OR 1   20 - DEC - 2021

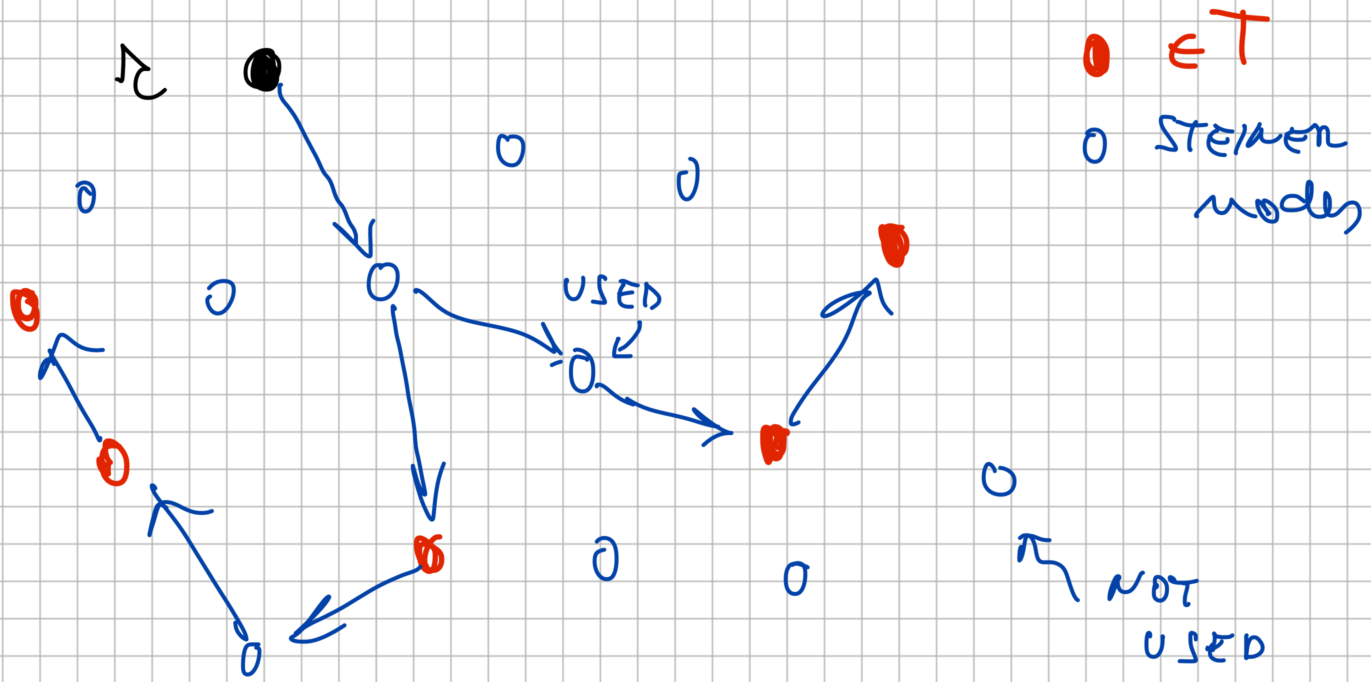
STEINER TREE PROBLEM

$G = (V, A)$ directed

$c : A \to \mathbb{R}$ (c_{ij} can be negative)

$r \in V$ ROOT node

$T \subseteq V \setminus \{r\}$ TERMINAL node set

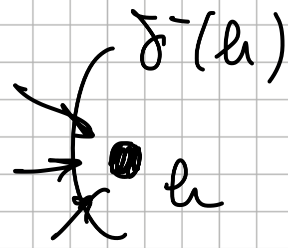


→ STEINER directed tree (ARBORESCENCE) rooted at node r

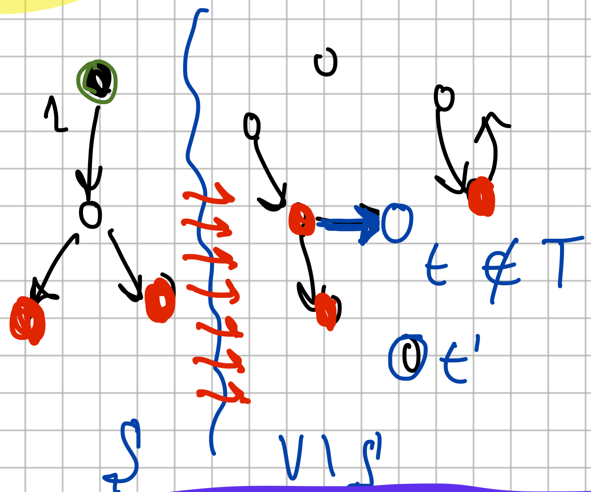
TOTAL COST → MINIMIZE

BRANCH-AND-CUT

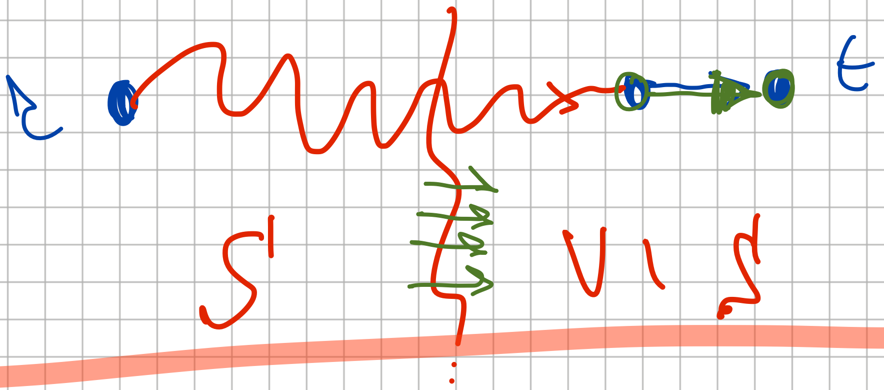
$$x_{ij} = \begin{cases} 1 & \text{if } (i,j) \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{cases} \min \displaystyle\sum_{(i,j)\in A} c_{ij} \cdot x_{ij} \\ \displaystyle\sum_{(i,j)\in \delta^-(h)} x_{ij} \begin{cases} = 1, & h \in T \\ = 0, & h = r \\ \leq 1, & h \in V \setminus (T \cup \{r\}) \end{cases} \\ 0 \leq x_{ij} \leq 1, \text{ INTEGER}, \quad \forall (i,j) \in A \end{cases}$$



$$\sum_{(i,j)\in \delta^+(S)} x_{ij} \geq \underbrace{\sum_{(i,j)\in \delta^-(t)} x_{ij}}_{\#\text{ of arcs entering } t} \quad \forall S \subsetneq V : r \in S, \; \forall t \in V \setminus S \qquad (*)$$



SEPARATION PR. FOR (*)

INPUT: $x^* \geq 0$

OUTPUT: $S^* \subsetneq V$: $r \in S^*$,

$t^* \in V \setminus S^*$:

$$\sum_{(i,j) \in \delta^+(S^*)} x^*_{ij} < \sum_{(i,j) \in \delta^-(t^*)} x^*_{ij}$$

(IF ANY)

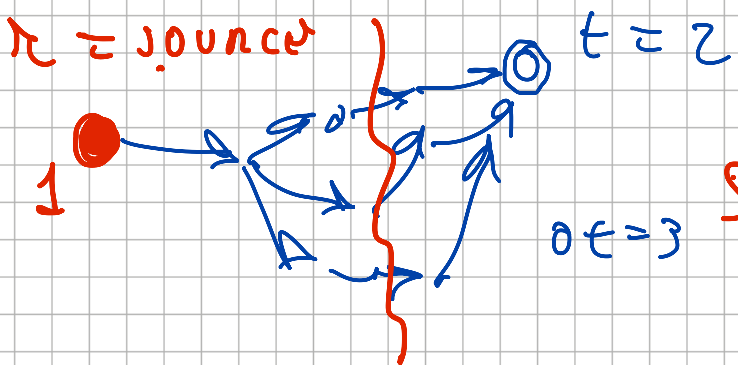
NETWORK $G = (V, A)$

$K_{ij} := x^*_{ij} \; (\geq 0)$

SOURCE NODE $s = r$

TERMINAL " $t \in V \setminus \{r\}$

TRY ALL!



or $t = 3$ $\quad S_t, V \setminus S_t$ MIN-CAPAC. CUT

$1 \to t = 2$: MAX-FLOW

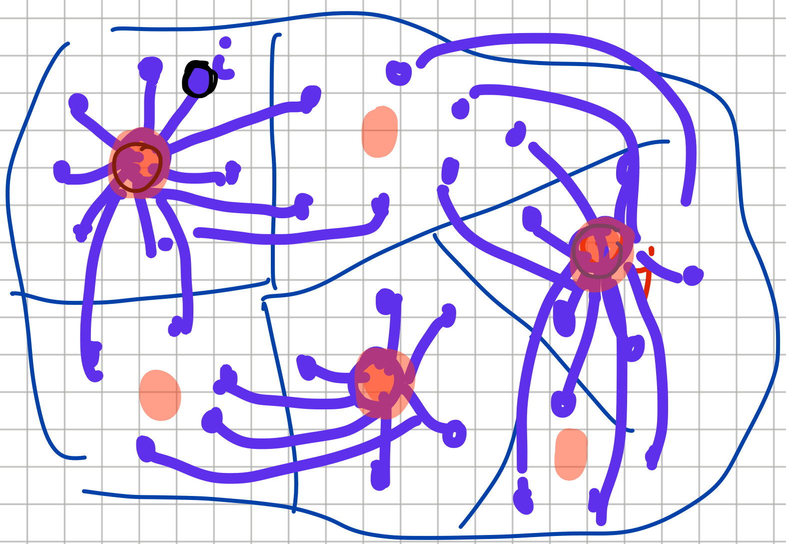
$1 \to t = 3$: MAX-FLOW

} $n - 1$ MAX-FLOW PR.

$1 \rightarrow t = n$ $\int$

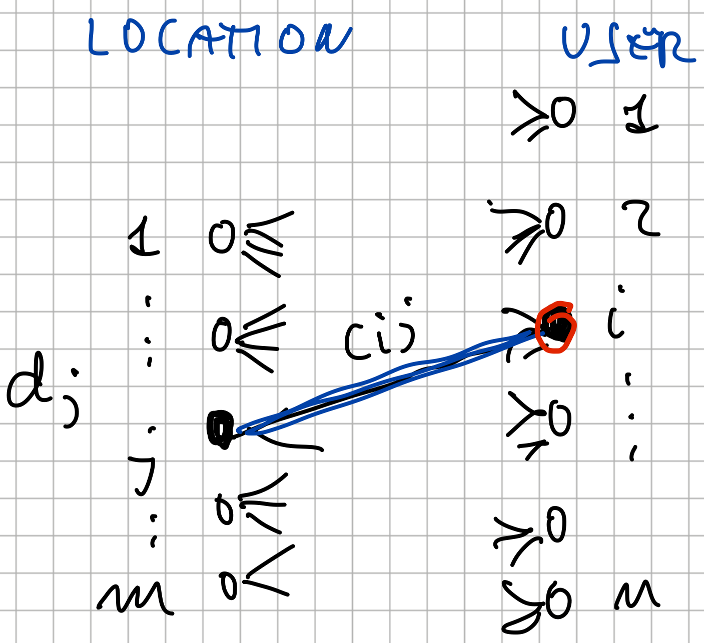
(PLANT FACILITY) LOCATION PR.

USERS
LOCATIONS



d_j = cost for building the facility in loc. j

— c_{ij} = connection cost user i to location j



n users
m locations

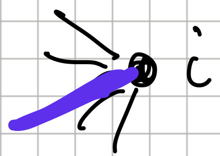
VARIABLES

$$y_j = \begin{cases} 1 & \text{if location } j \text{ is active} \\ 0 & \text{otherw.} \end{cases} \quad , j = 1, \dots, m$$

$$x_{ij} = \begin{cases} 1 & \text{if user } i \text{ connects to location } j \\ 0 & \text{otherwise} \end{cases}$$

$$\min \underbrace{\sum_{j=1}^{m} d_j y_j}_{\text{fixed cost}} + \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}}_{\text{connection cost}}$$

$$\underbrace{\sum_{j=1}^{m} x_{ij}}_{\text{n. of location connected to user } i} = 1 \quad , \forall i = 1, \dots, n$$



$$x_{ij} \in \{0,1\} \quad , \forall i = 1, \dots, n, \; j = 1, \dots, m$$

$$y_j \in \{0,1\} \quad , \forall j = 1, \dots, m$$

- MISSING CONSTRAINTS:

" $x_{ij} = 1 \Rightarrow y_j = 1$ "

$$x_{ij} \le y_j \quad , \forall i = 1, \dots, n, \; j = 1, \dots, m$$

↳ $x_{ij} = 0$ OK

$= 1$ $y_j = 1$

SET COVERING / PARTITIONING PROBLEM

$$A \in \{0,1\}^{m \times n}$$

$$c^T = [c_1, \dots, c_n]$$

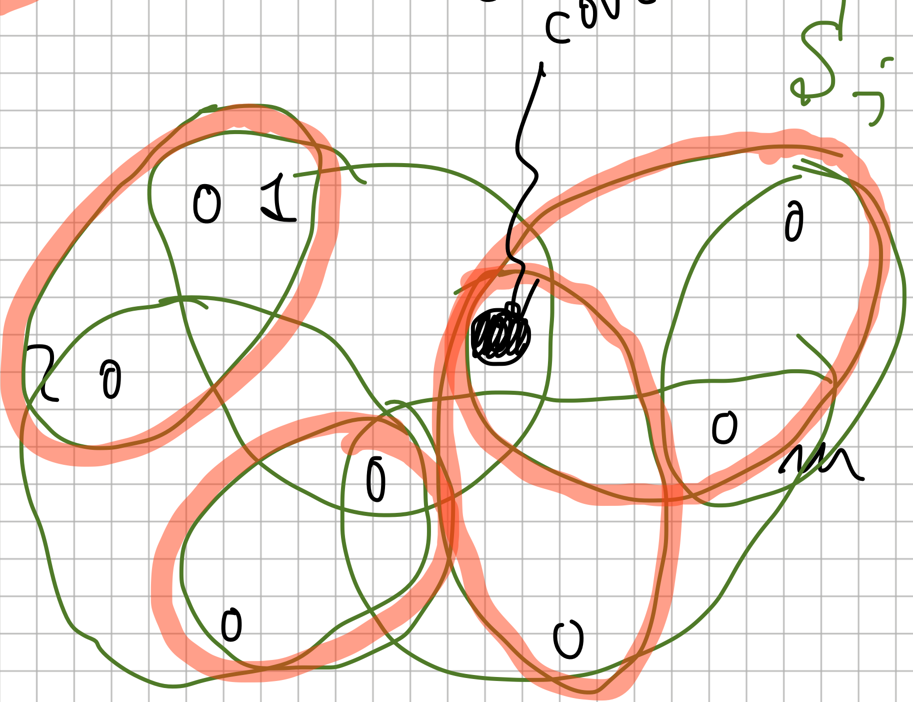
$$\begin{cases} \min & c^T x \\ & A x \geq \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ \\ m \end{matrix} \\ & 0 \leq x \leq 1 \quad \text{integer} \end{cases}$$

$=$ SET PARTITIONING

$\geq$ SET COVERING

$\Rightarrow$ NP-hard

COVERED TWICE

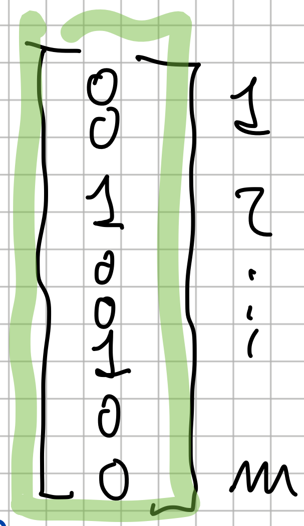


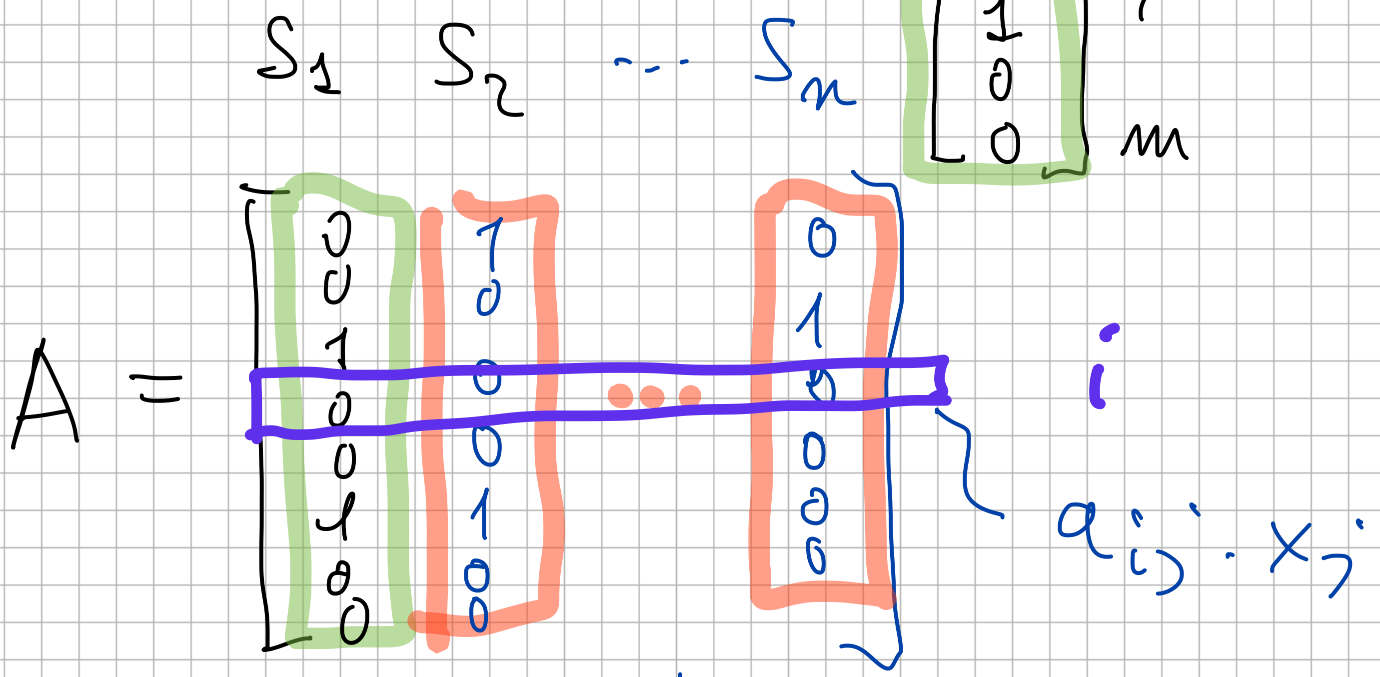
— m basic element ("nodes")

$$S_j \subseteq \{1, \dots, m\}$$

$$c_j \geq 0 \qquad j = 1, \dots, n$$

$S_j \leftrightarrow$ INCIDENCE VECTOR





$$x_j = \begin{cases} 1 & \text{if } S_j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$\rightarrow \begin{cases} \min \displaystyle\sum_{j=1}^{n} c_j x_j \\ \underbrace{\displaystyle\sum_{j=1}^{n} a_{ij} x_j}_{(*)} \geq 1, \ \forall i = 1, \dots, m \\ x_j \in \{0, 1\}, \ \forall j = 1, \dots, n \end{cases}$$

(*) n. of times item i is covered

Ex: $n = 10{,}000$

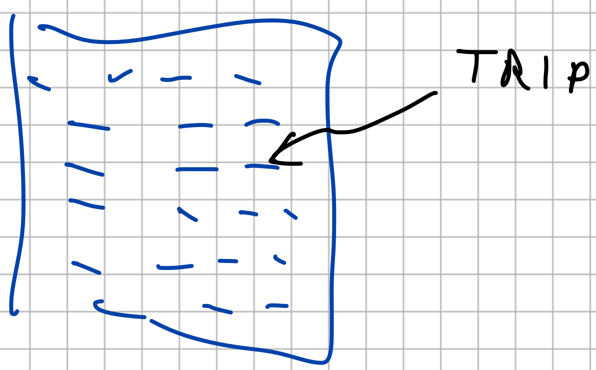
$m = 10{,}000{,}000 +$

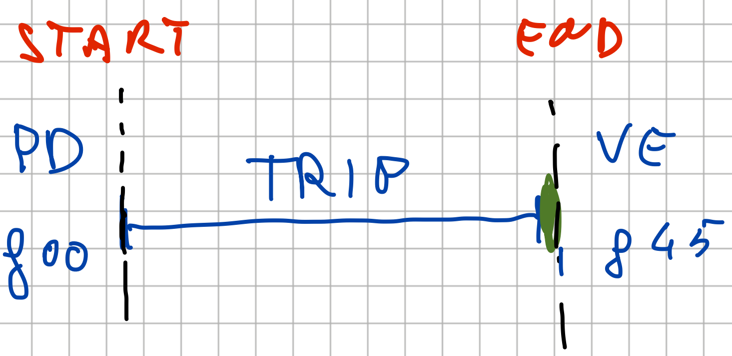
CREW SCHEDULING

BUS DRIVER

BUSITALIA VENETO

TIMETABLE

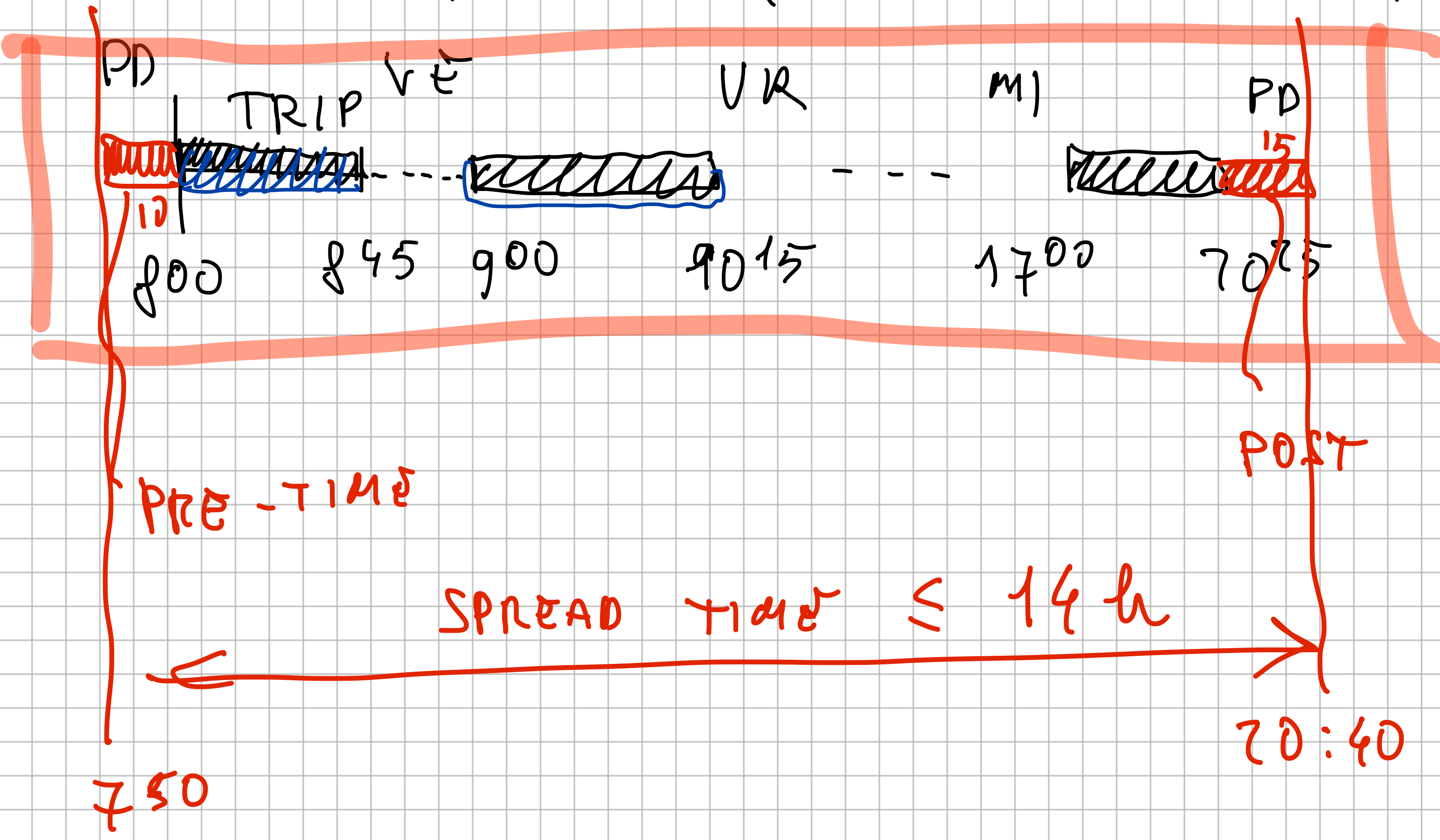




TYPICALLY:

$n = 10{,}000$ INPUT TRIPS

SHIFT / DUTY ("TURNO UOMO")



FEASIBILITY RULES:

1) # DRIVING TIME ≤ 7 h

2) CONTIN. DRIVING ≥ 4:30 h → BREAK ≥ 30'

...

PHASE I

GENERATE MILLIONS POSSIBLE DUTIES

FOR EACH DUTY: $j = 1, \dots, n$

S_j = set of trips covered by the duty

c_j = cost of duty

PHASE II

SOLVE THE SET COVERING PROBLEM:

$$\begin{cases} \min \sum_{j=1}^{n} c_j x_j \\ A x \geq \mathbb{1} \quad \text{"all trips must be covered"} \\ x \in \{0,1\}^n \end{cases}$$

$m = 10{,}000$ # trips

$n = 10M$ # of possible duties