OR1 21-DEC-2021

BIG-M constraints:

$a_i^T x \ge b_i$ not static in the model

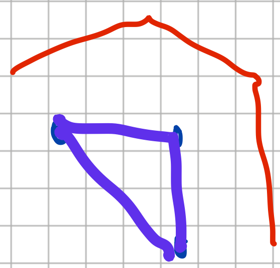
" $y_i = 0 \Rightarrow a_i^T x \ge b_i$ "

where $y_i \in \{0,1\}$ activation variable

$$a_i^T x \ge b_i - M y_i \quad \begin{cases} y_i = 0 \Rightarrow \dots \ge b_i \\ y_i = 1 \Rightarrow \ge \text{"}-\infty\text{"} \end{cases}$$

where $M \gg 0$ very large

Warning:
- the LP relaxation of the model can be very weak



"M as small as possible"

- large M can produce numerical problems

Disjunctive constr.s

Impose at least one of:

$$a_i^T x \geq b_i$$

$$a_k^T x \geq b_k$$

Introduce TWO binary var.s

y_i and y_k

$$\begin{cases} a_i^T x \geq b_i - M_i y_i \\ a_k^T x \geq b_k - M_k y_k \\ \boxed{y_i + y_k \leq 1} \\ y_i, y_k \in \{0,1\} \end{cases}$$

$y_i = 1$
deactivate
$a_i^T x \geq b_i$

$y_k = 1$
deact.
$a_k^T x \geq b_k$

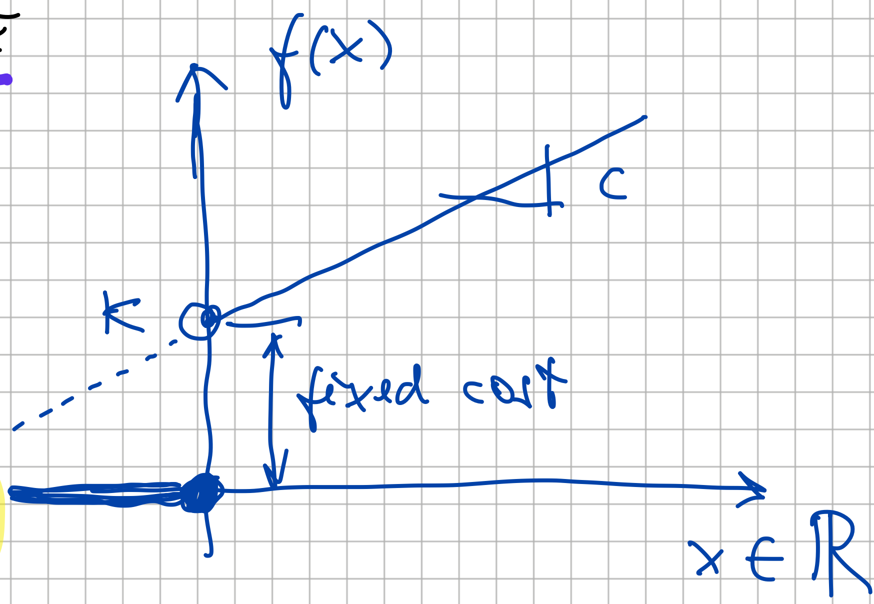
where $M_i, M_k \gg 0$

DISCRETE SETS

$$f^T x \in \{v_1, v_2, \dots, v_k\}$$

$$\begin{cases} f^T x = \sum_{i=1}^{k} v_i \cdot y_i \\ \sum_{i=1}^{k} y_i = 1 \\ y_1, \dots, y_k \in \{0,1\} \end{cases}$$

FIXED CHARGE



to be minimized

$K > 0$

$$f(x) = \begin{cases} K + cx & , \text{if } x > 0 \\ 0 & , \text{if } x \leq 0 \end{cases}$$

$$\begin{cases} f(x) = Ky + cx \\ x \leq My \\ y \in \{0,1\} \end{cases}$$

" $x \not> 0 \Rightarrow y = 1$ "

Assuming $K > 0$ and ...

where $M \gg 0$

$$\boxed{\text{INTEGER VAR.} \rightarrow \text{BINARY VAR.S}}$$

$0 \leq x \leq u$ integer

finite (e.g. 31)

$$x = \sum_{i=0}^{k} 2^i \cdot y_i$$

$y_i \in \{0,1\}, \quad i = 0, \dots, k$

Ex. 9-26 (p. 193)

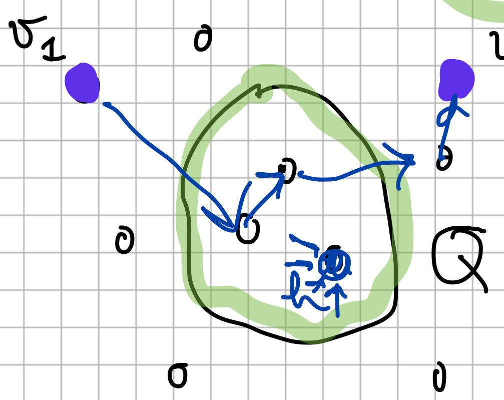
ILP model:

$G = (V, A)$ complete directed

$c : A \to \mathbb{R}_+$

$v_1, v_2 \in V, \quad v_1 \neq v_2$

$Q \subseteq V \setminus \{v_1, v_2\}$



SIMPLE PATH $v_1 \to v_2$

6 VISIT, AT LEAST, HALF OF THE NODES IN Q

NO LOOPS "$c_{ii} = +\infty$"

Sol. (p. 216)

$$\sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}$$

$$\begin{cases} \min \displaystyle\sum_{(i,j) \in A} c_{ij} x_{ij} \\ \displaystyle\sum_{(i,j) \in \delta^+(h)} x_{ij} - \sum_{(i,j) \in \delta^-(h)} x_{ij} = \begin{cases} +1, & h = v_1 \\ -1, & h = v_2 \\ 0, & h \in V \setminus \{v_1, v_2\} \end{cases} \\ \displaystyle\sum_{(i,j) \in A(S)} x_{ij} \leq |S| - 1, \quad \forall S \subseteq V : S \neq \emptyset \\ x_{ij} \in \{0,1\}, \quad \forall (i,j) \in A \end{cases}$$

$$\sum_{h \in Q} \underbrace{\sum_{(i,j) \in \delta^-(h)} x_{ij}}_{\substack{=0 \text{ if } h \text{ uncovered} \\ 1 \text{ if } h \text{ covered}}} \geq \lceil |Q|/2 \rceil$$

\# of nodes $h \in Q$ covered by the path

Ex. 9-2f

ILP model

$G = (V, E)$ undirected

(E_1, E_2) partition of E

c_e = cost, $\forall e \in E$
w_e = weight

SPANNING tree of $\boxed{\text{MAX}}$ cost s.t.

1) average weight of the edges in E_1 does not have to be greater than a given W

2) at least half of the chosen edges must belong to E_1

$$\left\{\begin{array}{l}
\max \; {\sum_{e \in E}}' c_e x_e \\[2ex]
\sum_{e \in E} x_e = |V| - 1 \\[2ex]
{\sum_{e \in E(S)}}' x_e \leq |S| - 1, \quad \forall S \subsetneq V : |S| \geq 3 \\[2ex]
x_e \in \{0, 1\}, \quad \forall e \in E
\end{array}\right.$$

1) $$\frac{{\sum_{e \in E_1}}' w_e x_e}{{\sum_{e \in E_1}}' x_e} \leq W$$

$= {\sum_{e \in E_1}}' W x_e$

i.e.

$${\sum_{e \in E_1}}' (W - w_e) x_e \geq 0$$

2) $$\underbrace{\sum_{e \in E_1} x_e}_{\text{\# of chosen edges in } E_1} \geq \left\lceil \frac{|V| - 1}{2} \right\rceil$$

\# of chosen edges