-1	-3	0	0	0
$x_4 =$	2	2	1	1	1	0	0
$x_5 =$	5	1	2	3	0	1	0
$x_6 =$	6	2	2	1	0	0	1

b $\quad A_1 \quad A_3 \quad A_6$

OPTIMAL TABLEAU

		x_1	x_2	x_3	x_4	x_5	x_6
$-z =$	$27/5$	0	$7/5$	0	$6/5$	$3/5$	0
$x_1 =$	$1/5$	1	$1/5$	0	$3/5$	$-1/5$	0
$x_3 =$	$8/5$	0	$3/5$	1	$-1/5$	$2/5$	0
$x_6 =$	4	0	1	0	-1	0	1

reduced cost

$\bar{b} = B^{-1} b$

B^{-1}

$$x_B = [\, x_1 ,\ x_3 ,\ x_6 \,]^T$$

$$x_F = [\, x_2 ,\ x_4 ,\ x_5 \,]^T$$

$$B = \left[\, A_1 \,\middle|\, A_3 \,\middle|\, A_6 \,\right]$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 3/5 & -1/5 & 0 \\ -1/5 & 2/5 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} 1/5 \\ 8/5 \\ 4 \end{bmatrix} =: \bar{b}$$

$$c_B^T = [c_1, c_3, c_6] =$$

$$[-3, -3, 0]$$

$$c_F^T = [-1, 0, 0]$$

$$u^T = c_B^T B^{-1} =$$

$$[-6/5, -3/5, 0]$$

1 $\quad b \to b + \Delta b$

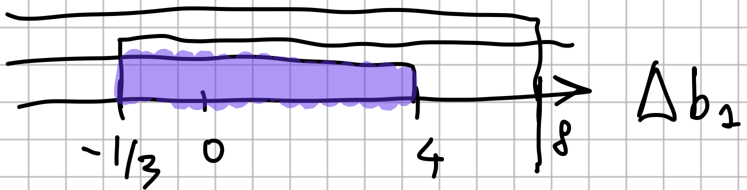
$$\boxed{B^{-1}\Delta b \geq -B^{-1}b = -\bar{b}}$$

$$\begin{bmatrix} 3/5 & -1/5 & 0 \\ -1/5 & 2/5 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} \Delta b_1 \\ \Delta b_2 \\ \Delta b_3 \end{bmatrix} \geq -\begin{bmatrix} 1/5 \\ 8/5 \\ 4 \end{bmatrix}$$

$$\begin{cases} 3/5\,\Delta b_1 - 1/5\,\Delta b_2 \geq -1/5 \\ -1/5\,\Delta b_1 + 2/5\,\Delta b_2 \geq -8/5 \\ -\Delta b_1 + \Delta b_3 \geq -4 \end{cases}$$

$\Delta b_2 = \Delta b_3 = 0$

$$\begin{cases} 3/5\,\Delta b_1 \geq -1/5 \\ -1/5\,\Delta b_1 \geq -8/5 \\ -\Delta b_1 \geq -4 \end{cases} \rightsquigarrow \begin{cases} \Delta b_1 \geq -1/3 \\ \Delta b_1 \leq 8 \\ \Delta b_1 \leq 4 \end{cases}$$



$$\boxed{-1/3 \leq \Delta b_1 \leq 4}$$

$\Delta b_1 = \Delta b_3 = 0 \rightsquigarrow \boxed{-4 \leq \Delta b_2 \leq 1}$

$\Delta b_1 = \Delta b_2 = 0 \rightsquigarrow \boxed{-4 \leq \Delta b_3}$

② $\boxed{c_F \to c_F + \Delta c_F}$

$$\boxed{\Delta c_F \geq -\bar{c}_F}$$

$$\begin{bmatrix} \Delta c_2 \\ \Delta c_4 \\ \Delta c_5 \end{bmatrix} \geq - \begin{bmatrix} \bar{c}_2 = 7/5 \\ \bar{c}_4 = 6/5 \\ \bar{c}_5 = 3/5 \end{bmatrix}$$

$$\begin{cases} \Delta c_2 \geq -7/5 \\ \Delta c_4 \geq -6/5 \\ \Delta c_5 \geq -3/5 \end{cases}$$

③ $c_B \to c_B + \Delta c_B$

$$\boxed{\Delta c_B^T \underbrace{B^{-1} F}_{\bar{F}} \leq \bar{c}_F^T}$$

$$\boxed{\bar{F} := B^{-1} F = \begin{bmatrix} 1/5 & 3/5 & -1/5 \\ 3/5 & -1/5 & 2/5 \\ 1 & -1 & 0 \end{bmatrix}}$$

$$[\Delta c_1, \Delta c_3, \Delta c_6] \cdot \begin{bmatrix} 1/5 & 3/5 & -1/5 \\ 3/5 & -1/5 & 2/5 \\ 1 & -1 & 0 \end{bmatrix} \leq [7/5, 6/5, 3/5]$$

$$\begin{cases} 1/5\, \Delta c_1 + 3/5\, \Delta c_3 + \Delta c_6 \leq 7/5 \\ 3/5\, \Delta c_1 - 1/5\, \Delta c_3 - \Delta c_6 \leq 6/5 \\ -1/5\, \Delta c_1 + 2/5\, \Delta c_3 \leq 3/5 \end{cases}$$

$$\Delta c_3 = \Delta c_6 = 0 \rightsquigarrow -3 \leq \Delta c_1 \leq 2$$

$$\Delta c_1 = \Delta c_6 = 0 \rightsquigarrow -6 \leq \Delta c_3 \leq 3/2$$

$$\Delta c_1 = \Delta c_3 = 0 \rightsquigarrow -\frac{6}{5} \leq \Delta c_6 \leq 7/5$$