OR 1 15-DEC-2021

# THE TRAVELING SALESMAN PROBLEM

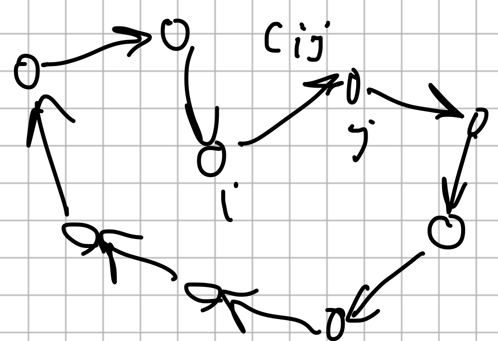

TSP

BILL COOK

$G = (V, A)$ complete directed gr.

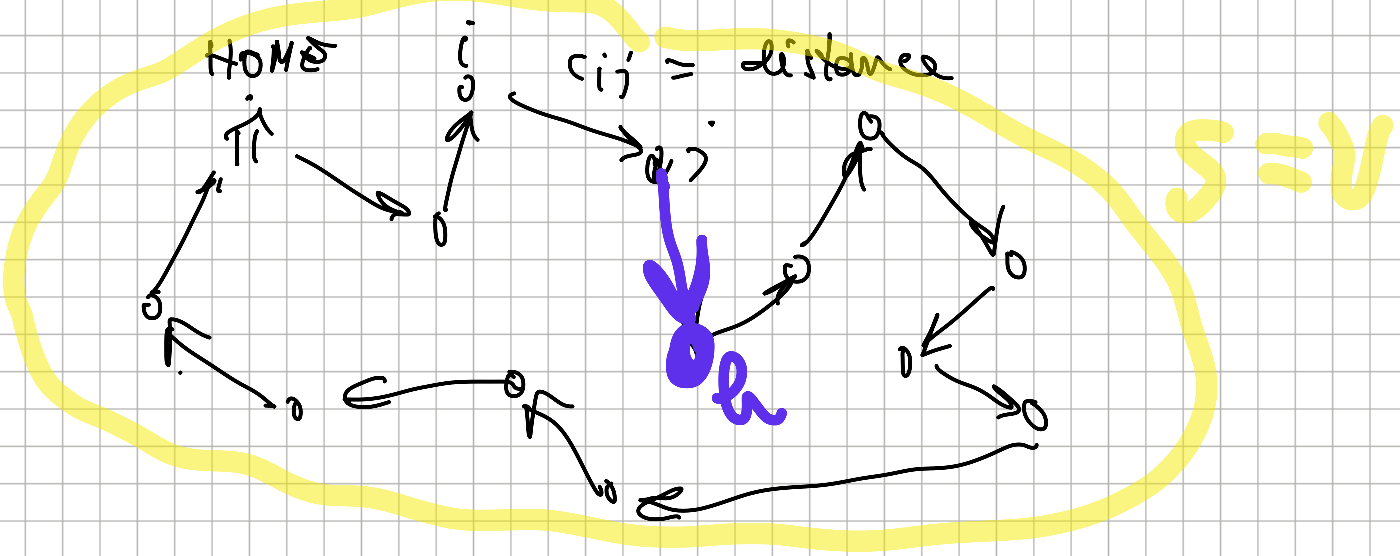
$c : A \to \mathbb{R}$

HAMILTONIAN circuit

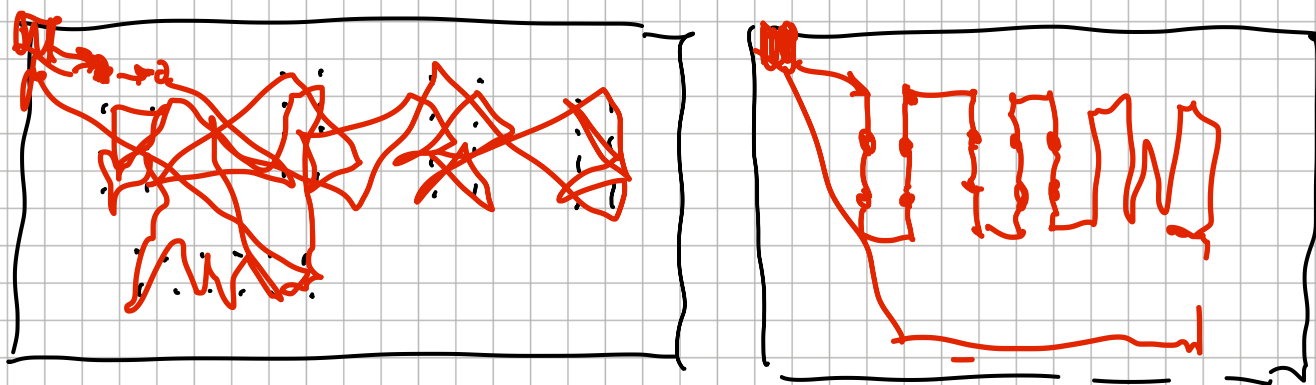



$C \subsetneq A$

$$\text{cost}(C) = \sum_{(i,j) \in C} c_{ij} \to \min$$

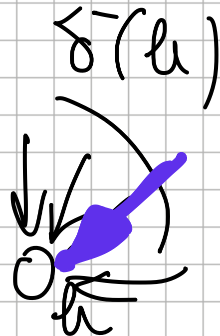



DISTRIBUTORIA PR.



ILP model : $(i,j) \in A$ :

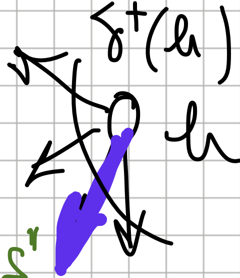
$$x_{ij} = \begin{cases} 1 & \text{if } (i,j) \text{ is in the optimal sol.} \\ 0 & \text{otherwise} \end{cases}$$



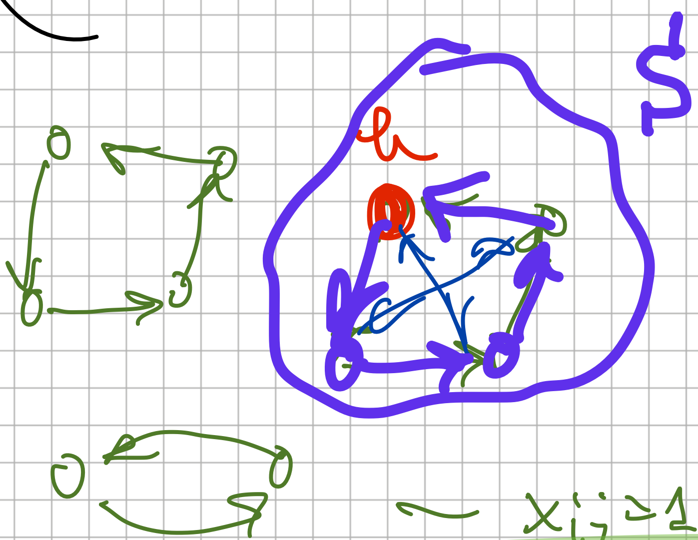

$$\begin{cases} \min \sum_{(i,j)\in A} c_{ij} \cdot x_{ij} \\ \sum_{(i,j)\in \delta^-(h)} x_{ij} = 1 \quad , \forall h \in V \end{cases}$$

"IN-DEGREE EQ.S"

$$\begin{cases} \displaystyle\sum_{(i,j)\in\delta^+(h)} x_{ij} = 1, & \forall h \in V \\ 0 \leq x_{ij} \leq \cancel{1} \ \text{INTEGER}, & \forall (i,j) \in A \end{cases}$$




"OUT-DEGREE EQ.S"



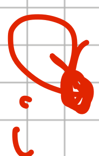

$\sim x_{ij} \geq 1$

SUBTOUR ELEM. CONSTR.S (SECs)

$$\boxed{\sum_{(i,j)\in A(S)} x_{ij} \leq |S| - 1, \quad \forall S \subsetneq V : |S| \geq 2}$$

where $A(S) := \{(i,j) \in A : i, j \in S\}$

(assuming that $G$ does not contain loops: $(i,i) \notin A$)




" $c_{ii} = +\infty$, $\forall i \in V$ "

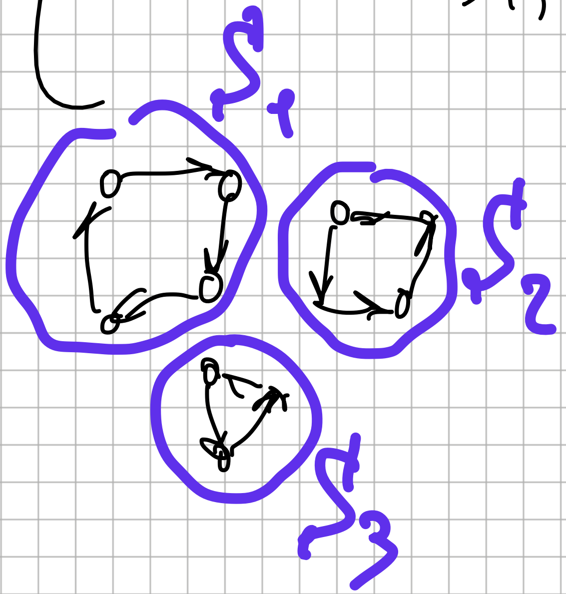
# of SEC's $\simeq 2^n$

$$\begin{array}{ccccccccccc} 1 & 2 & & & & & & & & & n \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}$$

# B & CUT

→ Forget about SEC's

$$
\begin{cases}
\min \sum c_{ij} x_{ij} \\
\sum_{(i,j)\in\delta^-(h)} x_{ij} = \sum_{(i,j)\in\delta^+(h)} x_{ij} = 1 \quad \forall h \in V \\
x_{ij} \geq 0 \quad \cancel{\text{INTEGER}}, \quad \forall (i,j) \in A
\end{cases}
$$

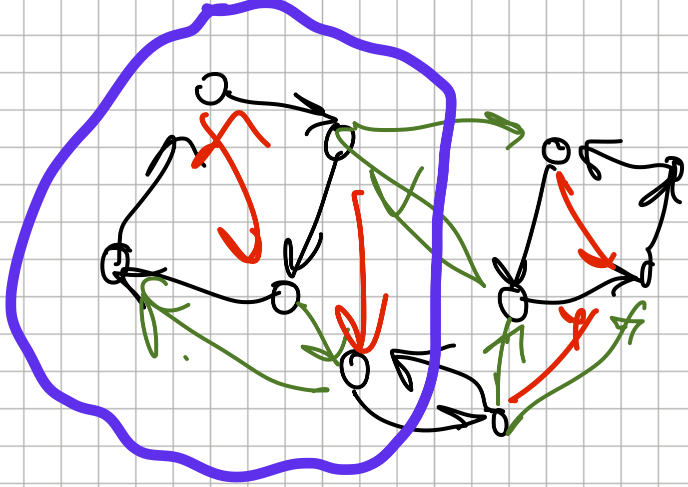



SUPPORT GRAPH $x^*$ of the LP relax.

⇒ SEC's on $S_1, S_2, S_3$

⇒ add to your LP model

INTEGER $x^*$ ⇒ SEPARATION IS TRIVIAL



$\rightarrow \quad x^* = 0.5$

$\rightarrow \quad x^* = 0.2$

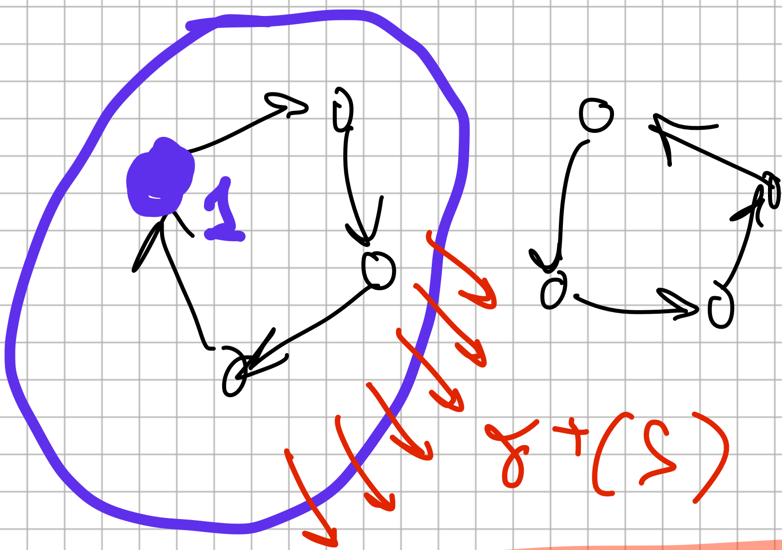
$\rightarrow \quad x^* = 0.1$

$x^*$ IS FRACT.

$\Rightarrow$

SEPARATION PROCEDURE FOR SEC's

## EQUIVALENT FORM OF SEC's




SEC's

$$\sum_{(i,j)\in\delta^+(S)} x_{ij} \geq 1, \quad \forall S \subsetneq V: \; |S| \geq 2, \; 1 \in S$$

" CUT FORM "

INPUT : $x^* \geq 0$

OUTPUT : Subset $S^* \subsetneq V : 1 \in S^*$ :

$$\boxed{\sum_{(i,j)\in\delta^+(S^*)}' x^*_{ij}} < 1 \rightarrow \text{MINIMIZE!}$$

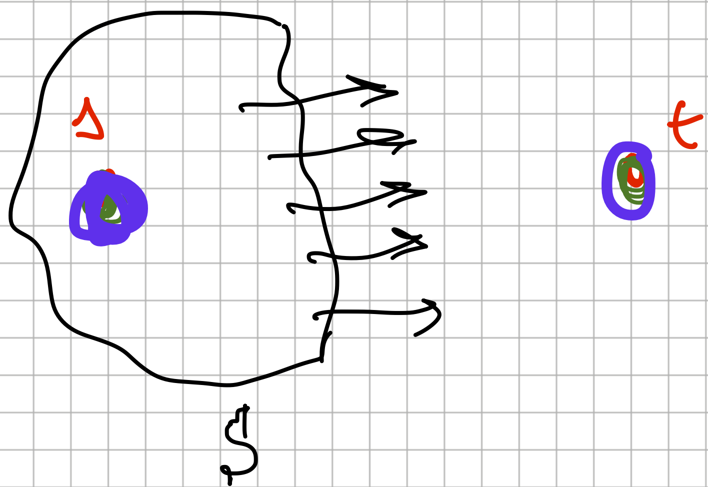
(IF ANY)

Network $G = (V, A)$

$k_{ij} = x^*_{ij} \quad (\geq 0)$

$s = ?$

$t = ?$




$$k(S) = \sum_{(i,j)\in\delta^+(S)}' k_{ij} = \sum_{(i,j)\in\delta^+(S)} x^*_{ij}$$

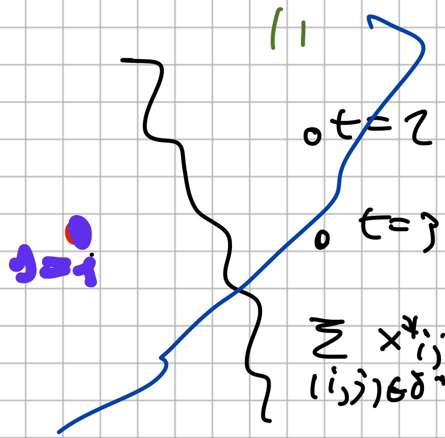
MIN-CAPACITY CUT
IN THE NETWORK !

$\Rightarrow$ MAX-FLOW
from $s$ to $t$ !!

$$\boxed{s = 1}$$

$$t = 2, 3, \dots, n$$

MAX-FLOW $s=1 \rightarrow t=2$
$\quad\parallel\quad$ $s=1 \rightarrow t=3$
$\quad\parallel\quad$ $s=1 \rightarrow t=n$




$$\sum_{(i,j)\in\delta^+(S)} x^*_{ij} < 1 \rightarrow \text{VIOLATED SEC.}$$