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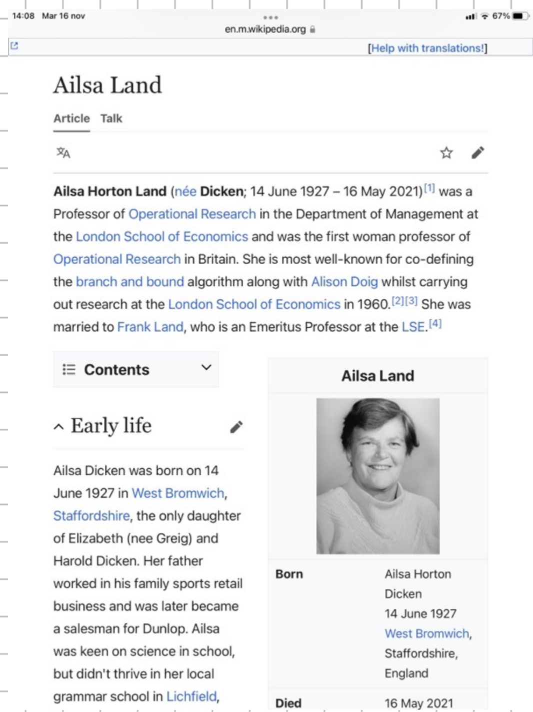
Cutting plane method

Ralph Gomory (1958)

BRANCH-AND-BOUND method

Land & Doig (1960)



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en.m.wikipedia.org

[Help with translations!]

Ailsa Land

Article Talk

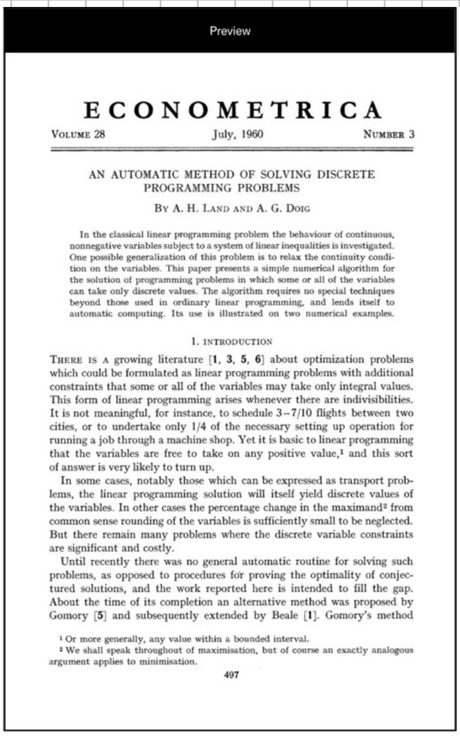
Ailsa Horton Land (née **Dicken**; 14 June 1927 – 16 May 2021)[1] was a Professor of Operational Research in the Department of Management at the London School of Economics and was the first woman professor of Operational Research in Britain. She is most well-known for co-defining the branch and bound algorithm along with Alison Doig whilst carrying out research at the London School of Economics in 1960.[2][3] She was married to Frank Land, who is an Emeritus Professor at the LSE.[4]

Contents

Early life

Ailsa Dicken was born on 14 June 1927 in West Bromwich, Staffordshire, the only daughter of Elizabeth (nee Greig) and Harold Dicken. Her father worked in his family sports retail business and was later became a salesman for Dunlop. Ailsa was keen on science in school, but didn't thrive in her local grammar school in Lichfield,

Ailsa Land	
Born	Ailsa Horton Dicken 14 June 1927 West Bromwich, Staffordshire, England
Died	16 May 2021



Preview

ECONOMETRICA

VOLUME 28 July, 1960 NUMBER 3

AN AUTOMATIC METHOD OF SOLVING DISCRETE PROGRAMMING PROBLEMS

BY A. H. LAND AND A. G. DOIG

In the classical linear programming problem the behaviour of continuous, nonnegative variables subject to a system of linear inequalities is investigated. One possible generalization of this problem is to relax the continuity condition on the variables. This paper presents a simple numerical algorithm for the solution of programming problems in which some or all of the variables can take only discrete values. The algorithm requires no special techniques beyond those used in ordinary linear programming, and lends itself to automatic computing. Its use is illustrated on two numerical examples.

1. INTRODUCTION

THERE IS A growing literature [**1**, **3**, **5**, **6**] about optimization problems which could be formulated as linear programming problems with additional constraints that some or all of the variables may take only integral values. This form of linear programming arises whenever there are indivisibilities. It is not meaningful, for instance, to schedule 3–7/10 flights between two cities, or to undertake only 1/4 of the necessary setting up operation for running a job through a machine shop. Yet it is basic to linear programming that the variables are free to take on any positive value,[1] and this sort of answer is very likely to turn up.

In some cases, notably those which can be expressed as transport problems, the linear programming solution will itself yield discrete values of the variables. In other cases the percentage change in the maximand[2] from common sense rounding of the variables is sufficiently small to be neglected. But there remain many problems where the discrete variable constraints are significant and costly.

Until recently there was no general automatic routine for solving such problems, as opposed to procedures for proving the optimality of conjectured solutions, and the work reported here is intended to fill the gap. About the time of its completion an alternative method was proposed by Gomory [**5**] and subsequently extended by Beale [**1**]. Gomory's method

[1] Or more generally, any value within a bounded interval.

[2] We shall speak throughout of maximisation, but of course an exactly analogous argument applies to minimisation.

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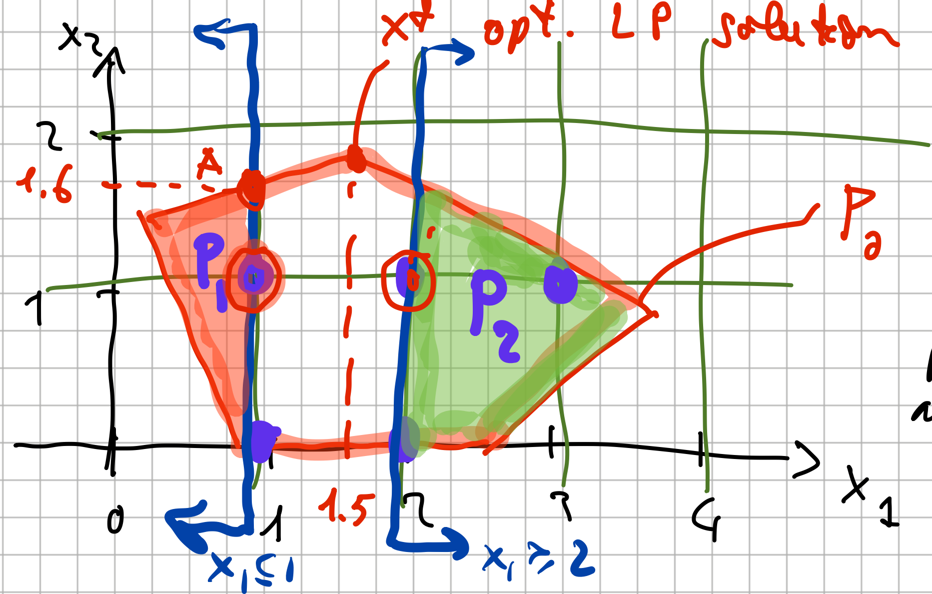
ILP :



$x_1^* = 1.5$

DISJUNCTION

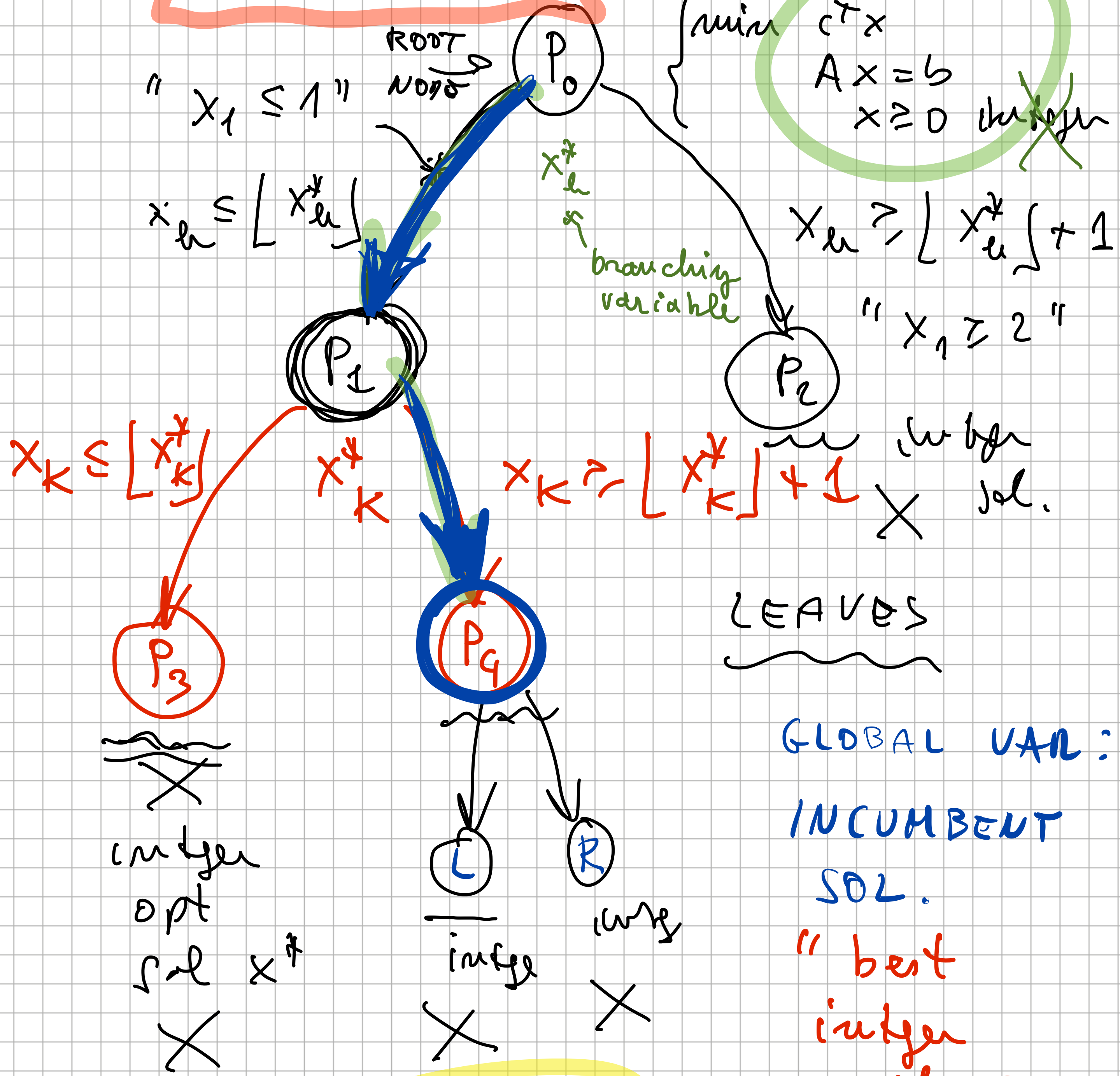
NOT A LINEAR INEQ.

$(x_1 \le 1) \lor (x_1 \ge 2)$

OR

BRANCHING TREE (ENUMERATION TREE)

$$\begin{cases} \min c^T x \\ Ax = b \\ x \geq 0 \ \cancel{\text{integer}} \end{cases}$$



GLOBAL VAR:

INCUMBENT SOL.

" best integer sol. so far "

x_{OPT} /

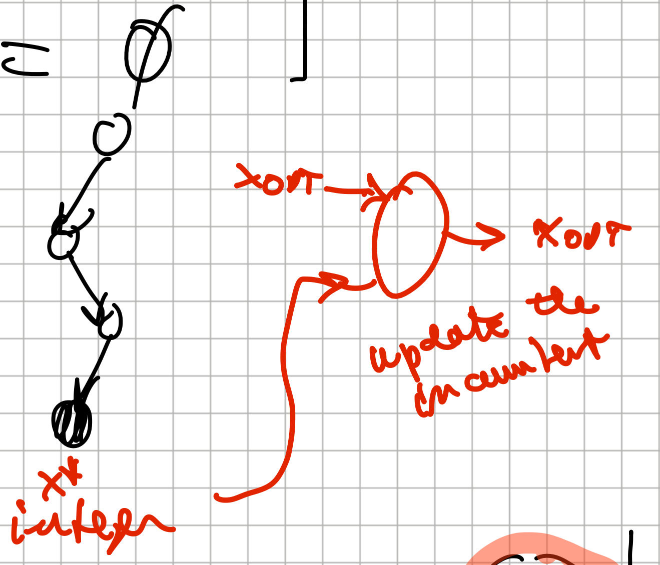
$c^t x_{OPT} = z_{OPT}$

value of the incumbent.

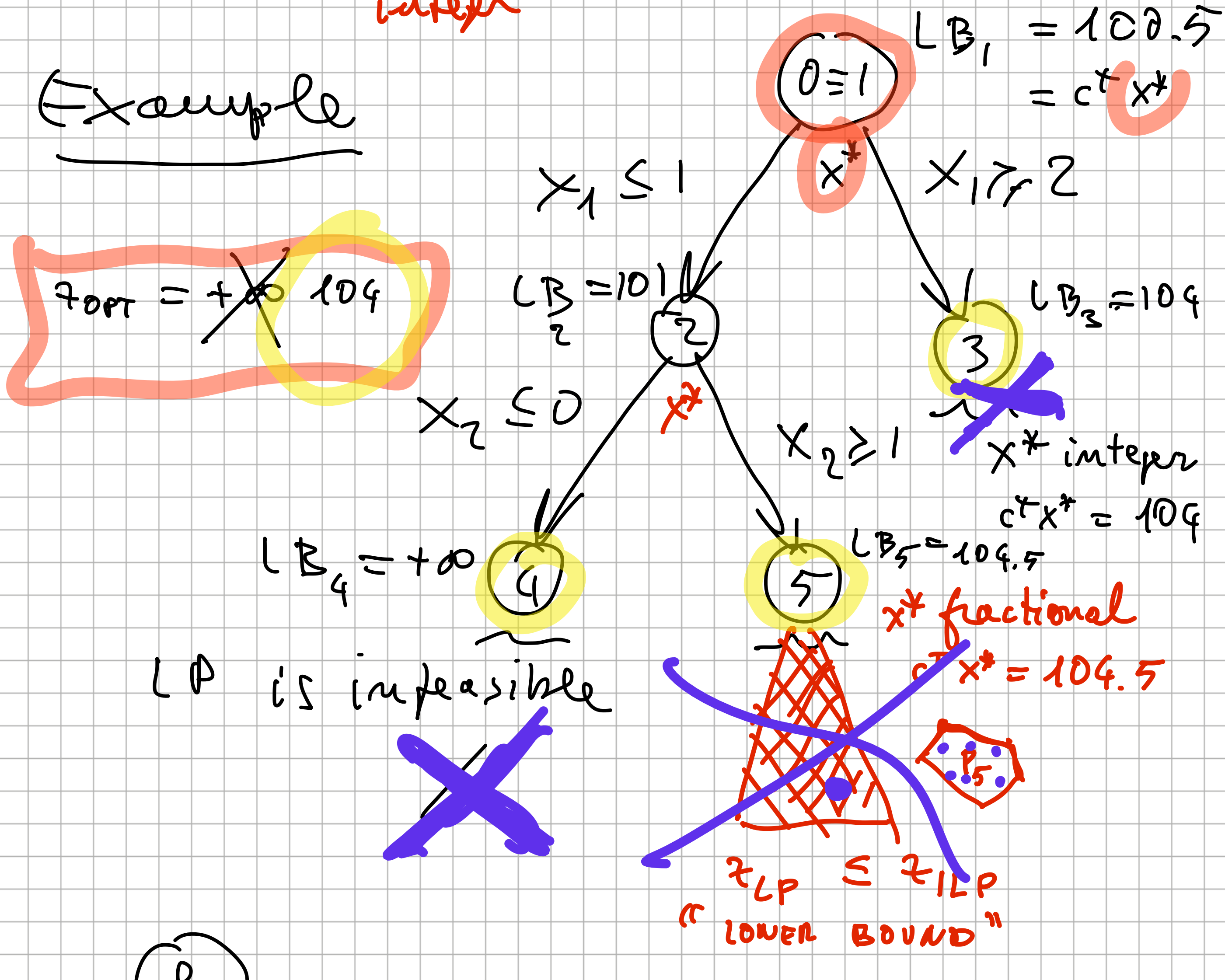
$z_{OPT} = +\infty$

$x_{OPT} = \emptyset$

} initialization



Example



ℓ

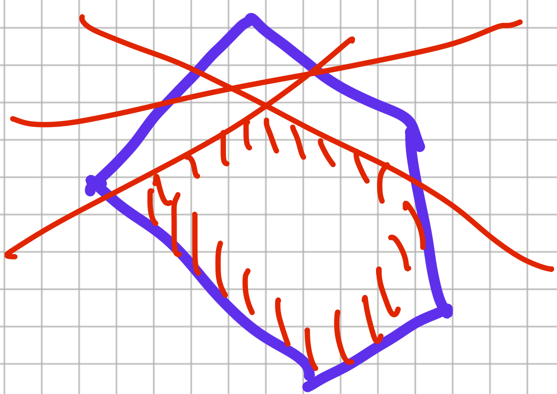
$$LB_\ell = c^T(x^*_{\text{node } \ell})$$

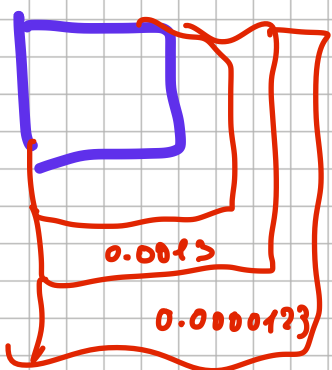
PRUNING / FATHOMING cond.

$$\text{if } \left(LB_h = c^T x^*_{NODE-h} > z_{OPT} \right)$$

then "PRUNE / KILL node h"

CUTTING PLANE

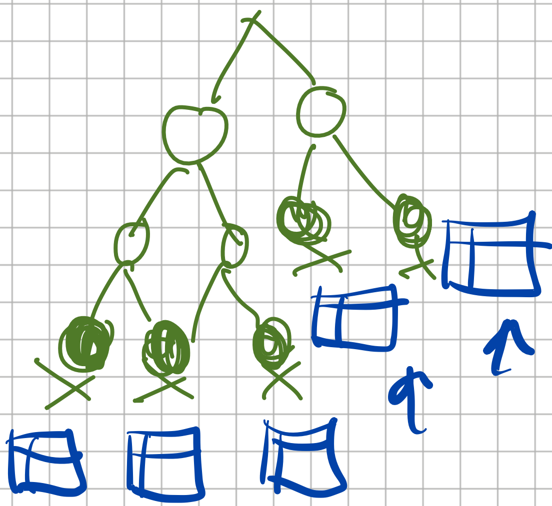




A SINGLE LP (larger and larger)

GOMORY proved finite converg.

B & BOUND

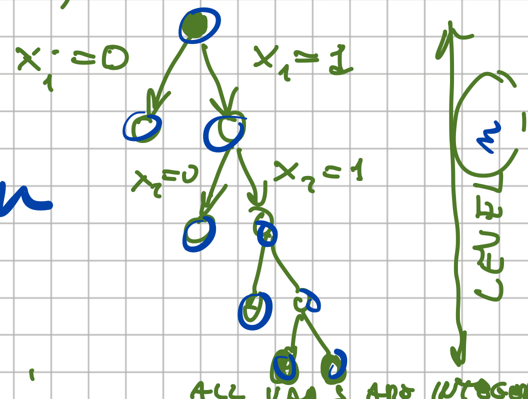


original tableau + new bounds on the variables

"$R_t = 2$"

ASSUME $x_i \in \{0,1\}$

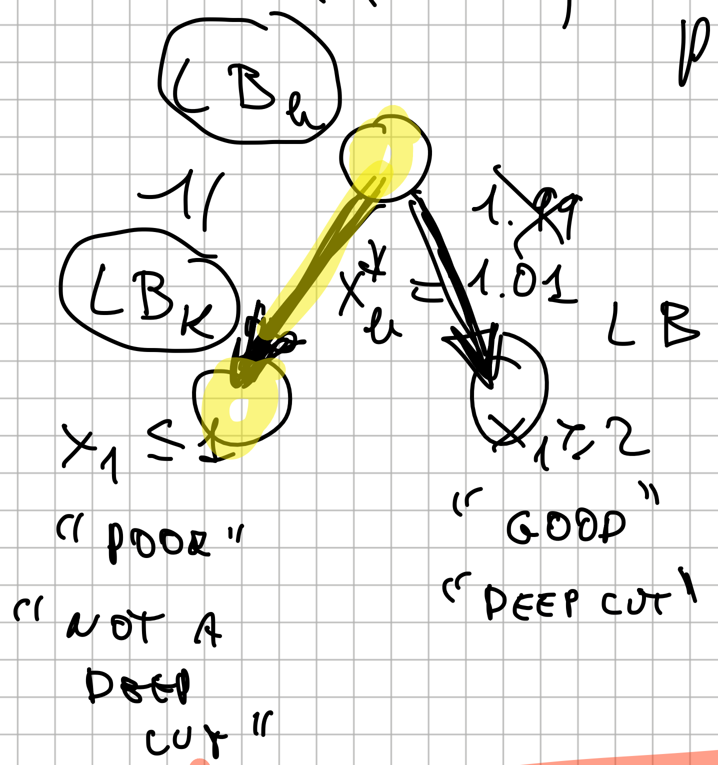
#nodes $\leq 2^n$

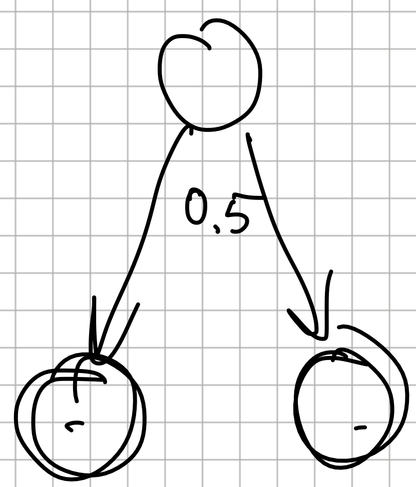


(under certain assumpt.s)

HOW TO CHOOSE THE BRANCHING VAR.?

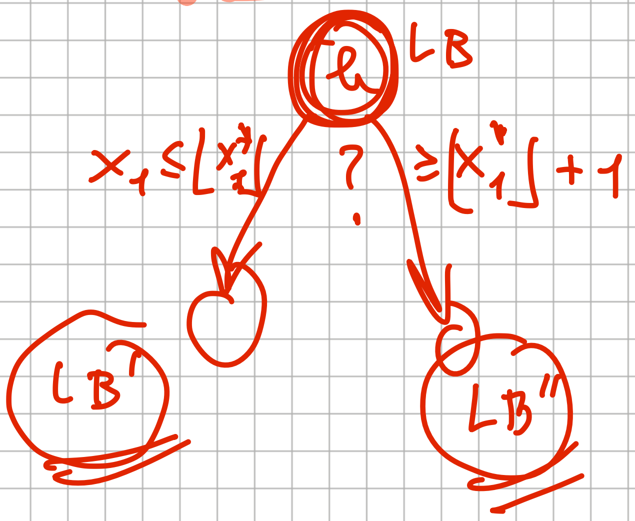
" $\varphi(x^*_h)$ is as close as possible to 0.5 "

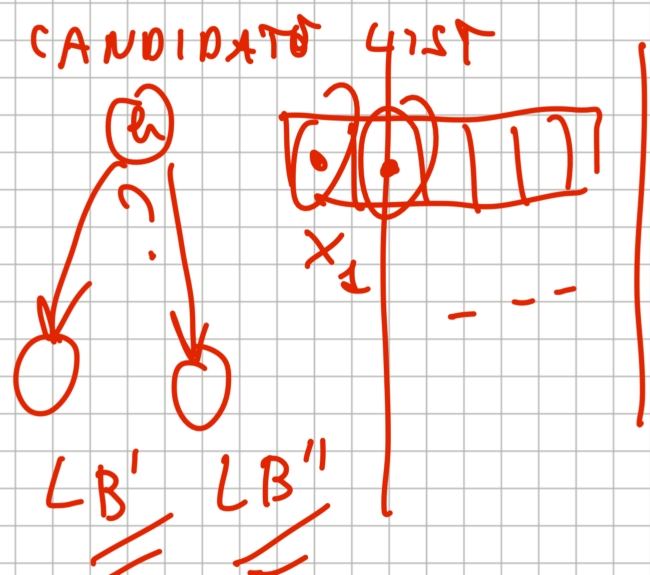




STRONG BRANCHING

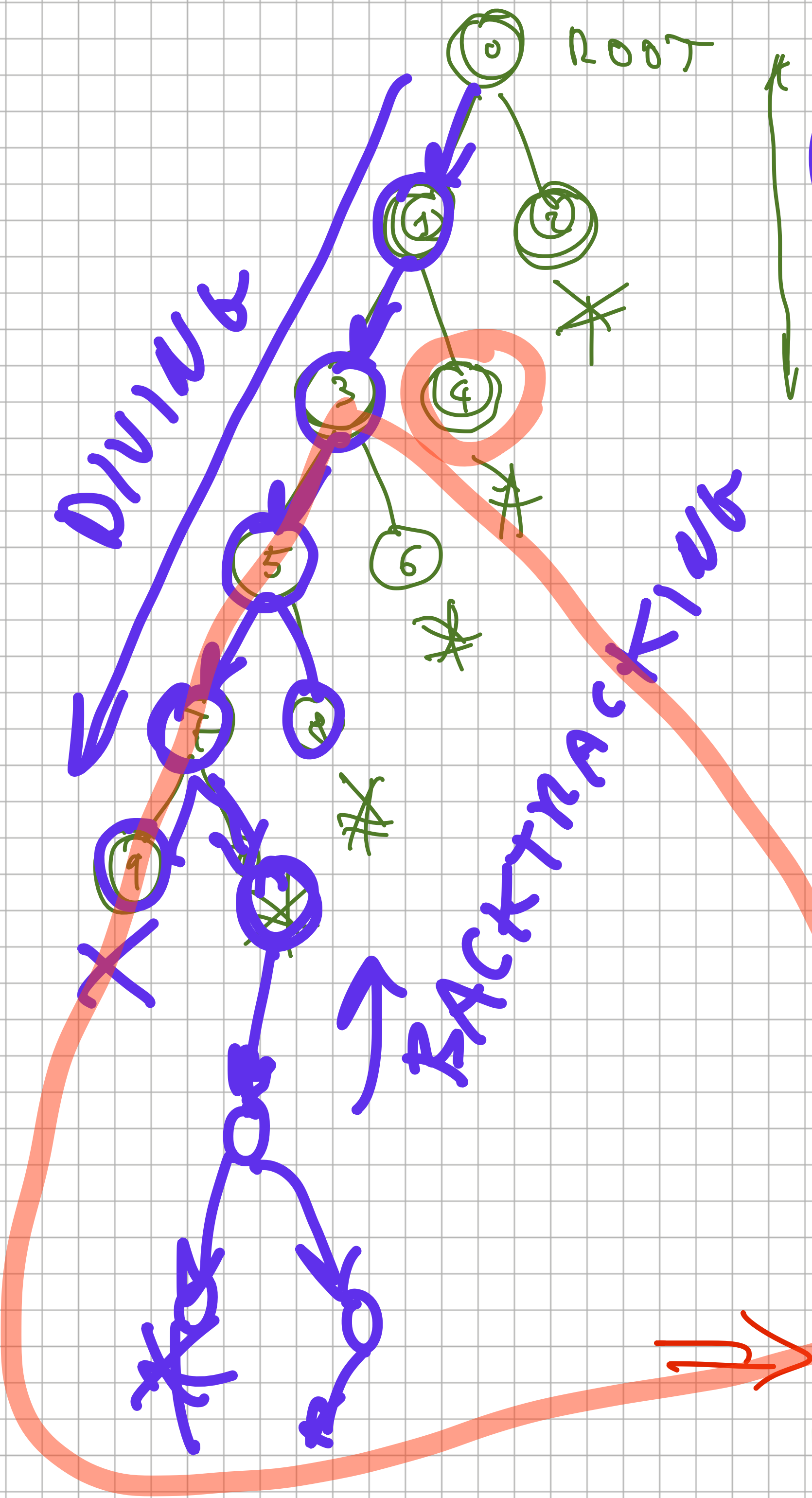
IBM CPLEX solver





HOW TO VISIT THE TREE?

DEPTH FIRST SEARCH

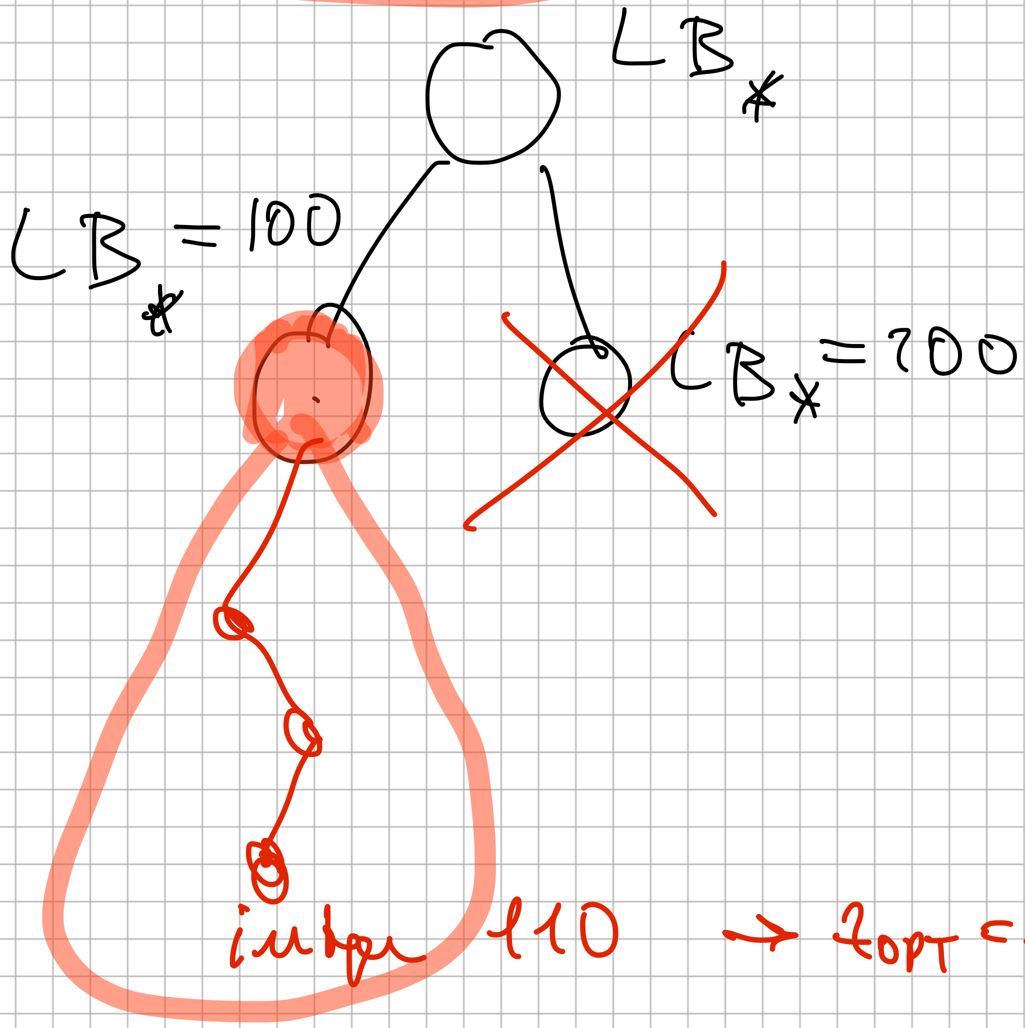


INCREASE THE PROBABILITY OF FINDING INTEGER SOL.S

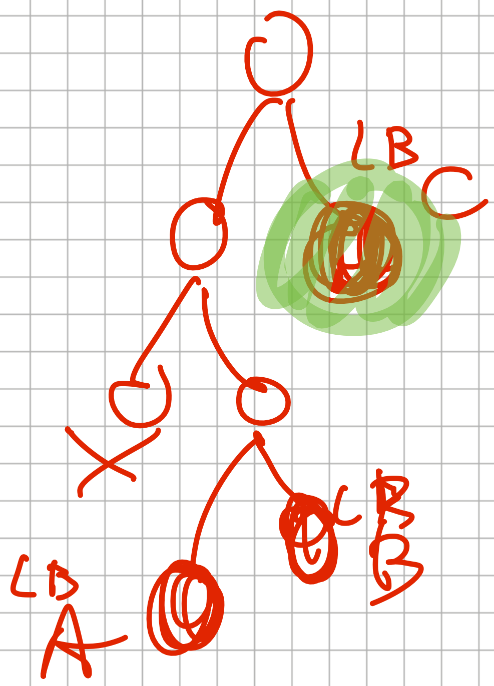
⇒ update z_{opt}

⇒ LB pruning test effective

BEST-BOUND FIRST



→ $z_{opt} = 110$



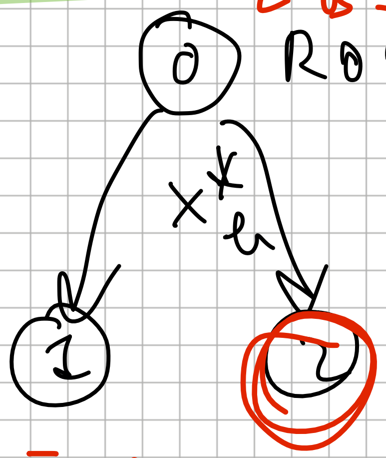
→ $\min \{ LB_A , LB_B , LB_C \}$

EXAMPLE

p. 112-113

$LB =$

LB   0   ROOT NODE



LB_2 ...

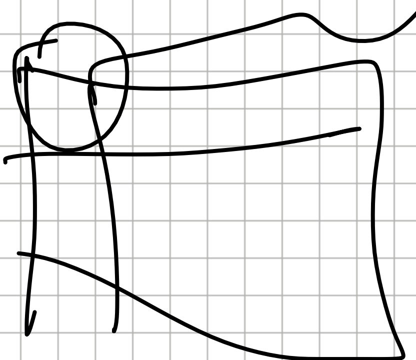
$LB_1 = \ldots$

z_{opt}

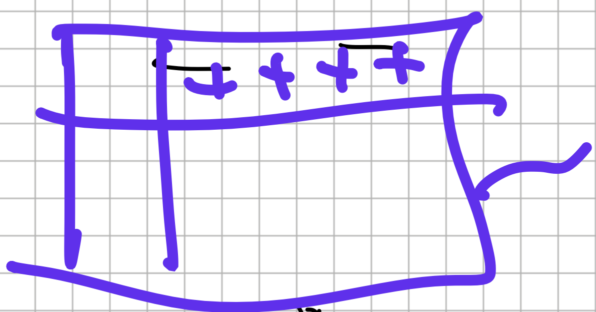
x_{opt}

ROOT NODE

LB

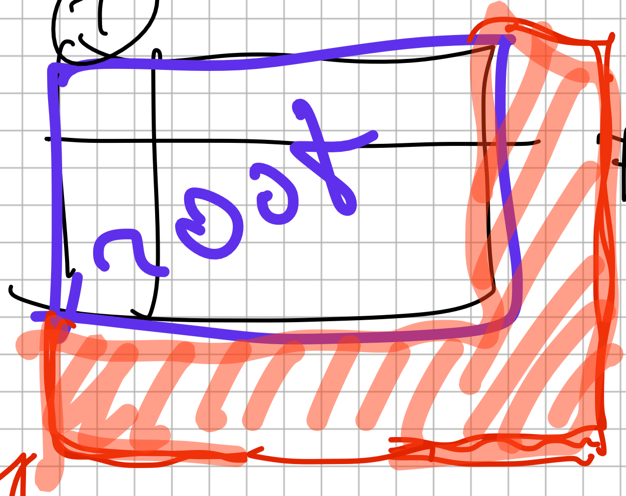


optimal tableau root node



x_k^* BRANCHING VAR.

NODE 1



FINAL TABLEAU

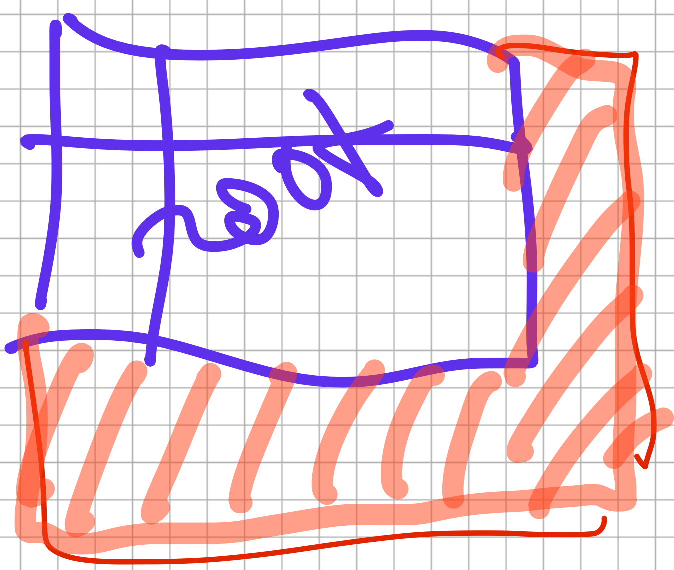
FATHOM NODE

BRANCHING AS A CUT

FINAL

$X^* =$ LB =

NODE 2



$X^* =$ LB =