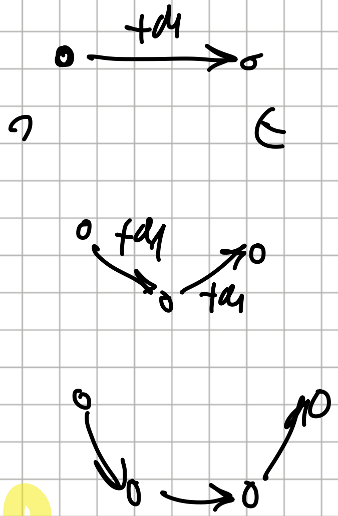
OR1 6-DEC-2021

CAN WE COMPUTE THE SHORTEST PATH ON MODIFIED COSTS?

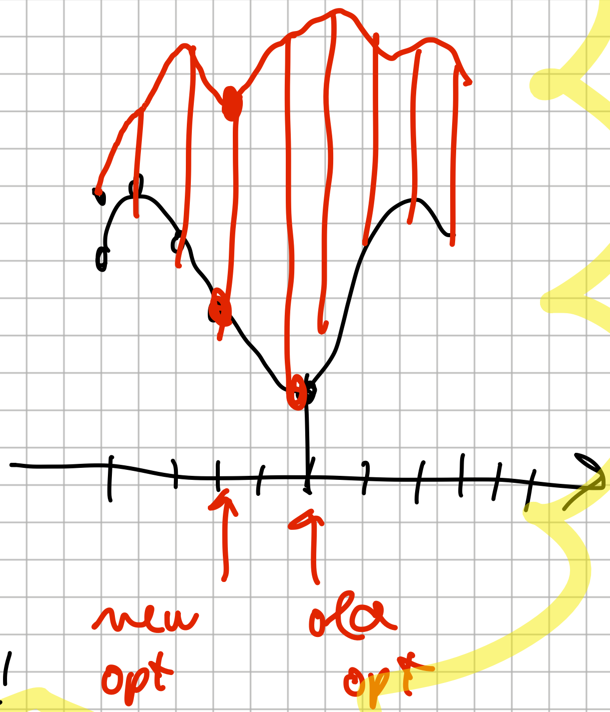
$c_{ij} \leftarrow c_{ij} + M, \forall i,j$, with $M \gg 0$



$+M$

$+2M$

$+3M$



OPTIMAL PATH DEPENDS ON M!!

PROJECT PLANNING

Project = set of ACTIVITIES

$$A_1, A_2, \dots, A_m$$

each having a given duration

$$d_1, d_2, \dots, d_m.$$

PRECEDENCE: $A_i \prec A_j$

"activity A_j can start only when A_i is finished"

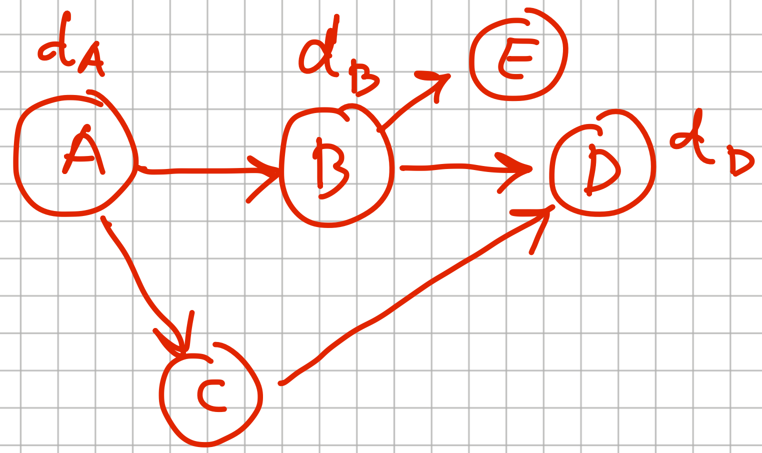
DIRECTED GRAPH $\Leftrightarrow$ project

Ex: activities A, B, C, D, E

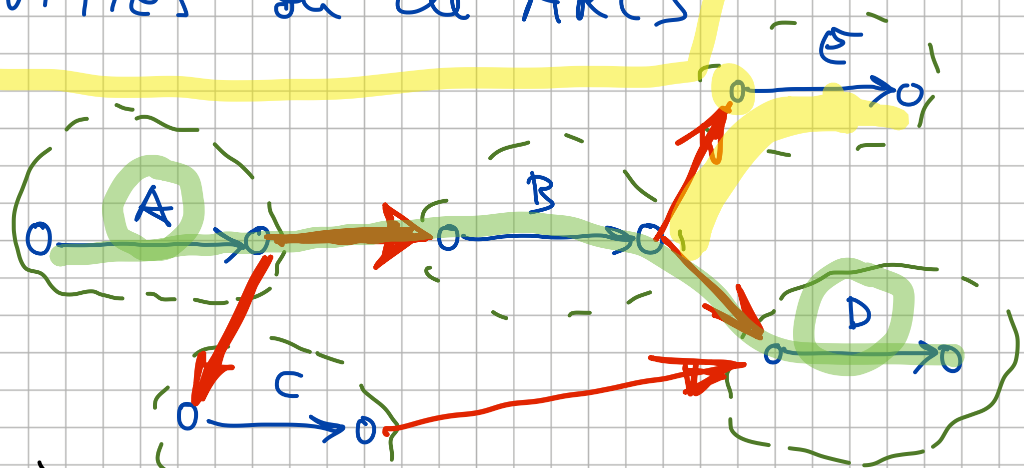
preced. $\boxed{A < B}$, $A < C$

$\boxed{B < D}$, $C < D$, $B < E$

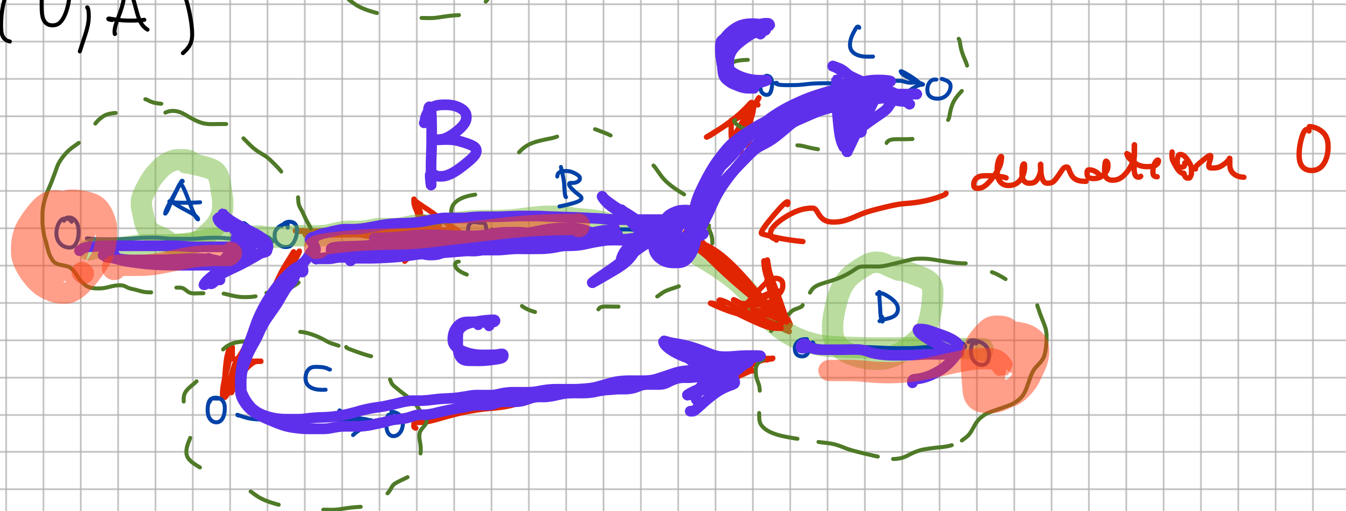
- ACTIVITIES on the NODES



- ACTIVITIES on the ARCS

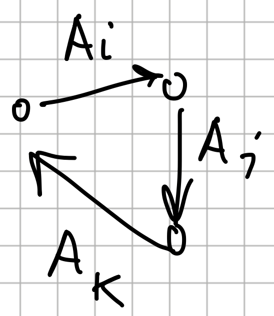


$G = (U, A)$



Properties

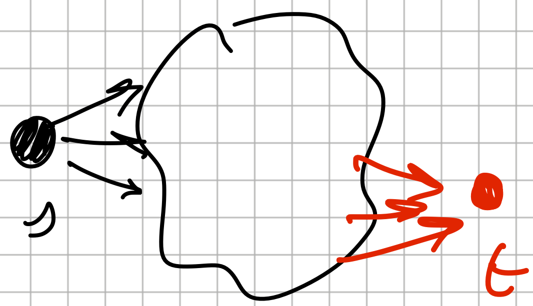
G does not contain any CIRCUIT



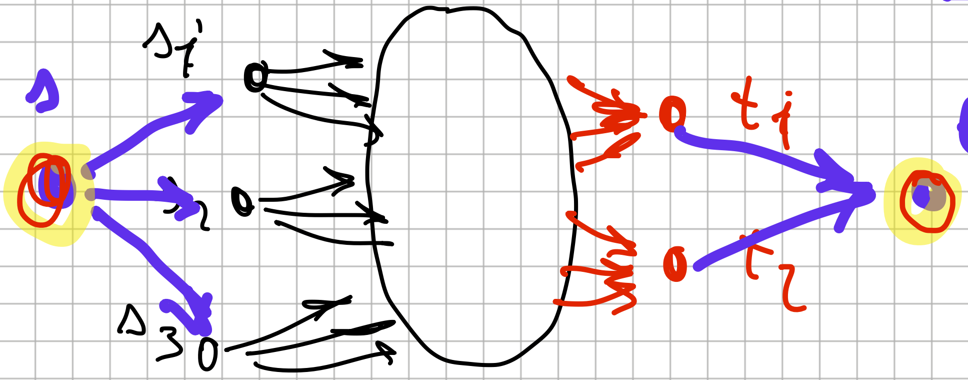
$A_i < A_j < A_K < A_i$

$\Rightarrow$ PR. INFEASIBLE

- $\exists$ a START NODE s : $\delta^-(s) = \emptyset$

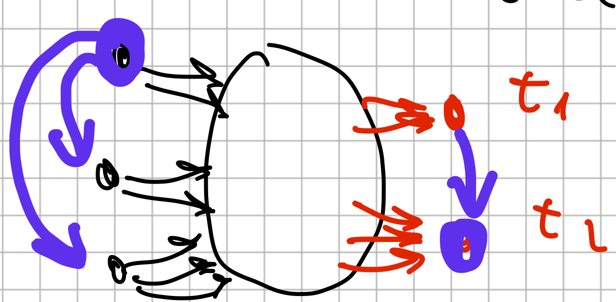


- $\exists$ a FINAL node t : $\delta^+(t) = \emptyset$



$\rightarrow$ duration 0

Hp: ONLY one START / FINAL node in G

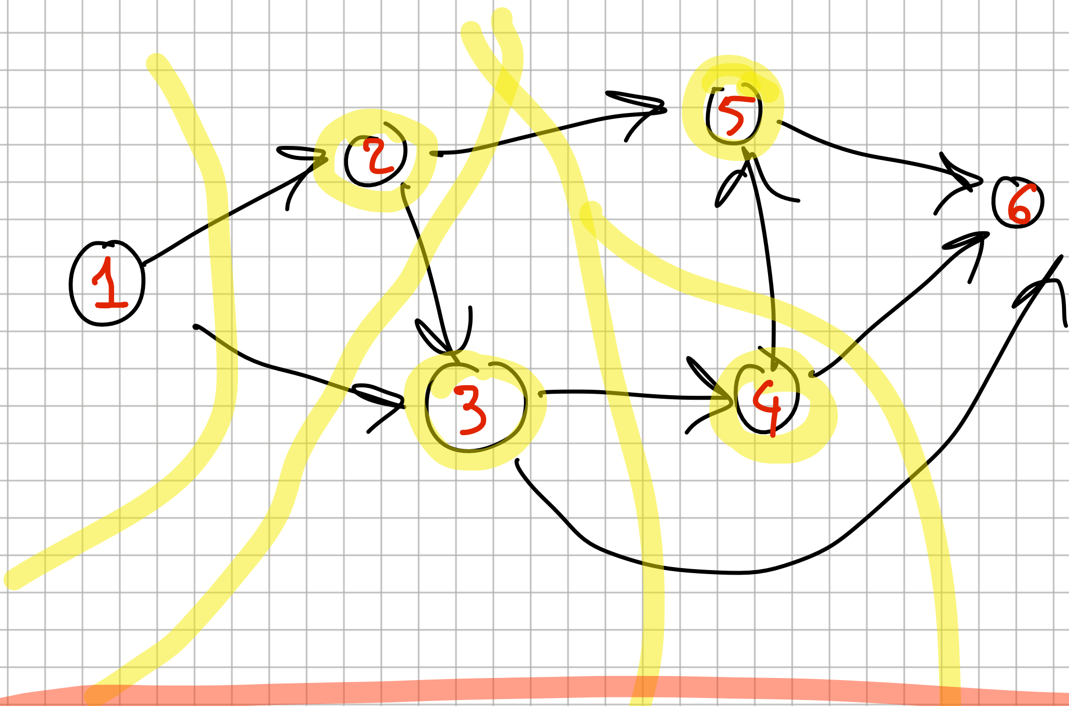


• **Hp**: Nodes are in TOPOLOGICAL ORDER

$(i) \rightarrow (j) \Rightarrow i < j$

SIMPLE (FAST) ALG.

$O(|A|)$ time



COMPUTE THE MINIMUM DURATION (MAKESPAN)

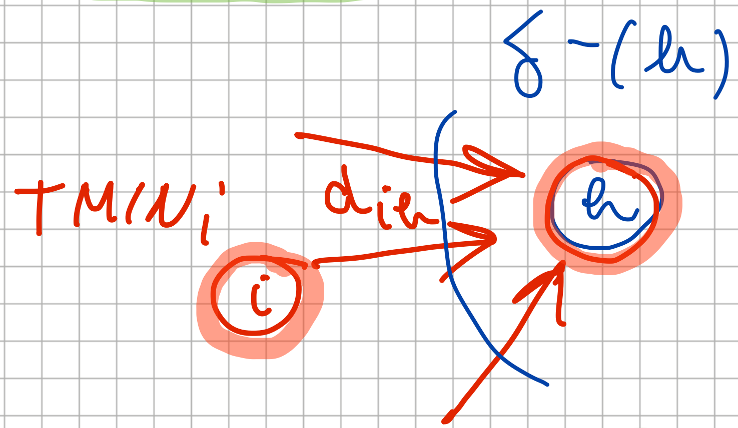
CPM alg. (CRITICAL PATH METHOD)

"PERT method"

Compute $TMIN_h$:= earliest possible time for "reaching" node h

$TMIN_1 = 0$

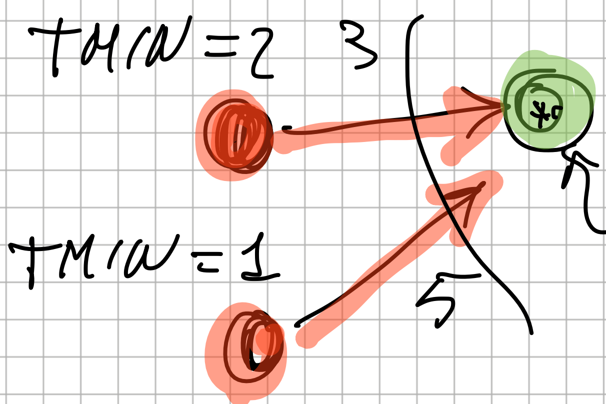
Scan nodes $h = \cancel{1}, 2, \dots, n$ in topological order



$$TMIN_h = \boxed{\max}\left\{ \boxed{TMIN_i} + d_{ih} : (i,h) \in \delta^-(h) \right\}$$

already computed

Why max?

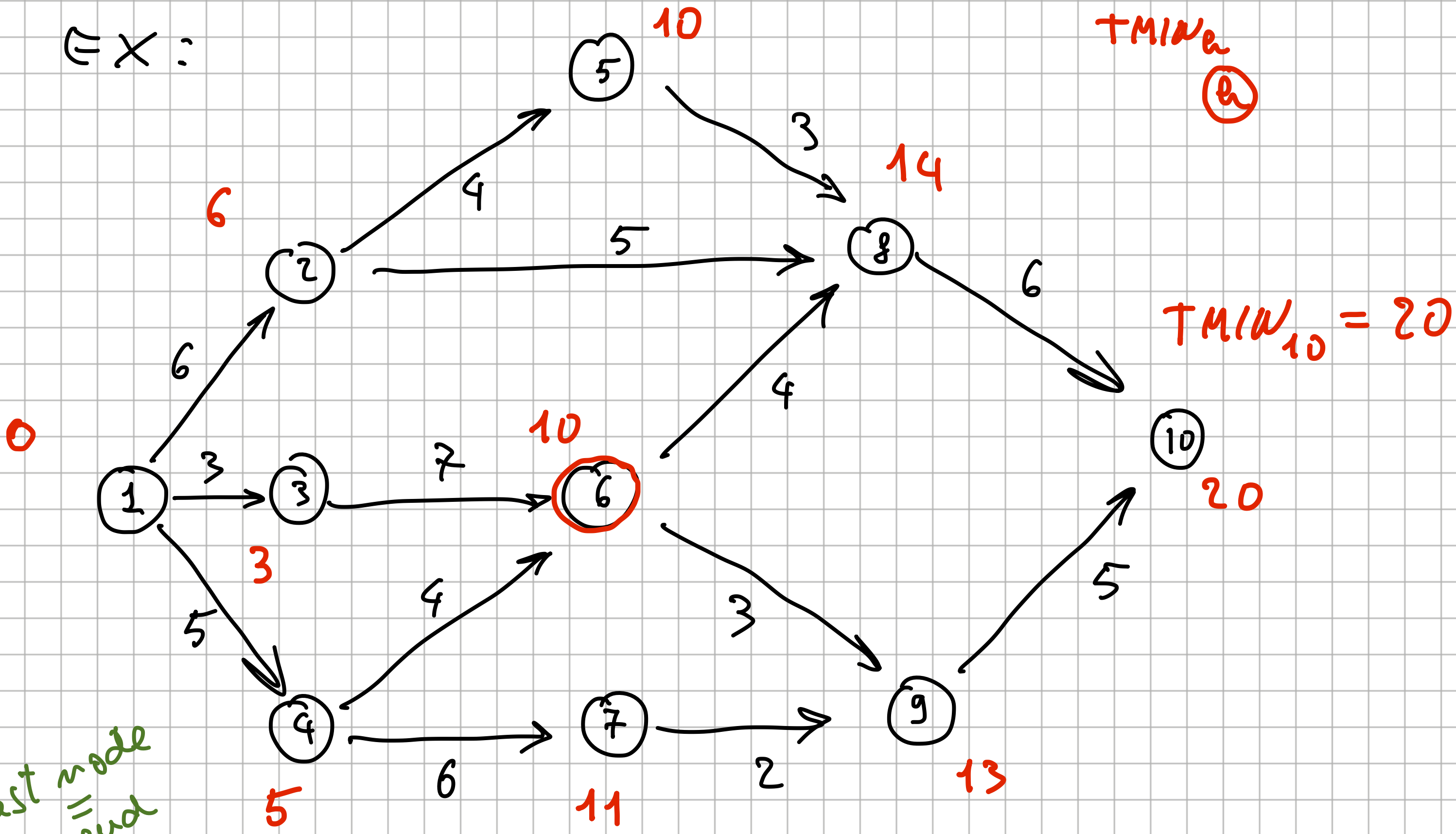


$2 + 3 = 5$

$1 + 5 = 6 \rightarrow$ max

cannot be "reached" before time 6 !

Ex:

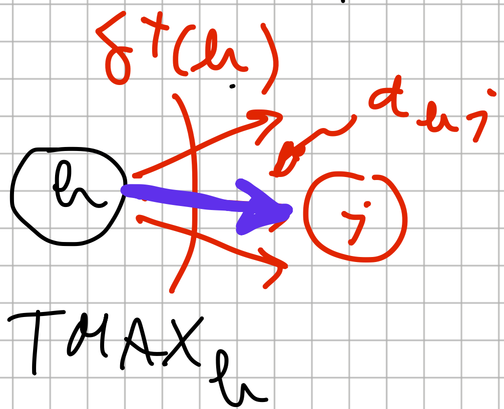


last node = project end

$TMIN_n$ = minimum duration of the project

Solution: start each activity $(i,j) \in A$ at the earliest possible time, namely $TMIN_i$

SENSITIVITY ANALYSIS:

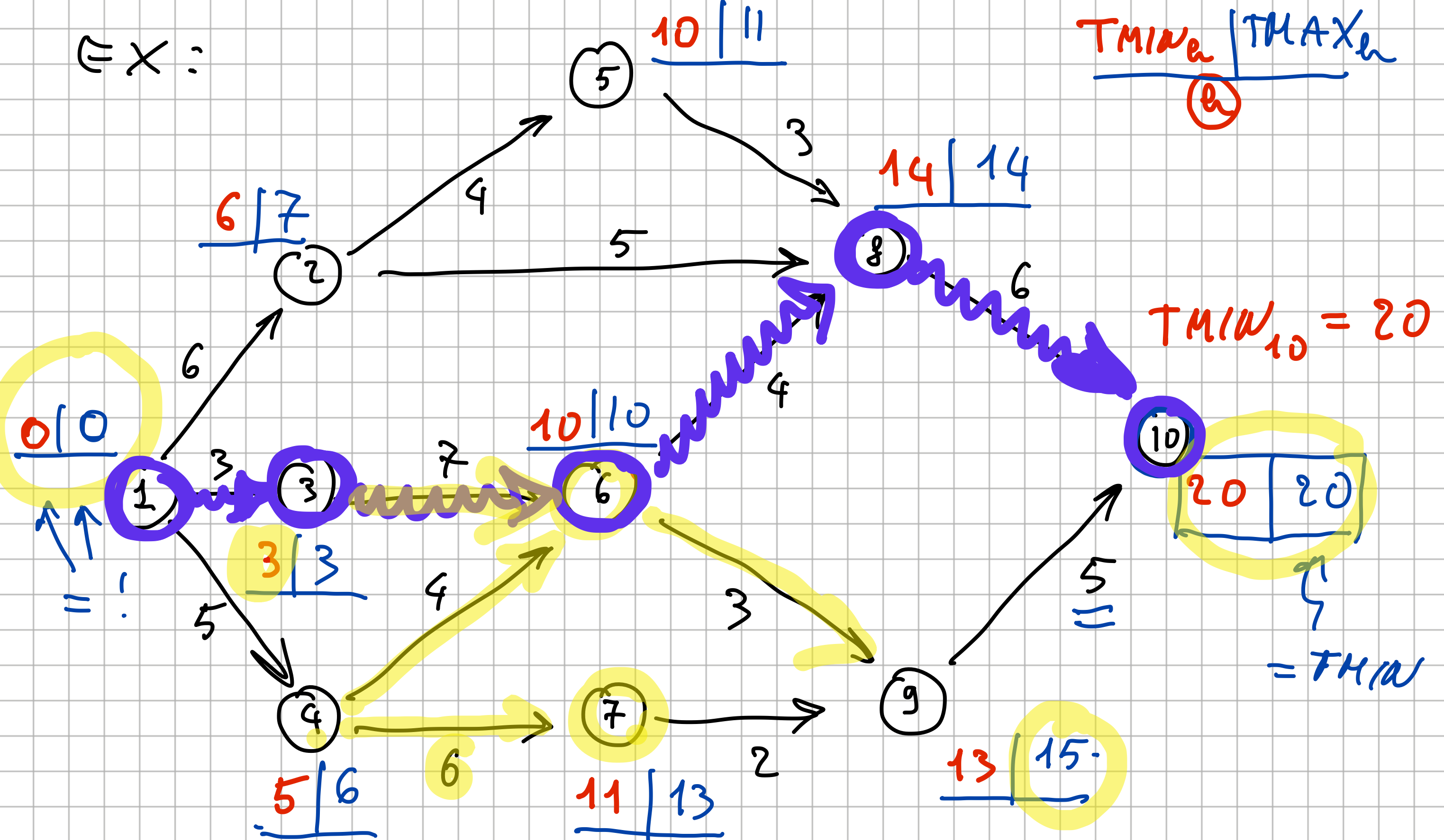


$$TMAX_h := \min\{TMAX_j - d_{hj},\ (h,j) \in \delta^+(h)\}$$

for $h = \cancel{n}, n-1, \dots, 1$

$TMAX_n := TMIN_n$

EX:



⊙ CRITICAL NODE $TMIN_h = TMAX_h$

→ " ACTIVITY (i,j)

$TMAX_j - TMIN_i = d_{ij}$

GANTT CHART