1 Applied Functional Analysis

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Aim: The course is intended to give a survey of the basic aspects of functional analysis, operator theory in Hilbert spaces, regularization theory and inverse problems.

Topics:

1. *Review of some notions on metric spaces and Lebesgue integration*: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.

2. *Banach and Hilbert spaces*: Normed spaces and Banach spaces. Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces. Hilbert adjoint operator. Self-adjoint operators, unitary operators.

3. *Fourier transform and convolution*: The convolution product and its properties. The basic L^1 and L^2 theory of the Fourier transform. The inversion theorem.

4. *Compact linear operators on normed spaces and their spectrum*: Spectral properties of bounded linear operators. Compact linear operators on normed spaces. Spectral properties of compact linear operators. Spectral properties of bounded self-adjoint operators, positive operators, operators defined by a kernel. Mercer Kernels and Mercer's theorem.

5. *Reproducing kernel Hilbert spaces, inverse problems and regularization theory*: Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Support vector regression and regularization networks. Representer theorem.

Course requirements:

1. The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence.

2. The arithmetic of complex numbers and the basic properties of the complex exponential function.

3. Some elementary set theory.

4. A bit of linear algebra.

All the necessary material can be found in W. Rudin's book Principles of Mathematical Analysis (3rd ed., McGraw-Hill, 1976). A summary of the relevant facts will be given in the first lecture.

References:

[1] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons , 1978.
[2] M. Reed and B. Simon, Methods of Modern Mathematical Physics, vol. I, Functional Analysis, Academic Press, 1980.
[3] G. Wahba. Spline models for observational data. SIAM, 1990.
[4] C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006.

Time table: Course of 28 hours (2 two-hours lectures per week): Classes on Tuesday and Thursday, 10:30 - 12:30. First lecture on Tuesday September 21, 2010. Room DEI/G (3-rd floor, Dept. of Information Engineering, via Gradenigo Building).

Examination and grading: Homework assignments and final test.