Nono test di autovalutazione di CONTROLLI AUTOMATICI
A.A. 2009/2010

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1. Si calcolino le trasformate zeta dei seguenti segnali (le domande (a)-(c) valgono 1.5 punti, le domande (d)-(f) 2 punti, le domande (g)-(l) 2.5 punti):

   (a) $v(t) = 1 + \delta(t) + \frac{1}{2^t}$; ....

   (b) $v(t) = t \cdot \delta_{-1}(t) + 3^t \delta_{-1}(t+4)$; ....

   (c) $v(t) = \delta(t+1) + \delta(t-1) + \binom{t}{2}$; ....

   (d) $v(t) = \delta_{-1}(t) - \delta_{-1}(t-5) + \binom{t}{1}5^t$; ....

   (e) $v(t) = \begin{cases} 1, & \text{per } t \in \mathbb{Z}_+, t \text{ pari;} \\ 0, & \text{altrimenti;} \end{cases}$ ....

   (f) $v(t) = \begin{cases} 2^t, & \text{per } t \in \mathbb{Z}_+, t \text{ dispari;} \\ 0, & \text{altrimenti;} \end{cases}$ ....

   (g) $v(t) = \begin{cases} 0, & \text{per } t = 0, 3; \\ 1, & \text{per } t = 1; \\ -1, & \text{per } t = 2; \\ 2, & \text{per } t \in \mathbb{Z}_+, t \geq 4; \end{cases}$ ....

   (h) $v(t) = \begin{cases} 1, & \text{per } t \in \mathbb{Z}_+, t \text{ pari;} \\ -2, & \text{per } t \in \mathbb{Z}_+, t \text{ dispari;} \end{cases}$ ....

   (i) $v(t) = \begin{cases} 1+t, & \text{per } t \in \mathbb{Z}_+, t \text{ pari;} \\ 0, & \text{altrimenti;} \end{cases}$ ....

   (l) $v(t) = \begin{cases} 2^t, & \text{per } t \in \mathbb{Z}_+, t \text{ multiplo di 3;} \\ -1, & \text{per } t \in \mathbb{Z}_+, t \text{ congruo a 1 modulo 3;} \\ 0, & \text{altrimenti.} \end{cases}$ ....

2. Si calcolino le antitrasformate zeta delle seguenti funzioni razionali (ogni domanda vale 2 punti):

   (a) $V(z) = z^{-2} + \frac{1}{(z+4)(z+1)}$; ....

   (b) $V(z) = \frac{z+1}{z(z-1)(z-2)}$; ....

   (c) $V(z) = \frac{z^2}{z^2+10}$; ....

   (d) $V(z) = \frac{2z}{z^2-\sqrt{3}z+1}$; ....

   (e) $V(z) = \frac{z^2+1}{z(z+1)^2}$. ....

RISPOSTE

1. Trasformata zeta:

   (a) $V(z) = \frac{z}{z-1} + 1 + \frac{z}{z-1/2}$.

   (b) $V(z) = \frac{z}{(z-1)^2} + \frac{z}{z-3}$.

   (c) $V(z) = z^{-1} + \frac{z}{(z-1)^3}$.

   (d) $V(z) = \sum_{i=0}^{4} z^{-i} + 5 \cdot \frac{z}{(z-5)^2}$.

   (e) $V(z) = \sum_{i=0}^{+\infty} 1z^{-2i} = \sum_{i=0}^{+\infty}(z^{-2})^i = \frac{1}{1-z^{-2}} = \frac{z^2}{z^2-1}$.

   (f) $V(z) = \sum_{i=0}^{+\infty} 2^{2i+1}z^{-2i-1} = 2z^{-1}\sum_{i=0}^{+\infty}(4z^{-2})^i = 2z^{-1} \cdot \frac{1}{1-4z^{-2}} = \frac{2z}{z^2-4}$.

   (g) $V(z) = 1 \cdot z^{-1} - 1 \cdot z^{-2} + \sum_{t=4}^{+\infty} 2z^{-t} = z^{-1} - z^{-2} + 2\Big(\sum_{i=0}^{+\infty} z^{-t} - 1 - z^{-1} - z^{-2} - z^{-3}\Big) = -2 - z^{-1} - 3z^{-2} - 2z^{-3} + 2\frac{z}{z-1}$.

   (h) $V(z) = \sum_{i=0}^{+\infty} 1 \cdot z^{-2i} - \sum_{i=0}^{+\infty} 2 \cdot z^{-2i-1} = (1-2z^{-1}) \cdot \sum_{i=0}^{+\infty} z^{-2i} = (1-2z^{-1}) \cdot \frac{1}{1-z^{-2}} = \frac{z(z-2)}{z^2-1}$.

   (i) $V(z) = \sum_{i=0}^{+\infty}(1+2i)z^{-2i} = -z^2\frac{d}{dz}\left(\sum_{i=0}^{+\infty} z^{-2i-1}\right) = -z^2\frac{d}{dz}\left(z^{-1} \cdot \frac{1}{1-z^{-2}}\right) = -z^2\frac{d}{dz}\left(\frac{z}{z^2-1}\right) = -z^2\frac{z^2-1-2z^2}{(z^2-1)^2} = \frac{z^2(z^2+1)}{(z^2-1)^2}$,
   dove si è usato il fatto che

   $$\frac{d}{dz}\left(\sum_{i=0}^{+\infty} z^{-2i-1}\right) = \sum_{i=0}^{+\infty}(-2i-1)z^{-2i-2} = -z^{-2} \cdot \sum_{i=0}^{+\infty}(2i+1)z^{-2i}.$$

   (l) $V(z) = \sum_{i=0}^{+\infty} 2^{3i}z^{-3i} - \sum_{i=0}^{+\infty} 1 \cdot z^{-3i-1} = \sum_{i=0}^{+\infty}(8z^{-3})^i - z^{-1} \cdot \sum_{i=0}^{+\infty} z^{-3i} = \frac{1}{1-8z^{-3}} - z^{-1} \cdot \frac{1}{1-z^{-3}} = \frac{z^3}{z^3-8} - \frac{z^2}{z^3-1}$.

2. Antitrasformata zeta:

   (a) $v(t) = \delta(t-2) + \frac{1}{4}\delta(t) + \frac{1}{12}(-4)^t - \frac{1}{3}(-1)^t$.

   (b) $v(t) = \frac{5}{4}\delta(t) + \frac{1}{2}\delta(t-1) - 2 + \frac{3}{4}2^t$.

   (c) $v(t) = \frac{1}{2}(j\sqrt{10})^t + \frac{1}{2}(-j\sqrt{10})^t = \frac{(\sqrt{10})^t}{2}\left(e^{jt\pi/2} + e^{-jt\pi/2}\right) = (\sqrt{10})^t \cos\left(\frac{t\pi}{2}\right)$.

   (d) $v(t) = -(2j)e^{j\pi t/6} + (2j)e^{-j\pi t/6} = 4\sin\left(t\frac{\pi}{6}\right)$.

   (e) $v(t) = \delta(t-1) - 2\binom{t-1}{1}(-1)^{t-2}$ oppure, equivalentemente, $v(t) = -2\delta(t) + \delta(t-1) + 2(-1)^t\delta_{-1}(t) + 2\binom{t}{1}(-1)^{t-1}$.