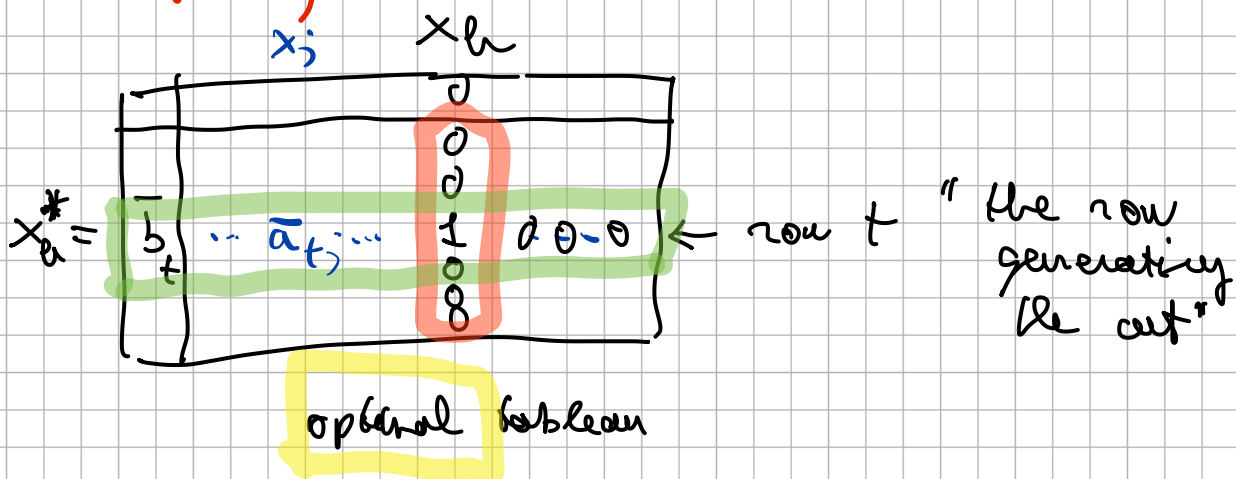


GOMORY'S CUTS ~ 1958

Separation for Chvátal cuts:

x^* \rightarrow **GOMORY** $\rightarrow u \in \mathbb{R}^m$:
 $[u^T A] x \leq [u^T b]$
 is violated by x^* (if any)
 a basis associated with x^*
 (i.e., if x^* is a vertex of P)



x_{h}^* which is fractional $\Rightarrow x_{h}$ is a basic variable

row t s.t. $\bar{b}_t = x_{h}^*$ is fractional!

$$1 \cdot x_h + \sum_{j \in J} \bar{a}_{tj} x_j = \bar{b}_t = x_h^*$$

only basic variable in row t

J index set of the nonbasic var.s

Chvátal cut:

$$1 \cdot x_h + \sum_{j \in J} \lfloor \bar{a}_{tj} \rfloor x_j \leq \lfloor \bar{b}_t \rfloor \quad (*)$$

(note: row t of the optimal tableau
 $u^t a_t = u^t b_t$
 where $u = t$ -th row
 of B^{-1})

(*) is valid for the integer sol.^s
 (by construction) and
 violated by x^* :

$$\left(x_r + \sum_{j \in \mathcal{F}} [\bar{a}_{tj}] x_j \right) \leq [\bar{b}_t]$$

$x = x^*$
 $x_r^* \neq$
 $x_j^* = 0 \forall j \in \mathcal{F}$
 $[x_r^*]$

(*) is called
 (CHUATAL-)GOMORY cut

$$(*) (*) \quad x_h + \sum_{j \in Q} [\bar{a}_{tj}] x_j + \sigma = \lceil \bar{b}_t \rceil$$

where $\sigma \geq 0$ is a slack variable

σ integer variable

$$(*) (*) (*) \quad x_h + \sum_{j \in Q} \bar{a}_{tj} x_j = \bar{b}_t$$

$(*) (*) - (*) (*) (*)$ produces:

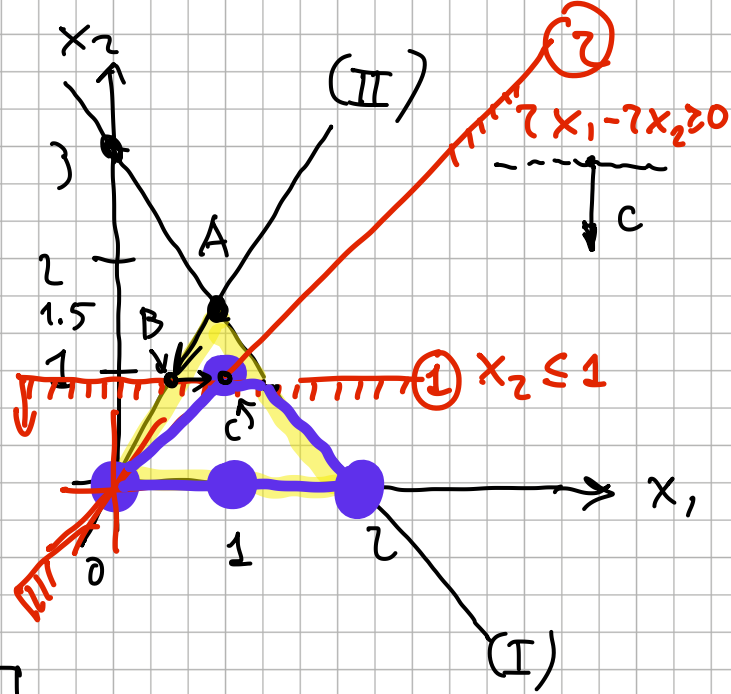
$$+ \sum_{j \in Q} - \underbrace{(\bar{a}_{tj} - \lceil \bar{a}_{tj} \rceil)}_{\varphi(\bar{a}_{tj})} x_j + \sigma = - \underbrace{(\bar{b}_t - \lceil \bar{b}_t \rceil)}_{\varphi(\bar{b}_t)}$$

where $\varphi(z) = z - \lceil z \rceil \geq 0$

FRACTIONAL FORM OF
THE GOMORY CUT

EX :

$$\begin{cases} \min & -x_2 \\ & 3x_1 + 2x_2 \leq 6 \quad (\text{I}) \\ & -3x_1 + 2x_2 \leq 0 \quad (\text{II}) \\ & x_1, x_2 \geq 0 \\ & \text{integer} \end{cases}$$



$$\begin{array}{l} -z = \\ x_3 = \\ x_4 = \end{array} \begin{array}{c} \downarrow \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \hline 0 & 0 & -1 & 0 \\ 6 & 3 & 2 & 1 \\ 0 & -3 & 2 & 0 \end{array} \end{array}$$

$$\begin{aligned} \leftarrow x_3 &= 6 - 3x_1 - 2x_2 \\ \leftarrow x_4 &= 0 + 3x_1 - 2x_2 \end{aligned}$$

$$\begin{array}{l} -z = \\ x_3 = \\ x_2 = \end{array} \begin{array}{c} \downarrow \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \hline 0 & -3/2 & 0 & 0 \\ 6 & 6 & 0 & -1 \\ 0 & -3/2 & 1 & 0 \end{array} \end{array}$$

$x^* = (1, 3/2)$ vertex A
 x_5 is integer var.

$$\begin{array}{l} -z = \\ x_1 = \\ x_2 = \\ x_5 = \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \hline 3/2 & 0 & 0 & 1/4 \\ 1 & 1 & 0 & 1/6 \\ 3/2 & 0 & 1 & 1/4 \\ -1/2 & 0 & 0 & -1/4 \end{array} \end{array}$$

\leftarrow row t

$$\leftarrow x_5 = -1/2 + \frac{1}{4}x_3 + \frac{1}{4}x_4$$

$$\begin{aligned} &= -\frac{1}{2} + \frac{1}{4} \underbrace{(6 - 3x_1 - 2x_2)}_{x_3} + \frac{1}{4} \underbrace{(3x_1 - 2x_2)}_{x_4} \\ &= \dots = 1 - x_2 \end{aligned}$$

Gomory cut: $x_5 \geq 0 \rightarrow 1 - x_2 \geq 0 \rightarrow x_2 \leq 1$

DUAL simplex:

$x^* = (2/3, 1)$

	x_1	x_2	x_3	x_4	x_5	vertex B
$-z =$	1	0	0	0	1	0
$x_1 =$	2/3	1	0	-1/3	2/3	0
$x_2 =$	1	0	1	0	1	0
$x_3 =$	2	0	0	1	-4	0
$x_6 =$	-2/3	0	0	-2/3	-2/3	1

Annotations: A yellow box highlights the first three columns. A green box highlights the first four rows. A blue circle highlights the $-2/3$ in the x_4 column of the x_6 row. A red arrow labeled 'row' points to the x_1 row, and another red arrow labeled 't' points to the $-1/3$ in the x_4 column of the x_1 row.

$\psi(-1/3) = 2/3$

$\psi(z) := z - [z]$

$\psi(1.9) = 1.9 - 1 = 0.9$

$\psi(-1.1) = -1.1 - (-2) = 0.9$

" $\psi(-|z|) = 1 - \psi(|z|)$ "

$x_6 = -2/3 + \frac{2}{3} \underbrace{(\dots)}_{x_4} + \frac{2}{3} \underbrace{(\dots)}_{x_5}$

$= \dots = 2x_1 - 2x_2$

cut $x_6 \geq 0 \Rightarrow 7x_1 - 2x_2 \geq 0$
 $x_1 - x_2 \geq 0$

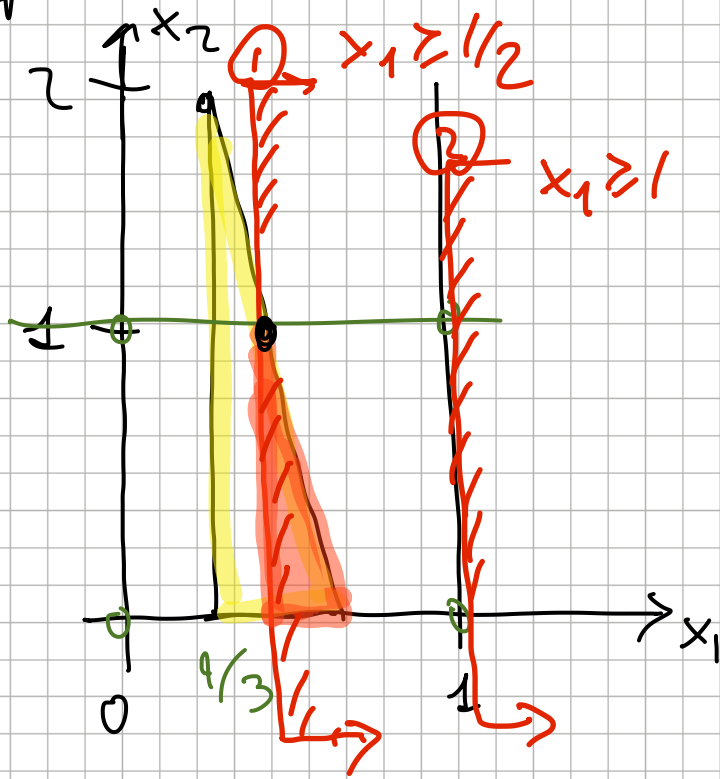
	x_1	x_2	x_3	x_4	x_5	x_6
$-z =$	1	0	0	0	1	0
$x_1 =$	1	0	0	0	1	$-1/2$
$x_2 =$	1	0	1	0	1	0
$x_3 =$	1	0	0	1	-5	$3/2$
$x_4 =$	1	0	0	1	1	$-3/2$

$x^* = (1, 1)$ vertex G'

x^* is integer (no Gomory cut can be produced)
 \Rightarrow STOP

EX

$$\begin{cases} \min x_1 + x_2 \\ 6x_1 + x_2 \leq 4 \\ 3x_1 \geq 1/3 \\ x_1, x_2 \geq 0 \\ \text{integer} \end{cases}$$



Gomory cuts ...

\Rightarrow after 2 cuts you realize infeasibility!

last tableau : INFEASIBLE

	x_1	x_2	x_3	x_4	x_5	x_6	
	-2	0	1	1	0	0	6

$\leftarrow (*)$

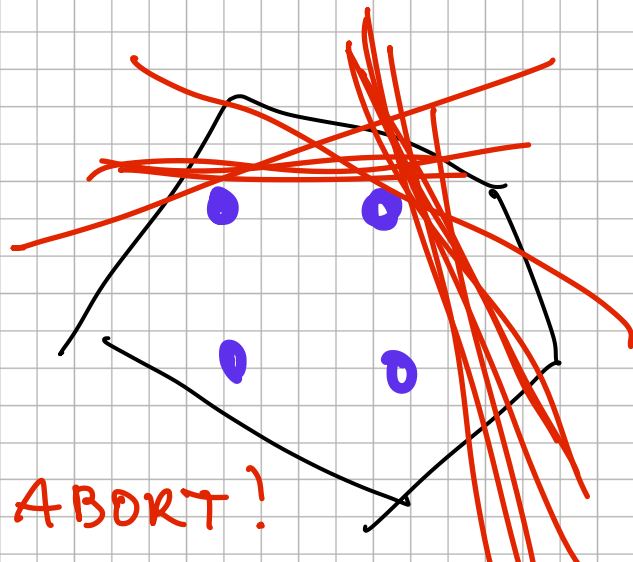
eq. (*) reads:

$$x_2 + x_3 + 6x_6 = -2$$

impossible as $x \geq 0$

\Rightarrow NO FEASIBLE x exists

IN PRACTICE:



\Rightarrow NUMERICAL ISSUES \Rightarrow ABORT!