L_1-Induced Norm Analysis of Positive Systems and Its Application to Stabilization of Large Scale Interconnected Positive Systems

Y. Ebihara

Department of Electrical Engineering,

Kyoto University, Japan.

Outline

- Analysis by Copositive Lyapunov Functions

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- Weighted L_1-induced Norm

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- Stability of Interconnected Positive Systems

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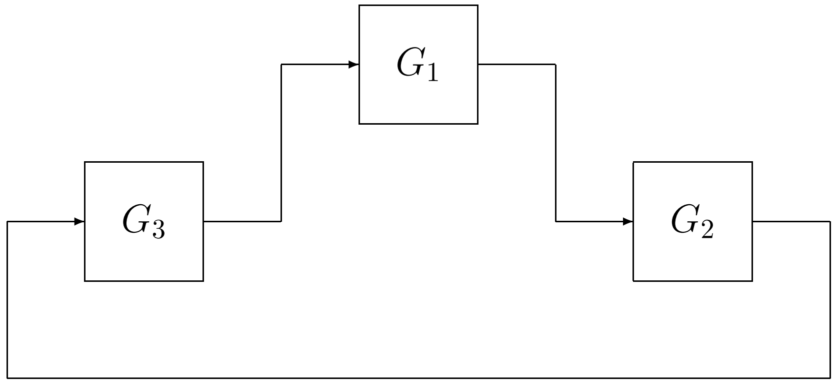

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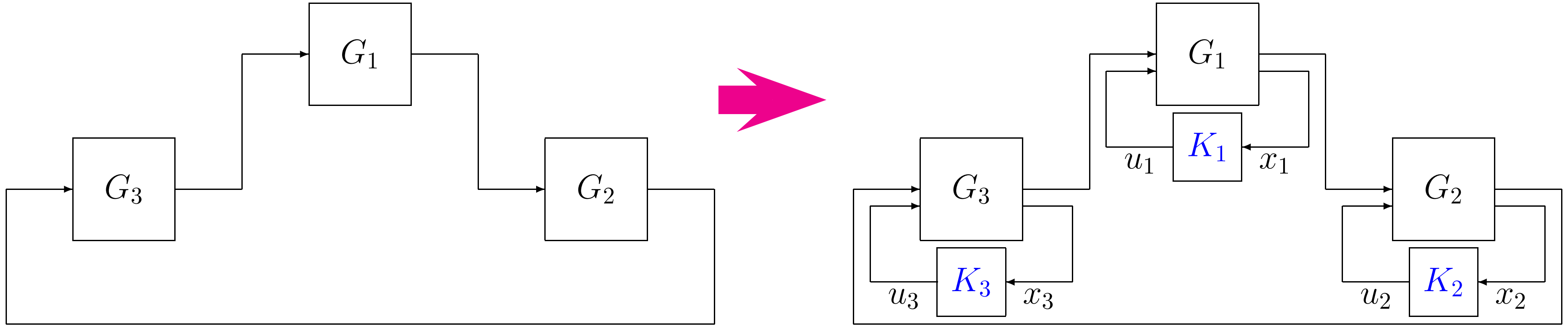

- Decentralized Stabilizing SF Synthesis
  - purely locally by solving LP
  - optimality: minimizing real dominant pole

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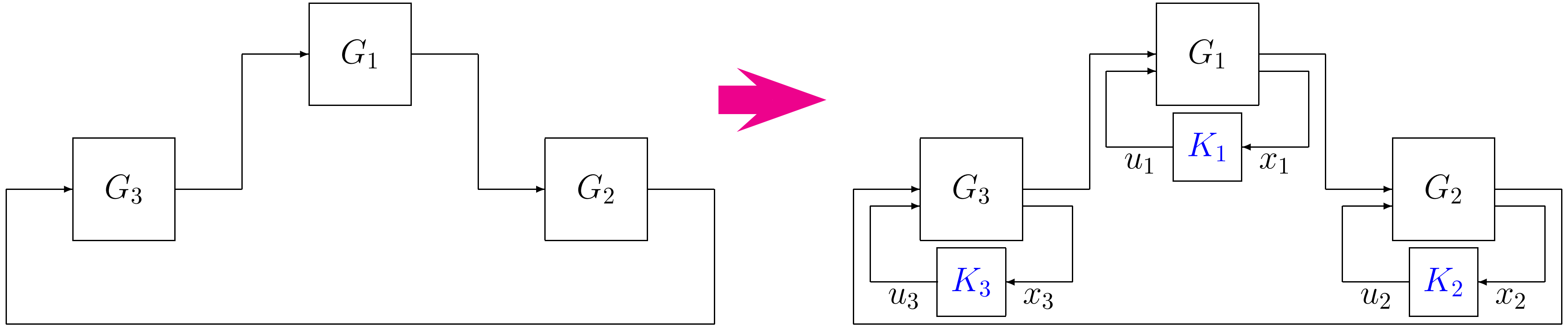
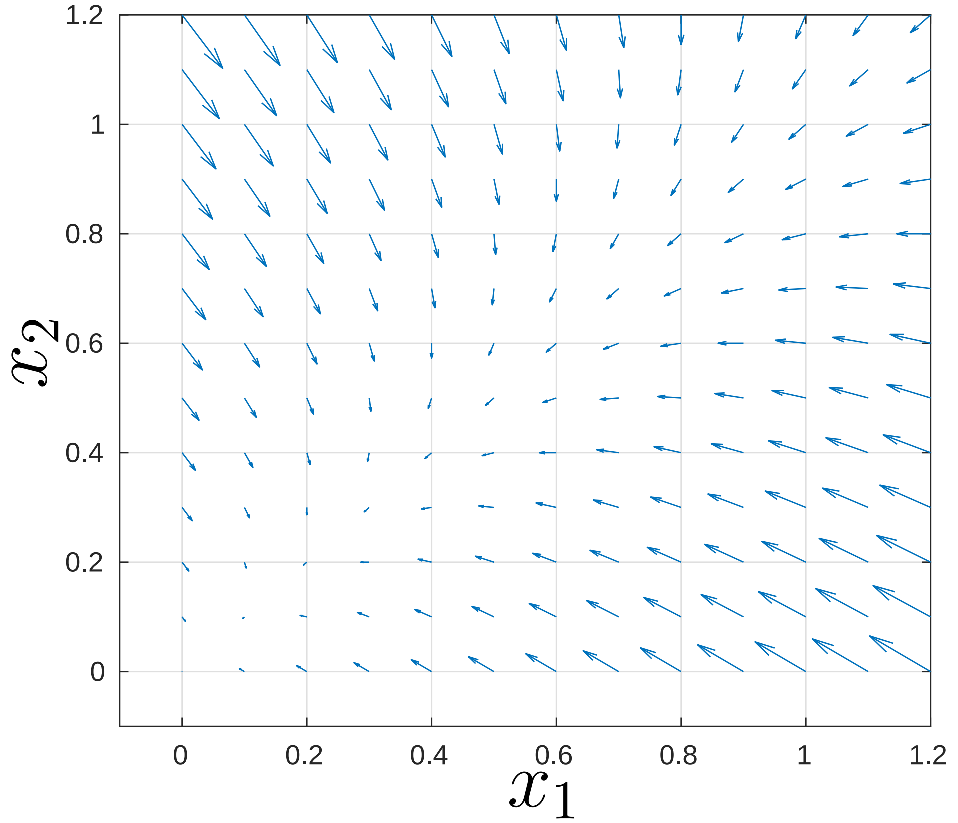


- Decentralized Stabilizing SF Synthesis
  - purely locally by solving LP
  - optimality: minimizing real dominant pole
- Conclusion

Stability Analysis by Copositive Lyapunov Functions

Positive System $\dot{x}(t) = Ax(t)$ $(A \in \mathbb{M}^n)$



$$x(0) \in \mathbb{R}_+^n \;\Rightarrow\; x(t) \in \mathbb{R}_+^n \; (\forall t \in \mathbb{R}_+)$$

Stability Analysis by Copositive Lyapunov Functions

$\boxed{\text{Positive System } \dot{x}(t) = Ax(t)\ (A \in \mathbb{M}^n)}$

Requirements for LFs

$$V(x(t)) > 0,\ -\dot{V}(x(t)) > 0\ (\forall x \in \mathbb{R}^n_+ \setminus \{0\})$$

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  - $h \geq \mathbf{1}_n^T,\ -h^T A \geq \mathbf{1}_n^T$ (LP)

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- quadratic: $V(x) = x^T P x, \ P \in \mathbb{S}^n_{++}$
  - ➤ $P \succeq I_n, \ -(PA + A^T P) \succeq I_n$ (SDP)

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  - ➤ $P \succeq_{\mathcal{COP}} I_n, -(PA + A^T P) \succeq_{\mathcal{COP}} I_n$ (COP)

$$P \succeq_{\mathcal{COP}} 0 \ \Leftrightarrow x^T P x \geq 0\ (\forall x \in \mathbb{R}^n_+)$$

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COP is hard (tractable if $n \leq 4$)

Stabilizing SF Synthesis by LP

$\boxed{\text{Problem}}$ For given $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$, find $K \in \mathbb{R}^{m\times n}$ such that $A + BK \in \mathbb{H}^n \cap \mathbb{M}^n$.

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Reduction to LP

Find $g \in \mathbb{R}^n_{++}$, $K \in \mathbb{R}^{m\times n}$ s.t. $A + BK \in \mathbb{M}^n$, $(A + BK)g \ll 0$.

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Find $g \in \mathbb{R}^n_{++}$, $K \in \mathbb{R}^{m\times n}$ s.t. $(A + BK)G \in \mathbb{M}^n$, $(A + BK)G\mathbf{1}_n \ll 0$.

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$\Updownarrow$ $Y := KG$ (change of variables)

Find $g \in \mathbb{R}^n_{++}$, $Y \in \mathbb{R}^{m\times n}$ s.t. $AG + BY \in \mathbb{M}^n$, $(AG + BY)\mathbf{1}_n \ll 0$.

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- stabilizing SF gain: $K = YG^{-1}$
- synthesis under structural constraint

$$K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \Leftrightarrow Y = \begin{bmatrix} Y_1 & 0 \\ 0 & Y_2 \end{bmatrix}$$

Weighted L_1-induced Norm Analysis

Positive System

$$G:\ \begin{cases} \dot{x} = Ax + Bw, & x(0)=0, \\ z = Cx + Dw \end{cases}$$

$A \in \mathbb{M}^n$, $B \in \mathbb{R}_+^{n \times n_w}$, $C \in \mathbb{R}_+^{n_z \times n}$, $D \in \mathbb{R}_+^{n_z \times n_w}$

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Definition

For given $q_z \in \mathbb{R}_{++}^{n_z}$ and $q_w \in \mathbb{R}_{++}^{n_w}$, we define

$$\|G_{q_z,q_w}\|_{1+} := \sup_{\|q_w^T w\|_1 = 1,\ w \in L_{1+}} \|q_z^T z\|_1.$$

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Here,

$$\|v\|_1 := \int_0^\infty \|v(t)\|_1 dt,$$

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$$L_{1+} := \{v(t) : \|v\|_1 < \infty, \quad v(t) \geq 0\ \forall t \in [0, \infty)\}.$$

the set of L_1-bounded and positive signals

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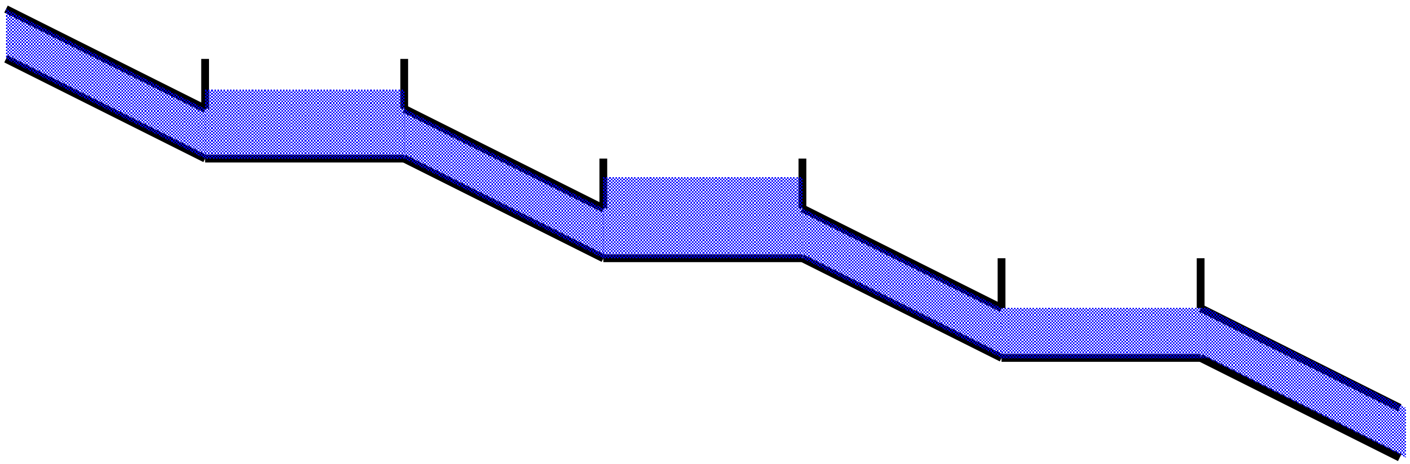
Example: Tank System



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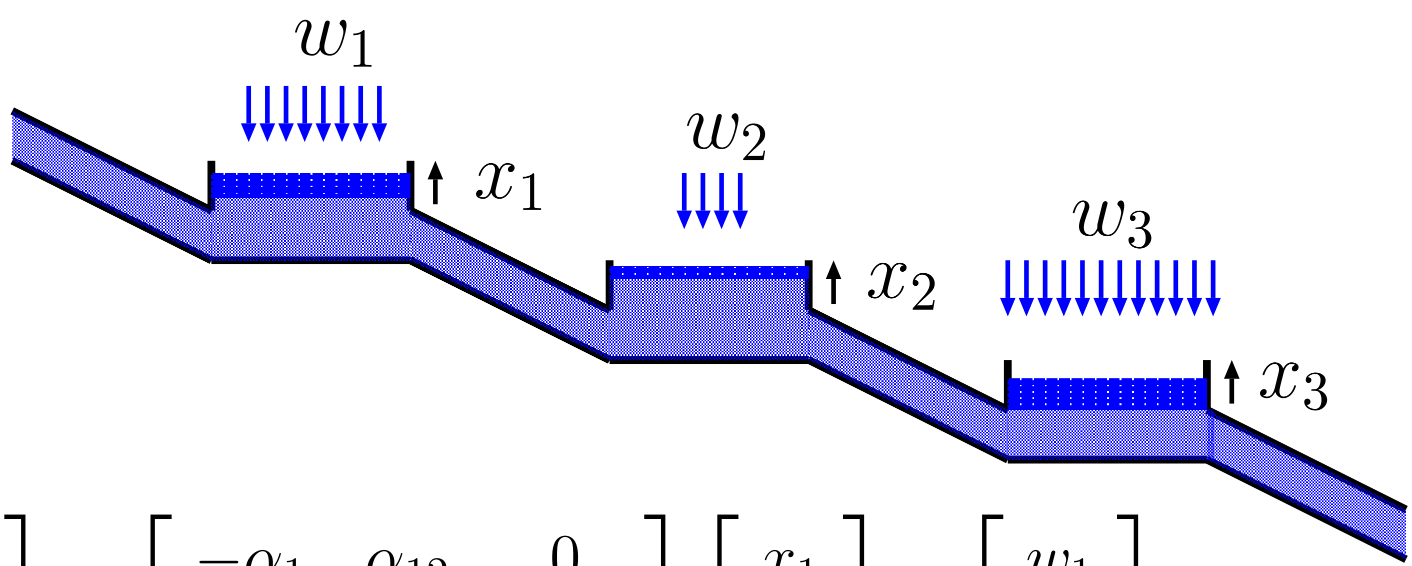

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\alpha_1 & \alpha_{12} & 0 \\ \alpha_{21} & -\alpha_2 & \alpha_{23} \\ 0 & \alpha_{32} & -\alpha_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}.$$

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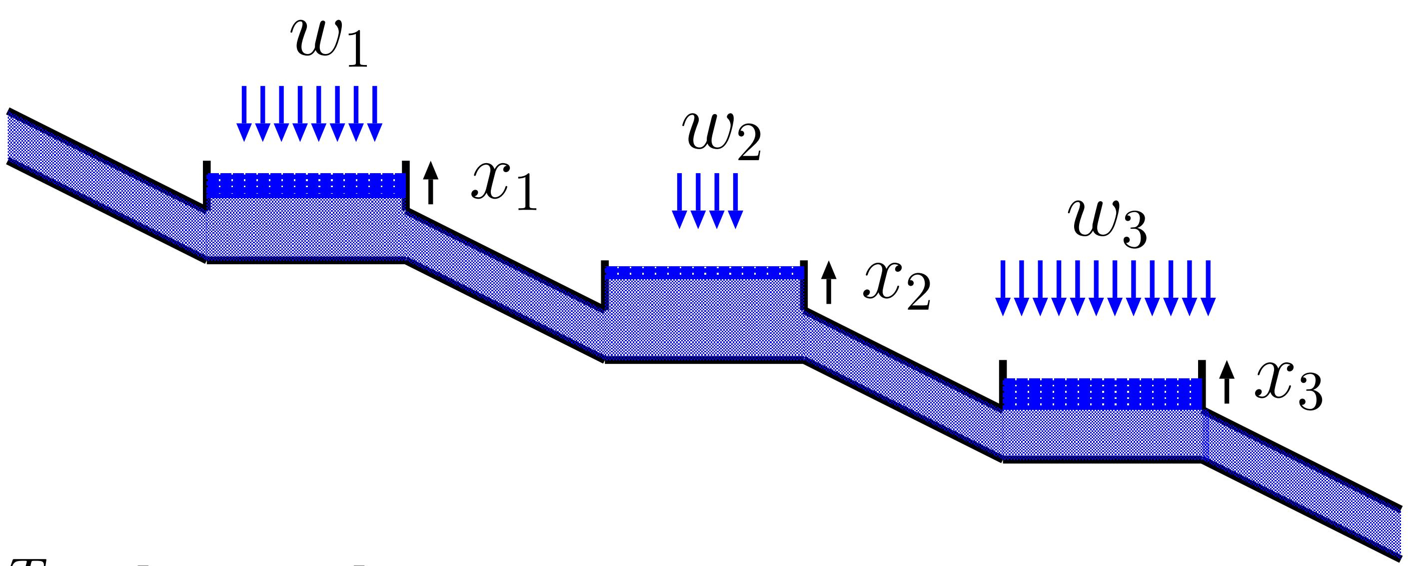

- $q_z^T = [\ 1\ 1\ 1\ ]$ ➤ $z = x_1 + x_2 + x_3$ (total fluctuation)
- $q_{z,i}$: weigthts for x_i, $q_{w,i}$: rain strengh

Weighted L_1-induced Norm Analysis

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$$\|G_{q_z,q_w}\|_{1+} := \sup_{\|q_w^T w\|_1=1,\ w\in L_{1+}} \|q_z^T z\|_1.$$

Facts

$$\|G_{q_z,q_w}\|_{1+} = \|Q_z G Q_w^{-1}\|_1,$$

$$Q_z = \mathrm{diag}(q_z),\ Q_w = \mathrm{diag}(q_w).$$

L_1-induced norm with weightings on the input and output signals

Weighted L_1-induced Norm Analysis

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How to compute?

Weighted L_1-induced Norm Analysis

Theorem (Ebihara et al., CDC11,TAC17)

For given $q_z \in \mathbb{R}_{++}^{n_z}$, $q_w \in \mathbb{R}_{++}^{n_w}$ and $\gamma > 0$, the followings are equivalent:

(i) $A \in \mathbb{H}^n$ and $\|G_{q_z,q_w}\|_{1+} < \gamma$.

(ii) $\exists h \in \mathbb{R}_{++}^n$ such that

$$\begin{bmatrix} h^T A + q_z^T C & h^T B + q_z^T D - \gamma q_w^T \end{bmatrix} \ll 0.$$

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Characterization by Linear Programming

$$\inf_{h \in \mathbb{R}_{++}^n, \gamma} \gamma \text{ s. t. } \begin{bmatrix} h^T A + q_z^T C & h^T B + q_z^T D - \gamma q_w^T \end{bmatrix} \ll 0.$$

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Weighted L_1-induced Norm Analysis

Characterization by Linear Programming

$$\inf_{h\in\mathbb{R}^n_{++},\gamma} \gamma \text{ s. t. } \begin{bmatrix} h^TA+q_z^TC & h^TB+q_z^TD-\gamma q_w^T \end{bmatrix} \ll 0.$$

LP-based Positive System Analysis

- M. Rami and F. Tadeo, MTNS 2006
- C. Briat, Int. J. Robust Nonlin. Cont. 2013
- J. Shen and J. Lam, Automatica 2014
- A. Rantzer, EJC 2015

C. Briat

J. Lam

A. Rantzer

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$\boxed{\text{Application?}}$

- stability analysis of interconnected systems
- decentralized controller synthesis

Stability Analysis of Interconnected Positive Systems

Subsystem G_i (positive and stable)

$$G_i : \begin{cases} \dot{x}_i = A_i x_i + \displaystyle\sum_{j=1, j\neq i}^{N} B_{ij} w_{ij}, \\ z_{ji} = C_{ji} x_i \ (j \neq i) \end{cases} \quad (i = 1, \cdots, N)$$

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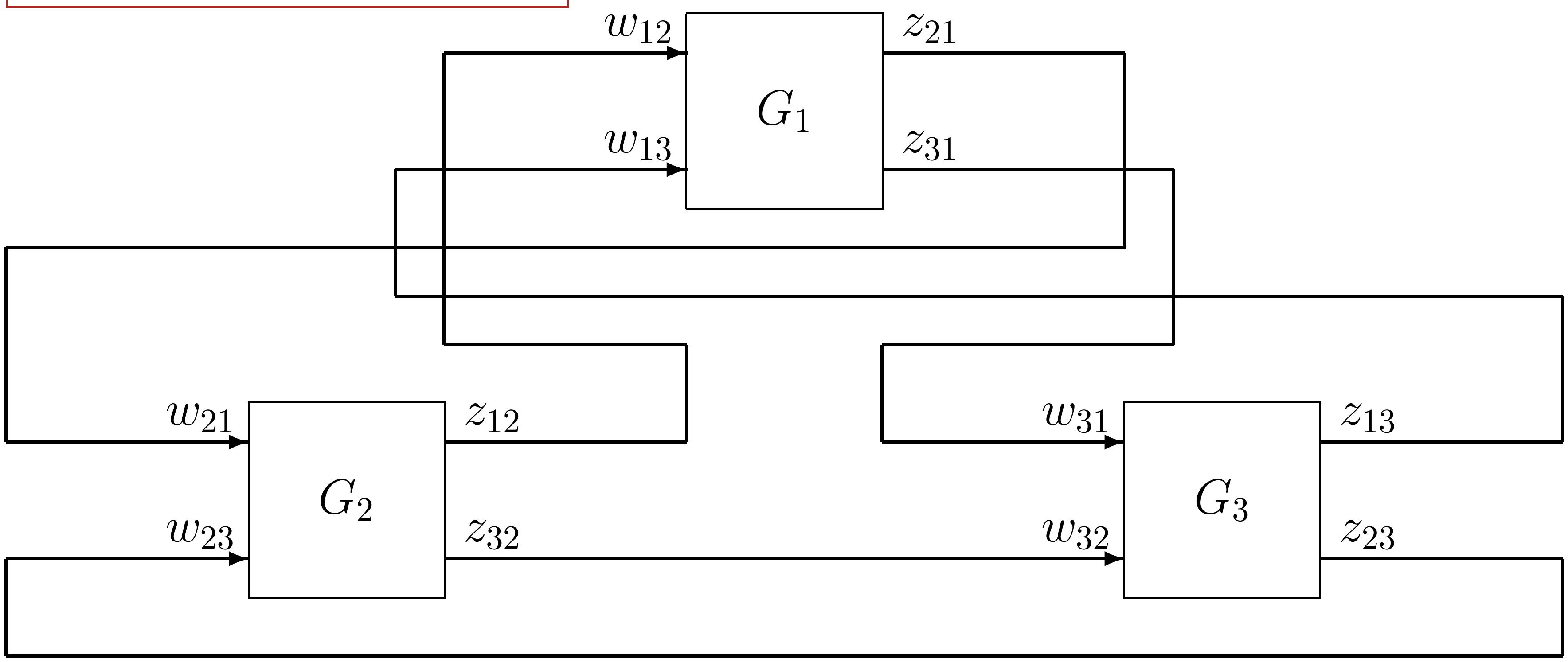
Interconnection

$$w_{ij} = z_{ij} \ (i, j = 1, \cdots, N, \ i \neq j)$$

Stability Analysis of Interconnected Positive Systems

Example ($N = 3$)



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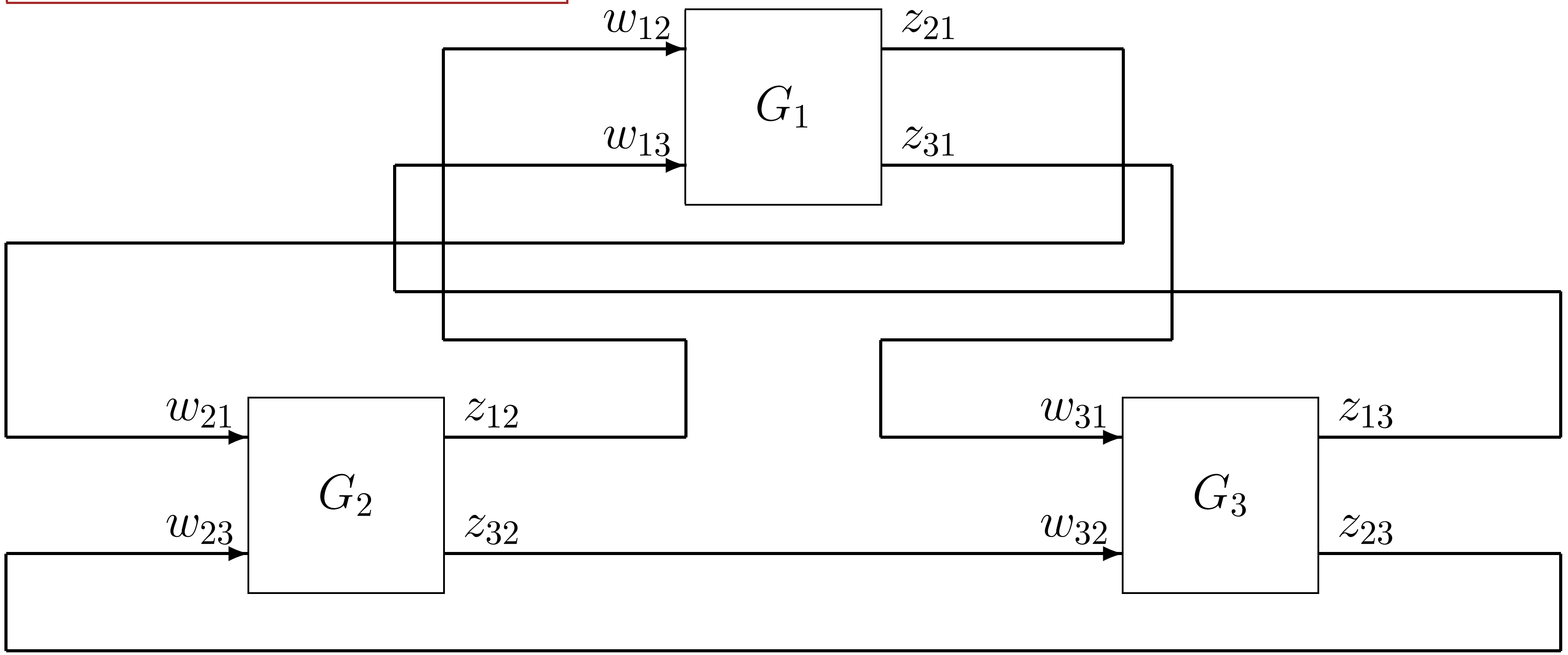

most general interconnection structure

Stability Analysis of Interconnected Positive Systems

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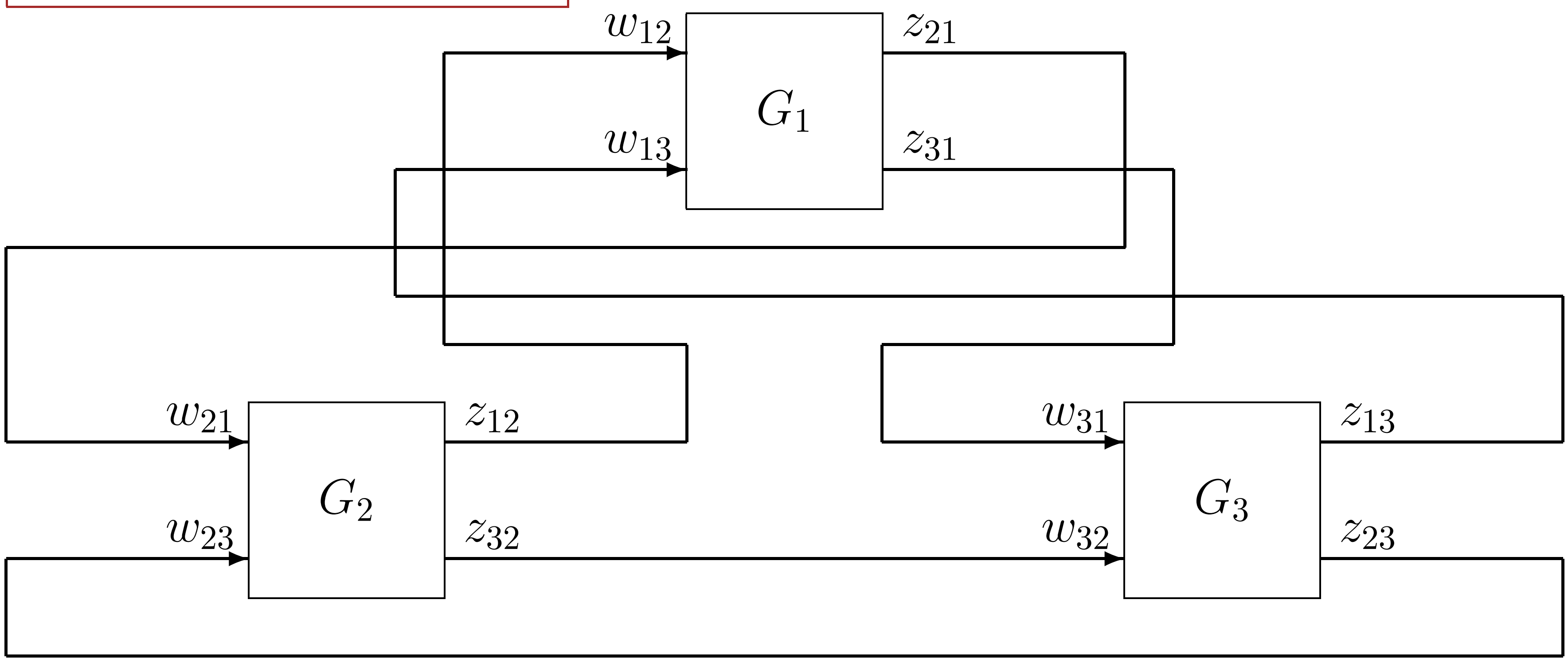

stability characterization via
weighted L_1-induced norm?

Stability Analysis of Interconnected Positive Systems

Theorem (Ebihara et al., CDC2011, TAC2017)

The interconnected system is stable iff there exists $q_{ij} \in \mathbb{R}_{++}^{n_{w_{ij}}}$ $(i,j = 1,\cdots,N, i \neq j)$ such that $\|G_{i,q_{i,z},q_{i,w}}\|_{1+} < 1 \quad (i = 1,\cdots,N)$ where

$$\begin{aligned} q_{i,z} &= \begin{bmatrix} q_{1i}^T & \cdots & q_{i-1,i}^T & q_{i+1,i}^T & \cdots & q_{Ni}^T \end{bmatrix}^T, \\ q_{i,w} &= \begin{bmatrix} q_{i1}^T & \cdots & q_{i,i-1}^T & q_{i,i+1}^T & \cdots & q_{iN}^T \end{bmatrix}^T. \end{aligned}$$

stability characterization via weighted L_1-induced norm

Stability Analysis of Interconnected Positive Systems

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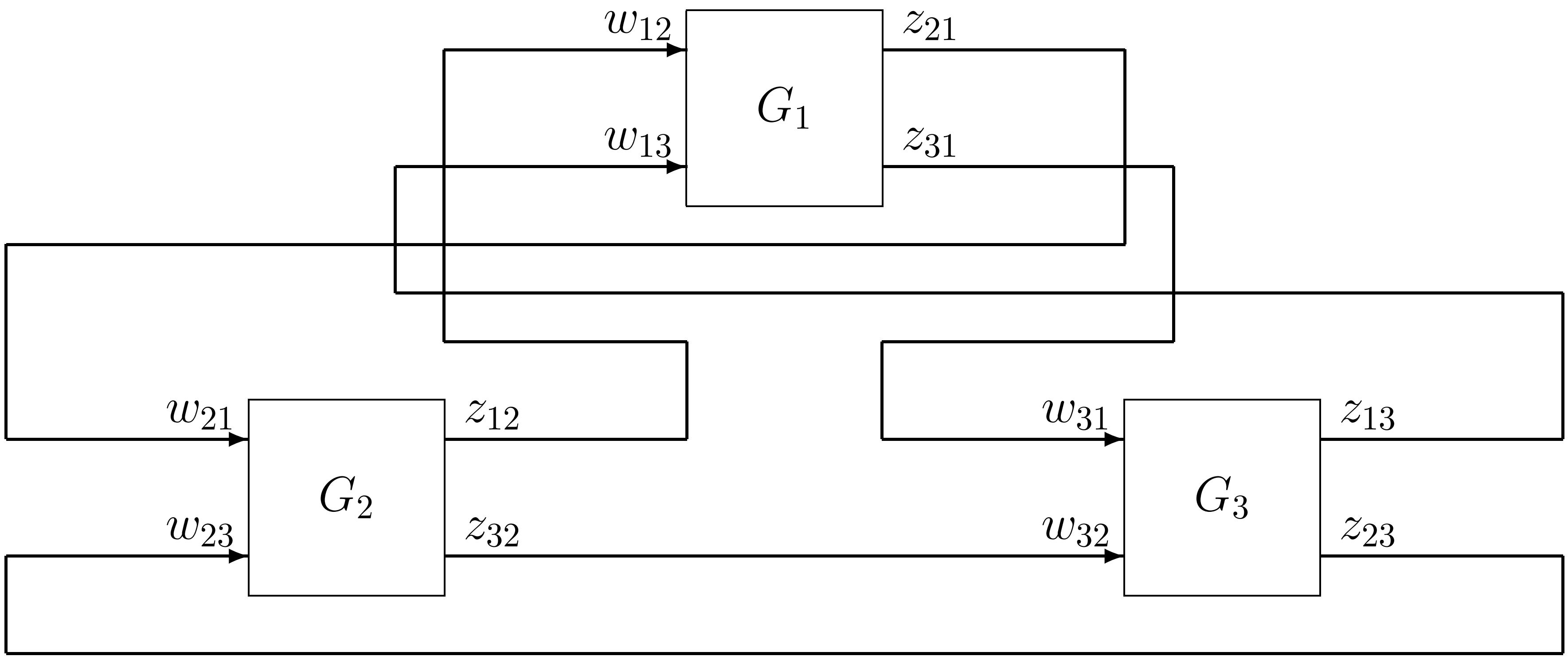

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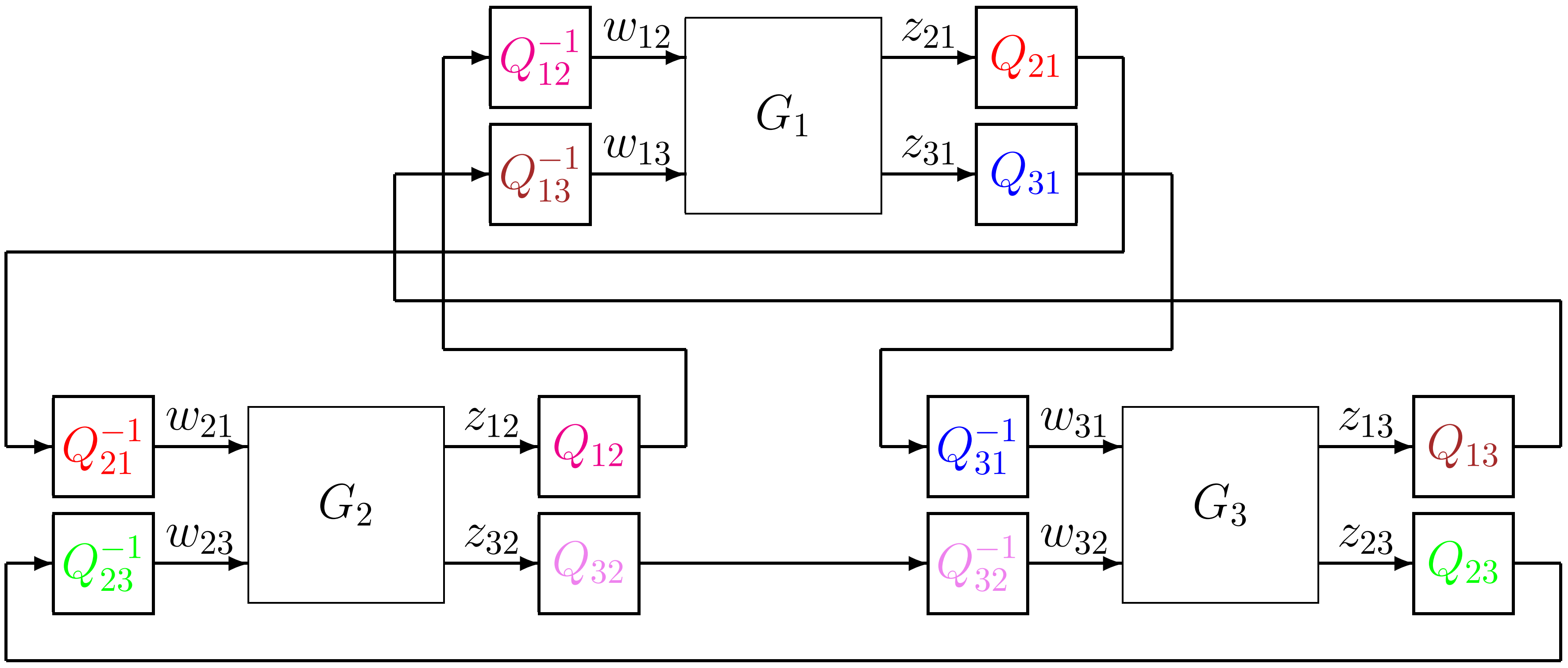

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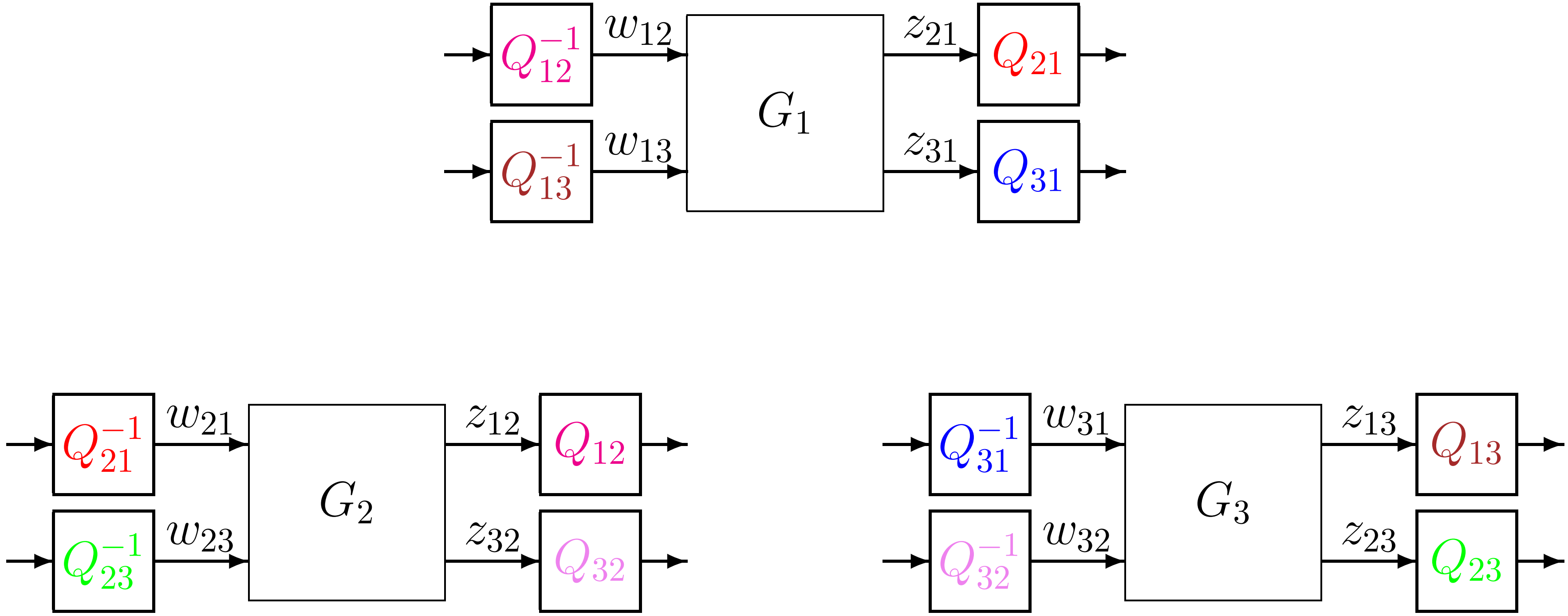


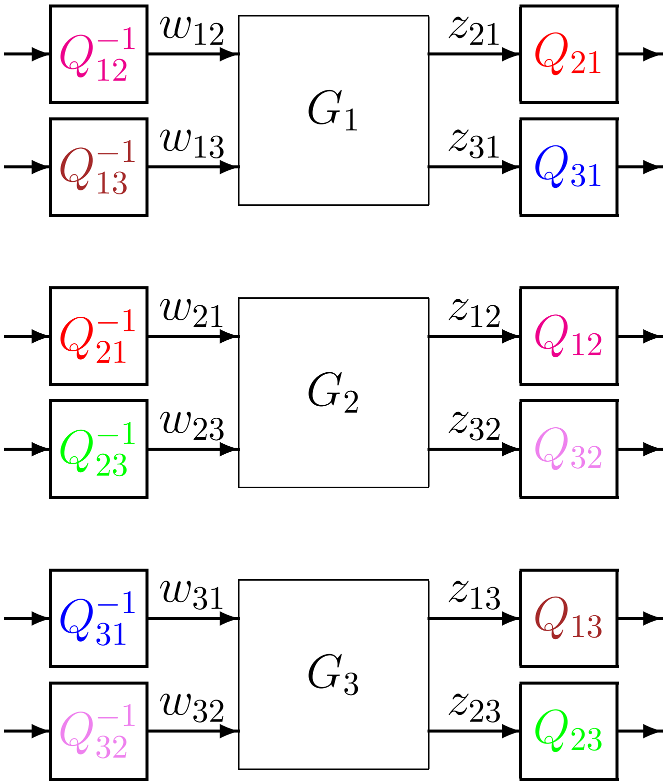
Stability Analysis of Interconnected Positive Systems

Example ($N = 3$)

$$\|G_{1,[q_{21}^T\ q_{31}^T]^T,[q_{12}^T\ q_{13}^T]^T}\|_{1+} < 1,$$

$$\|G_{2,[q_{12}^T\ q_{32}^T]^T,[q_{21}^T\ q_{23}^T]^T}\|_{1+} < 1,$$

$$\|G_{3,[q_{13}^T\ q_{23}^T]^T,[q_{31}^T\ q_{32}^T]^T}\|_{1+} < 1.$$



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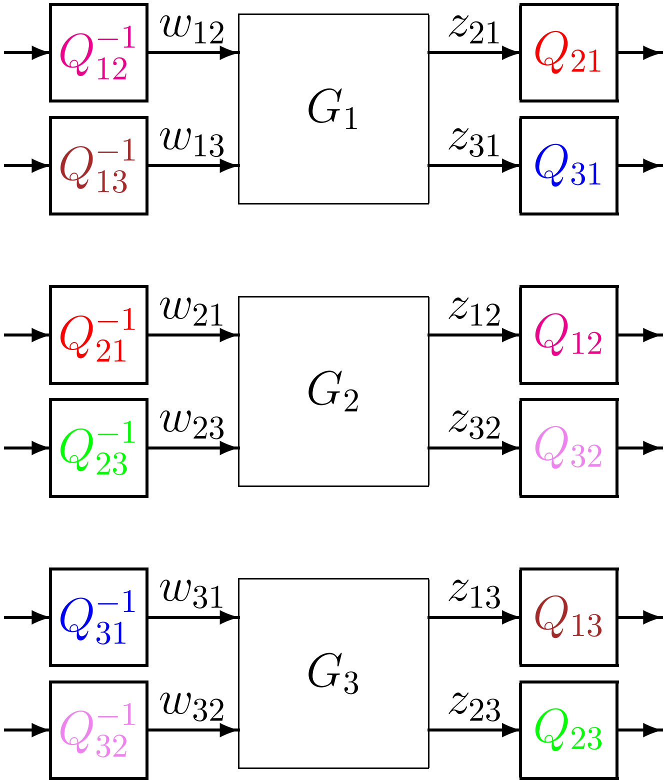

- necessary and sufficient condition for stability with L_1-induced norm of each subsystem
- coupled over subsystems through q_{ij}

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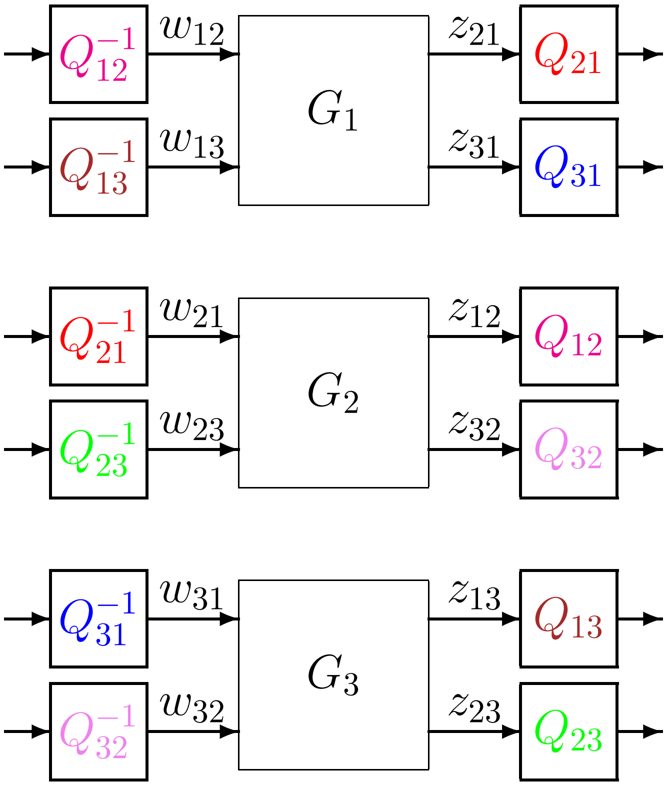

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Decentralized Stabilizing Synthesis

Decentralized Control of Interconnected Positive Systems

Subsystem G_i $(i = 1, \cdots, N)$

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Decentralized Control of Interconnected Positive Systems

Subsystem $G_i \ (i = 1, \cdots, N)$

$$G_i : \begin{cases} \dot{x}_i = A_i x_i + \displaystyle\sum_{j=1, j\neq i}^{N} B_{w_{ij}} w_{ij} + B_{u_i} u_i, \\ z_{ji} = C_{z_{ji}} x_i + D_{z_{ji} u_i} u_i \ (j \neq i) \end{cases}$$

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- achieve overall stability and positivity
- K_i can be designed purely locally

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practical but intractable requirement

Decentralized Control of Interconnected Positive Systems

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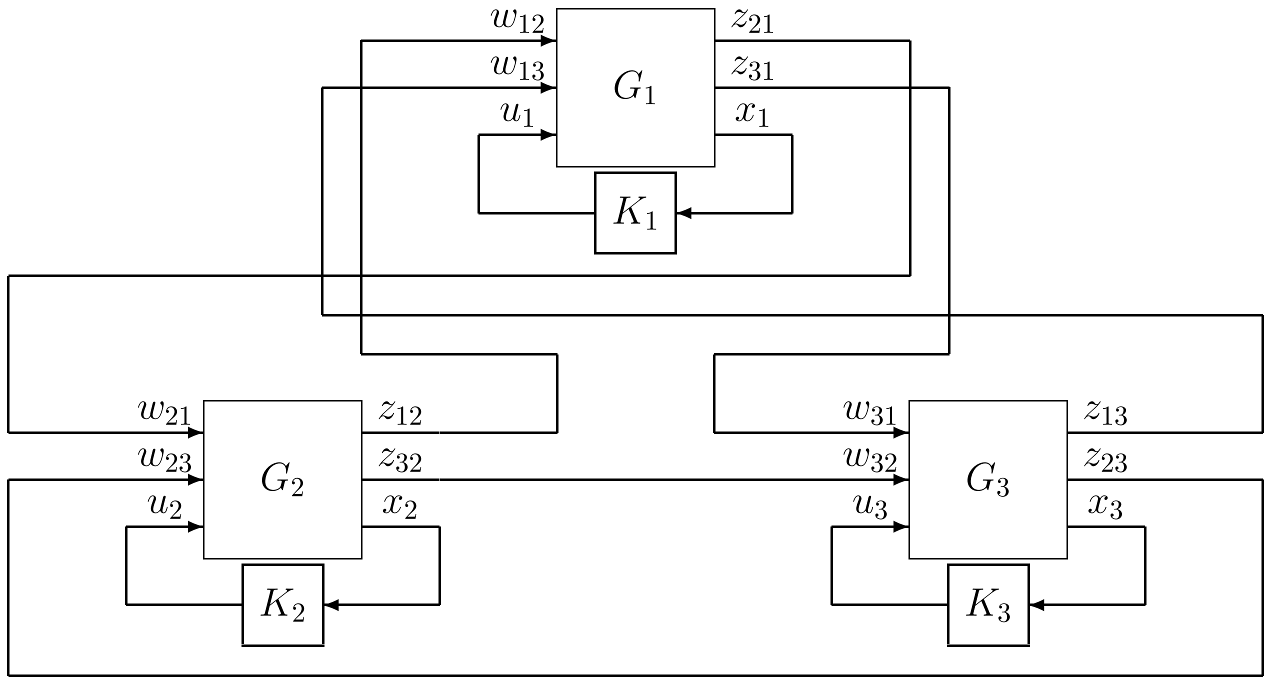

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Stability Condition

$$\|G_{i,K_i,q_{i,z},q_{i,w}}\|_{1+} < 1 \quad (i = 1, \cdots, N).$$

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How to solve?

Decentralized Control of Interconnected Positive Systems

Stability Condition of Interconnected PS's

$$\|G_{i,K_i,q_{i,z},q_{i,w}}\|_{1+} < 1 \quad (i = 1, \cdots, N).$$

What is the tractable network structure?

Decentralized Control of Interconnected Positive Systems

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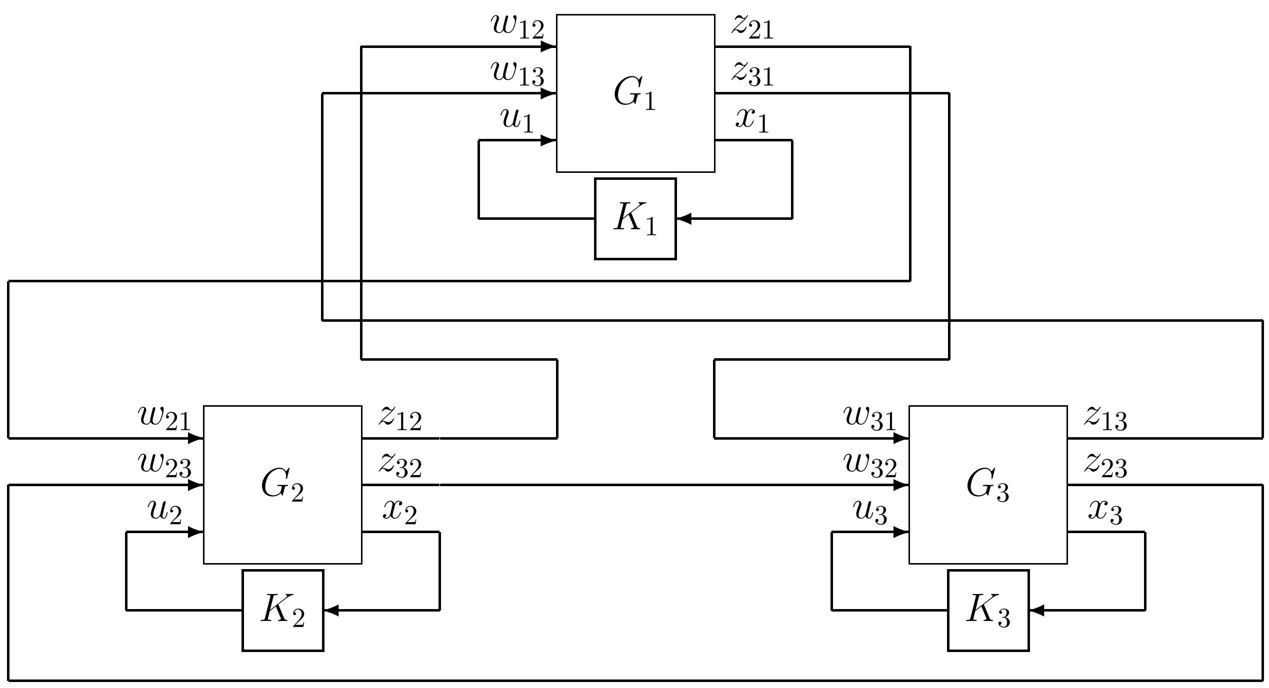
Tractable Network Structure

G_i provides the same scalar output to G_j $(j \neq i)$



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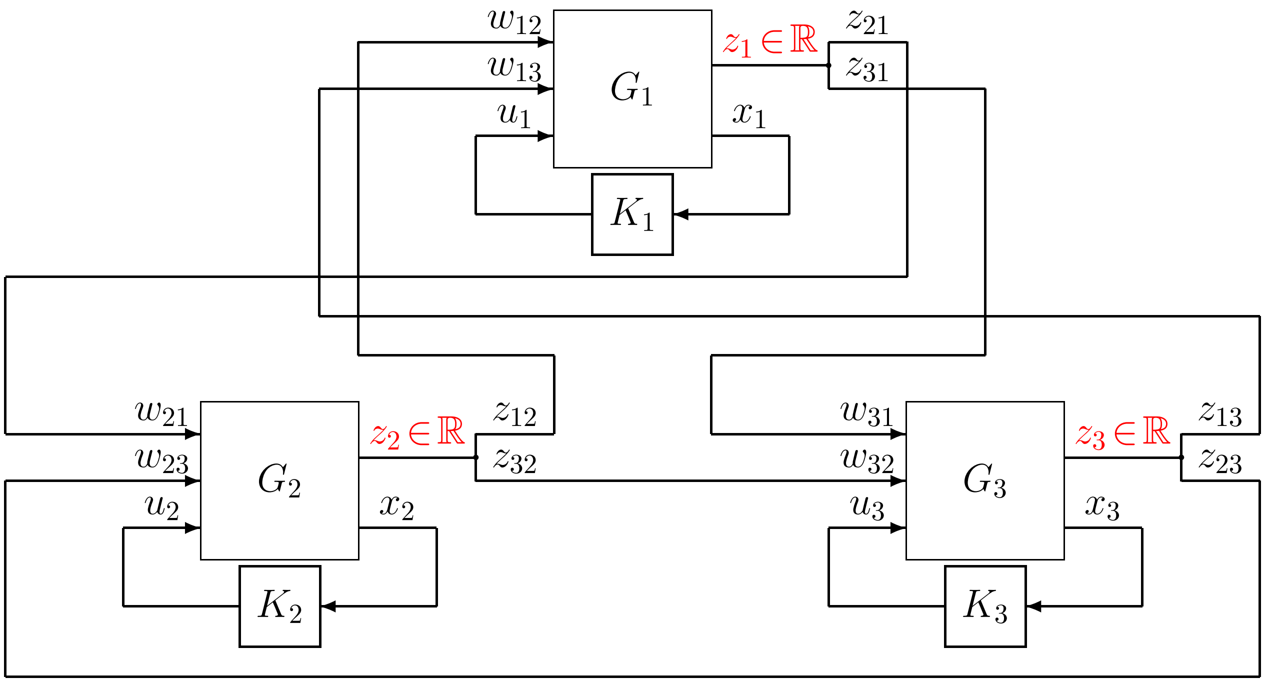

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Simplest Case: G_i $(i = 1, 2, 3)$ are all SISO and

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \Omega \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \ \Omega = \begin{bmatrix} 0 & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & 0 & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & 0 \end{bmatrix} \in \mathbb{R}_+^{3\times 3}.$$

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$$\Downarrow$$

$$\|q_{i,z}\|_1 \|G_{i,K_i,1,q_{i,w}}\|_{1+} < 1 \quad (i = 1, \cdots, N).$$

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Synthesis Problem

$$\inf_{K_i \in \mathcal{K}_{\mathrm{p},i}} \|G_{i,K_i,1,q_{i,w}}\|_{1+}$$ ($K_i \in \mathcal{K}_{\mathrm{p},i}$: positivity constraint)

L_1-induced Norm Optimal Control Problem

(optimal $q_{i,w}$ is not locally available)

Decentralized Control of Interconnected Positive Systems

L_1-induced Norm Optimal Control

$$\inf_{K_i \in \mathcal{K}_{\mathrm{p},i}} \|G_{i,K_i,1,q_{i,w}}\|_{1+},\ G_{i,K_i} = \left[\begin{array}{c|c} A_i + B_{u_i}K_i & B_{w_i} \\ \hline C_{z_i} + D_{z_iu_i}K_i & 0 \end{array}\right]$$

Decentralized Control of Interconnected Positive Systems

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Optimal Control for Normalized System

$$\inf_{K_i \in \mathcal{K}_{\mathrm{p},i}} \|G^{\mathrm{n}}_{i,K_i,1,\mathbf{1}}\|_{1+}, \quad G^{\mathrm{n}}_{i,K_i} = \left[\begin{array}{c|c} A_i + B_{u_i} K_i & \mathbf{1}_{n_i} \\ \hline C_{z_i} + D_{z_i u_i} K_i & 0 \end{array}\right]$$

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Preceding Results (Ebihara et al., ACC2012)

- $K_i^\star$ can be computed purely locally via LP
- $K_i^\star$ is robustly optimal for any $B_{w_i} \geq 0, \; q_{i,w} \geq 0$

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our primary objective has been achieved!!

Decentralized Control of Interconnected Positive Systems

L_1-induced Norm Optimal Control

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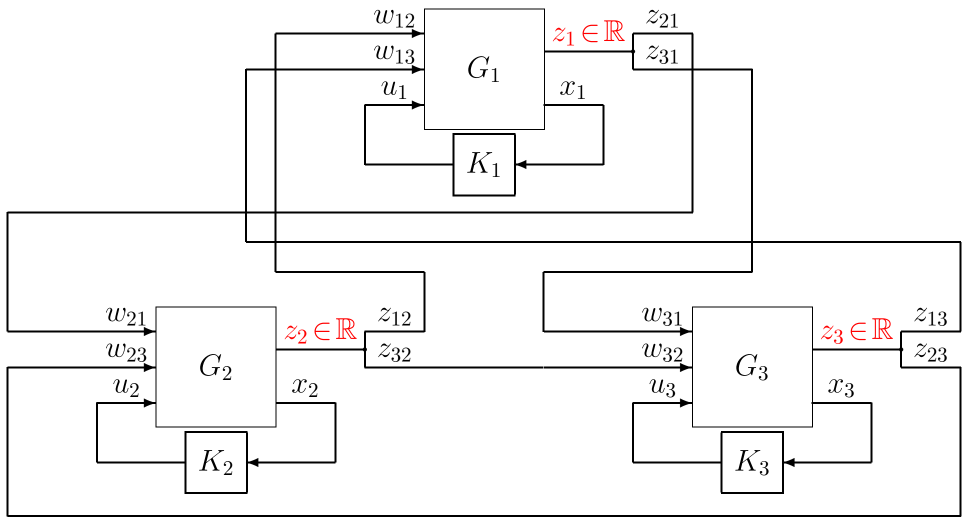
optimality of $K^\star = \{K_1^\star, \cdots, K_N^\star\}$ in global sense?

Main Result



$$\dot{x}(t) = A_K x(t),$$
$$K = \{K_1, \cdots, K_N\}$$

Main Result

Performance Index

$\lambda_{\mathrm{F}}(A_K)$: maximal real part of the eigenvalues of A_K

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$$\lambda_{\mathrm{F}}^{\star} := \inf_{K \in \mathcal{K}_{\mathrm{d}} \cap \mathcal{K}_{\mathrm{p}}} \lambda_{\mathrm{F}}(A_K)$$

$\mathcal{K}_{\mathrm{d}}$: decentralized structural constraint

$\mathcal{K}_{\mathrm{p}}$: positivity constraint

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$$\lambda_{\mathrm{F}}(A_{K^\star}) = \lambda_{\mathrm{F}}^{\star}$$

$K^\star = \{K_1^\star, \cdots, K_N^\star\}$ is optimal for $\lambda_{\mathrm{F}}(A_K)$

($K_i^\star$: locally L_1-ind.-norm-optimal controller)

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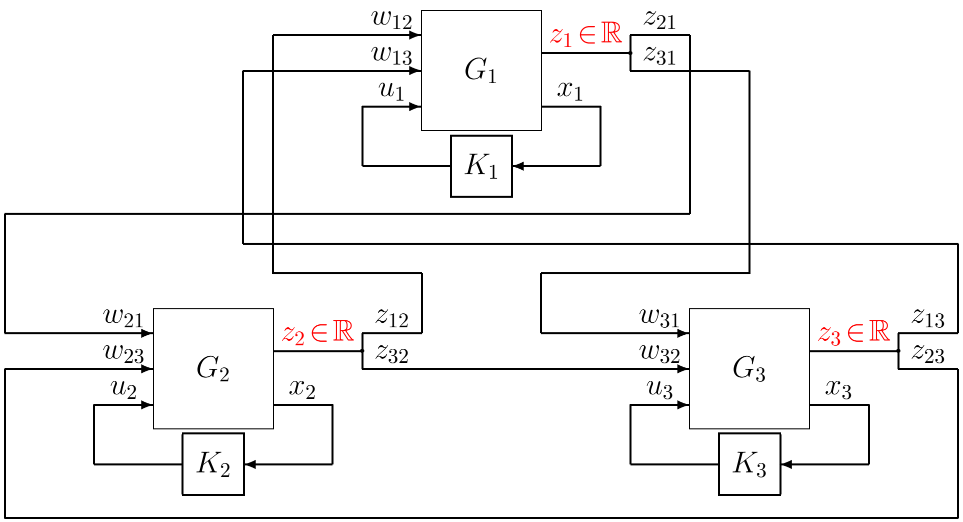
Remark

- $K^\star$ can also be designed by centralized approach

$$\sup_{K\in\mathcal{K}_{\mathrm{d}}\cap\mathcal{K}_{\mathrm{p}}, g, \alpha} \alpha \text{ s.t. } g \gg 0, \ (A_K + \alpha I)g \ll 0.$$

  - needs bisection search over $\alpha \in \mathbb{R}$
  - computationally demanding ($g \in \mathbb{R}^{\sum n_i}$)

Numerical Example 1 (Toy Example)



- $n_1 = 2, n_2 = 3, n_3 = 4$
- open-loop unstable ($\lambda_{\mathrm{F}}(A) = 1.1547$)

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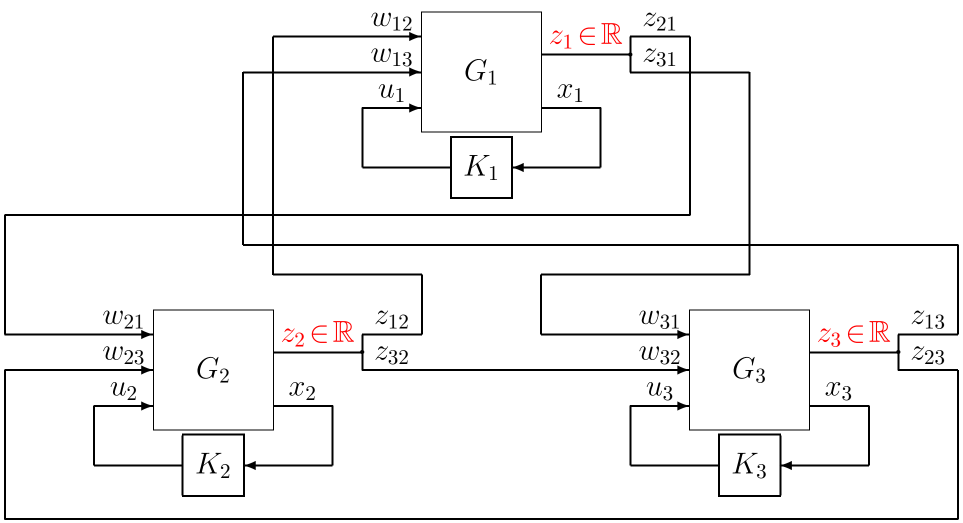

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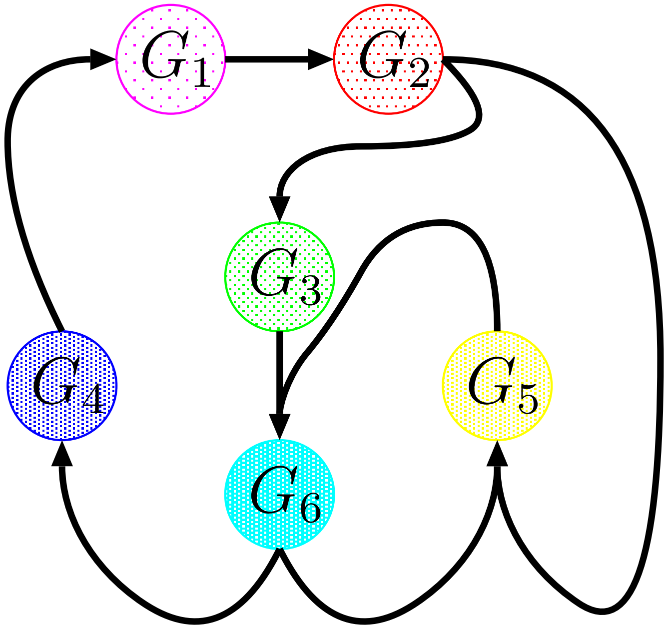
L_1-ind.-norm Optimal Gains

$$K_1^\star = \begin{bmatrix} -0.4444 & -2.0000 \end{bmatrix},$$
$$K_2^\star = \begin{bmatrix} -0.8000 & -0.3333 & -1.0000 \end{bmatrix},$$
$$K_3^\star = \begin{bmatrix} -0.5000 & -0.1111 & -0.1111 & -0.2222 \end{bmatrix}.$$

- $\lambda_{\mathrm{F}}^\star = \lambda_{\mathrm{F}}(A_{K^\star}) = -0.3056 \ (K^\star = \{K_1^\star, K_2^\star, K_3^\star\})$
- coincides with centralized approach with computationally demanding bisection search

Numerical Example 2 (Control Node Selection)

Interconnected PS with Directed Network



- node i:
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- edge (i, j): weights $\Omega_{i,j}$

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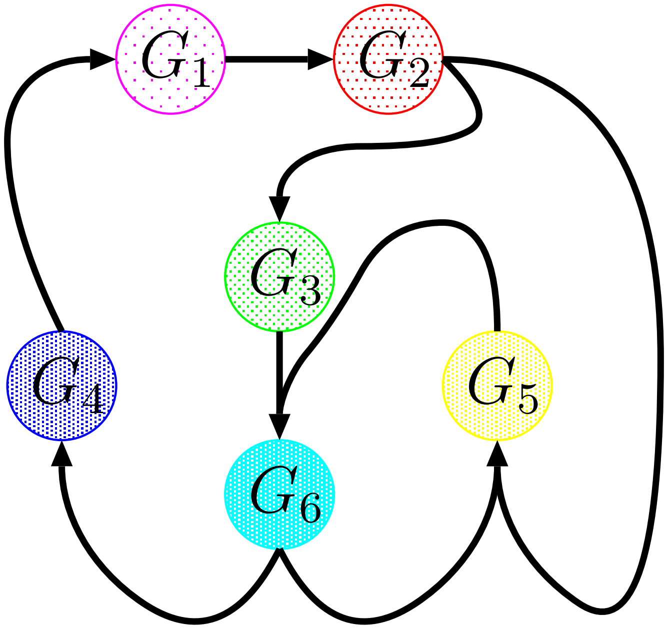

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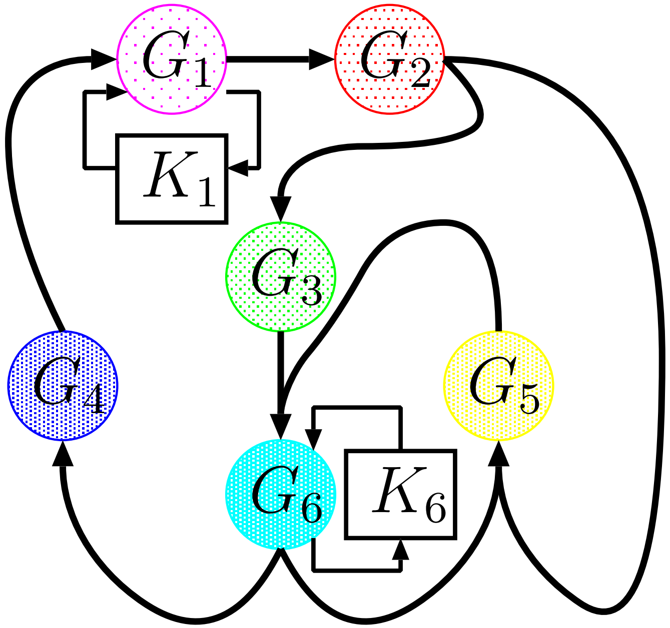

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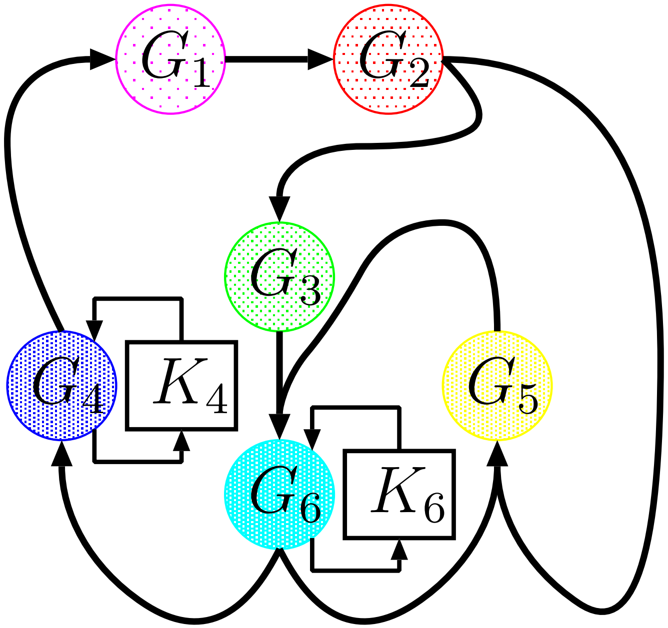

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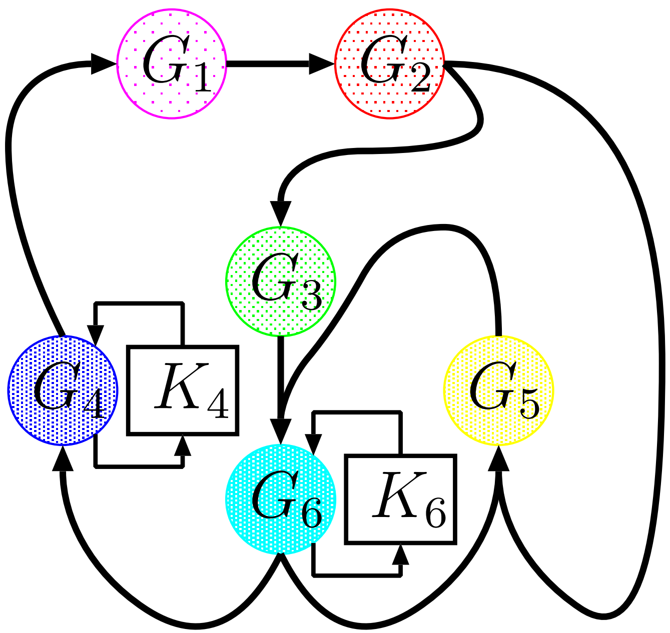

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Problem Find N_c nodes $\mathcal{N}_c = \{n_1, \cdots, n_{N_c}\}$ and SF gains K_{n_i} $(i = 1, \cdots, N_c)$ such that controlled interconnected system is positive and stable

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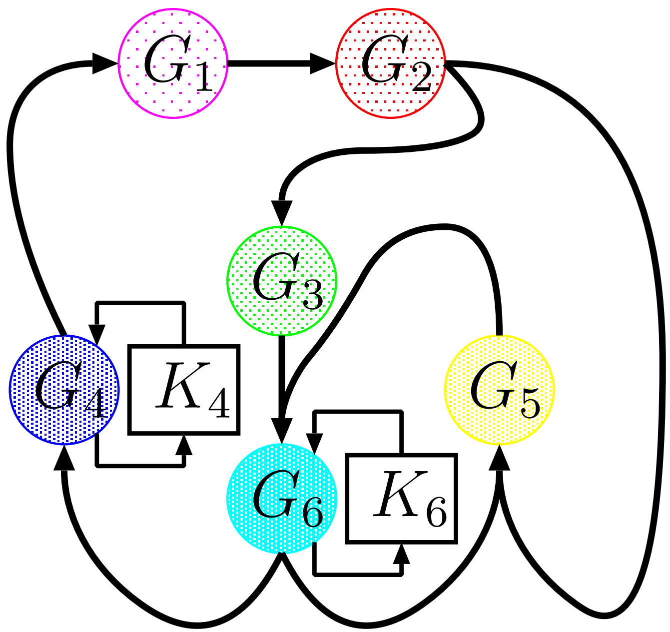

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Centralized Approach

For each selected N_c nodes $\mathcal{N}_c$, solve

find $K_{\mathcal{N}_c} \in \mathcal{K}_\mathrm{d} \cap \mathcal{K}_\mathrm{p}$ and g s.t. $g \gg 0, \ A_{K_{\mathcal{N}_c}} g \ll 0.$

$$K_{\mathcal{N}_c} = \begin{bmatrix} K_1 & 0 & \cdots & 0 \\ 0 & K_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & K_N \end{bmatrix} \quad (K_i = 0 \text{ if } i \notin \mathcal{N}_c)$$

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change of variable $Y = K_{\mathcal{N}_c}\mathrm{diag}(g)$ ➤ LP

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LP is feasible ➤ stability and positivity achieved

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need to solve synthesis problem ${}_N C_{N_c}$ times

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Proposed Approach

Step 1: Compute $K_i^\star \; (i = 1, \cdots, N)$

- $K_i^\star$: locally L_1-ind.-norm-optimal controller
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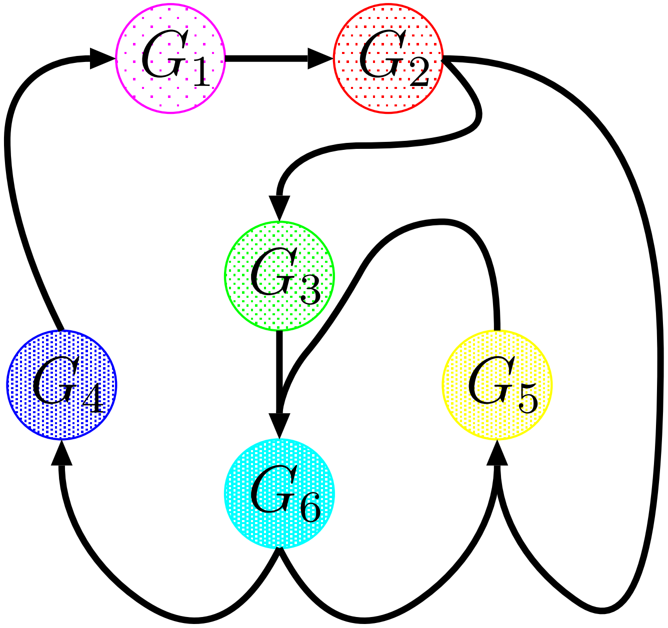

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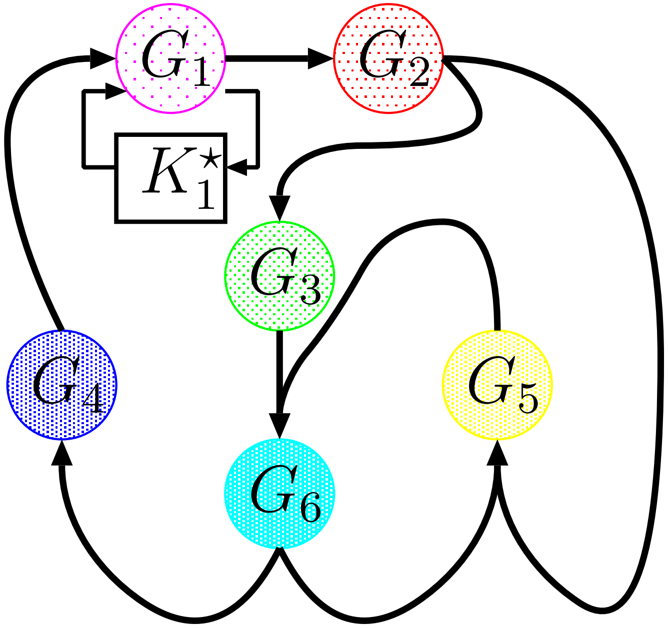

- If node 1 is selected, $K_1^\star$ is the best for G_1

Numerical Example 2 (Control Node Selection)

Proposed Approach

Step 1: Compute $K_i^\star \ (i = 1, \cdots, N)$

- $K_i^\star$: locally L_1-ind.-norm-optimal controller
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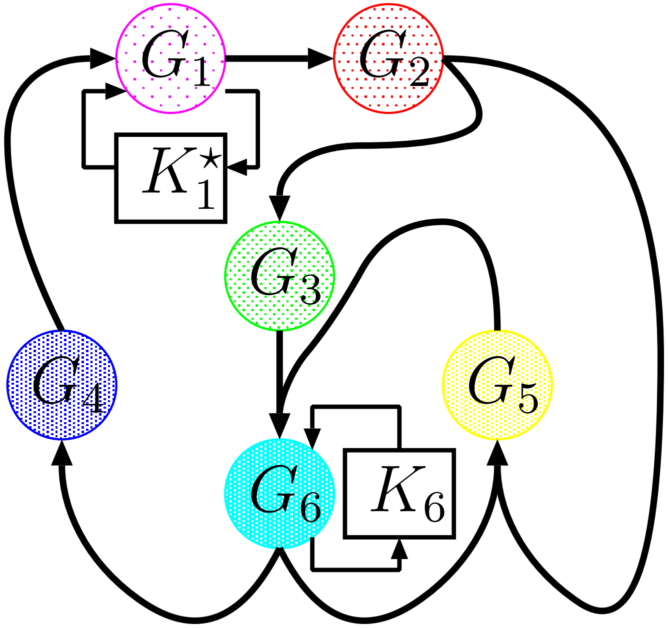

- If node 1 is selected, $K_1^\star$ is the best for G_1
- If nodes 1 and 6 are selected, $K_1^\star$ is still the best for G_1 irrespective of K_6

Numerical Example 2 (Control Node Selection)

Proposed Approach

Step 1: Compute $K_i^\star \ (i = 1, \cdots, N)$

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- computationally efficient (LP with $g_i \in \mathbb{R}^{n_i}$)

For each selected N_c nodes $\mathcal{N}_c$, the best controller is

$$K_{\mathcal{N}_c}^\star = \begin{bmatrix} K_1^\star & 0 & \cdots & 0 \\ 0 & K_2^\star & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & K_N^\star \end{bmatrix} \quad (K_i^\star = 0 \text{ if } i \notin \mathcal{N}_c)$$

Numerical Example 2 (Control Node Selection)

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Step 2: For each selected N_c nodes $\mathcal{N}_c$,
compute $\lambda_{\mathrm{F}}(A_{K^\star_{\mathcal{N}_c}})$

Numerical Example 2 (Control Node Selection)

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$\lambda_{\mathrm{F}}(A_{K^\star_{\mathcal{N}_c}}) < 0$ ➤ stability and positivity achieved

Numerical Example 2 (Control Node Selection)

Proposed Approach

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Step 2: For each selected N_c nodes $\mathcal{N}_c$,
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- computationally efficient

suffice to solve analysis problem ${}_N C_{N_c}$ times

Numerical Example 2 (Control Node Selection)

Comparison

Centralized Approach

- LP of each size $\sum n_i$ for $K^{\star}_{\mathcal{N}_c}$: ${}_N C_{N_c}$ times

Numerical Example 2 (Control Node Selection)

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- LP of each size $\sum n_i$ for $K^{\star}_{\mathcal{N}_c}$: ${}_N C_{N_c}$ times

Proposed Approach

- LP of each size n_i for $K^{\star}_i$: N times
- computation of $\lambda_{\mathrm{F}}(A_{K^{\star}_{\mathcal{N}_C}})$: ${}_N C_{N_c}$ times

Numerical Example 2 (Control Node Selection)

Comparison

Centralized Approach

- LP of each size $\sum n_i$ for $K^\star_{\mathcal{N}_c}$: ${}_N C_{N_c}$ times

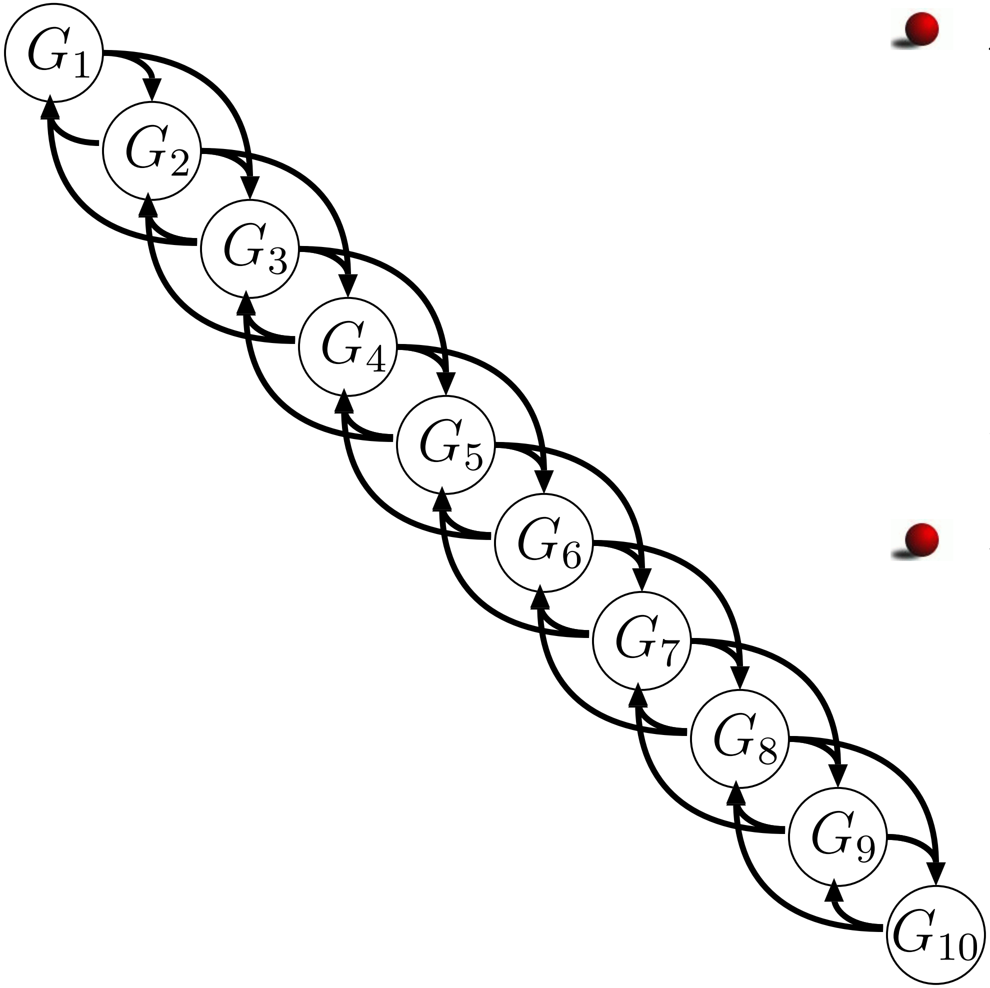
Proposed Approach

- LP of each size n_i for $K^\star_i$: N times
- computation of $\lambda_{\mathrm{F}}(A_{K^\star_{\mathcal{N}_C}})$: ${}_N C_{N_c}$ times

proposed approach is much more efficient

Numerical Example 2 (Control Node Selection)

Concrete Example

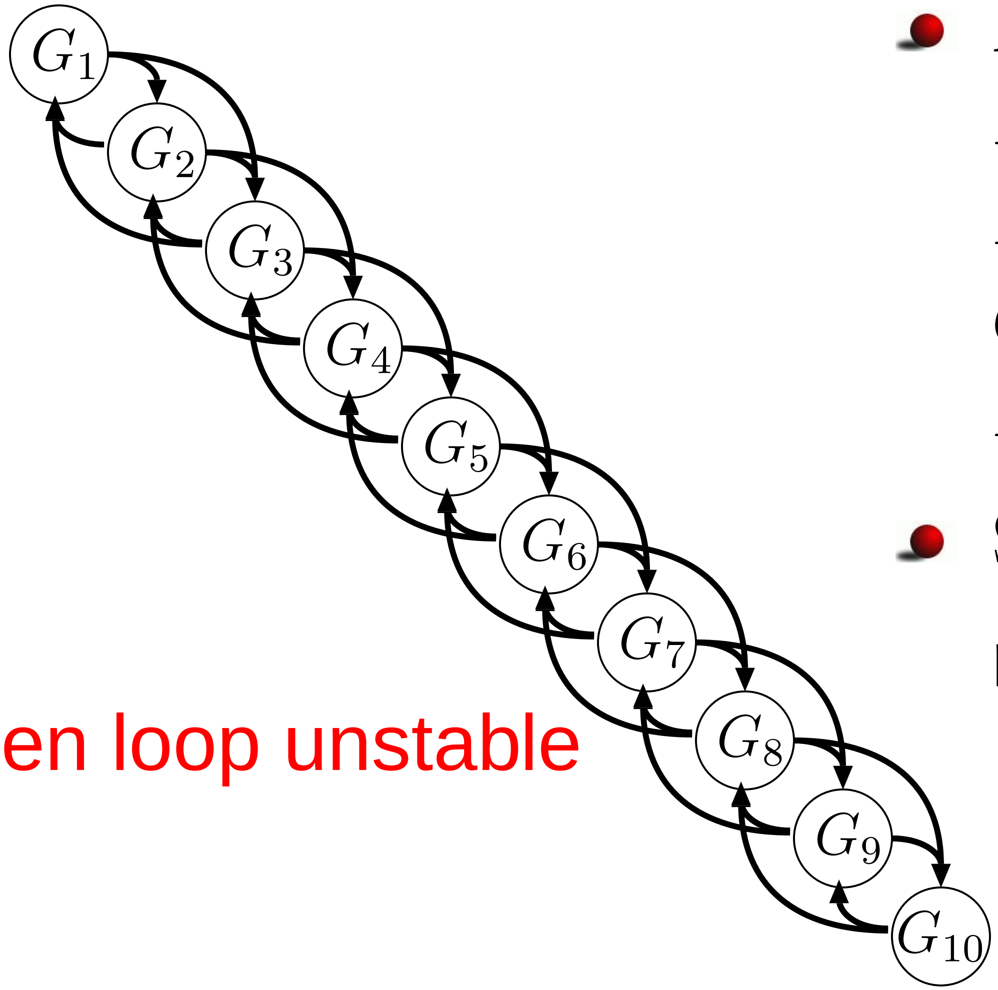


- $A_i \in \mathbb{M}^3 \cap \mathbb{H}^3$,
  $B_{w_i} \in \mathbb{R}_+^{3\times 1}$,
  $B_{u_i} \in \mathbb{R}^{3\times 1}$,
  $C_{z_i} \in \mathbb{R}_+^{1\times 3}$,
  $D_{z_i u_i} = 0$
- $\Omega_{i,j} > 0 \ (|i-j| \le 2)$

  randomly generated

Numerical Example 2 (Control Node Selection)

Concrete Example



open loop unstable

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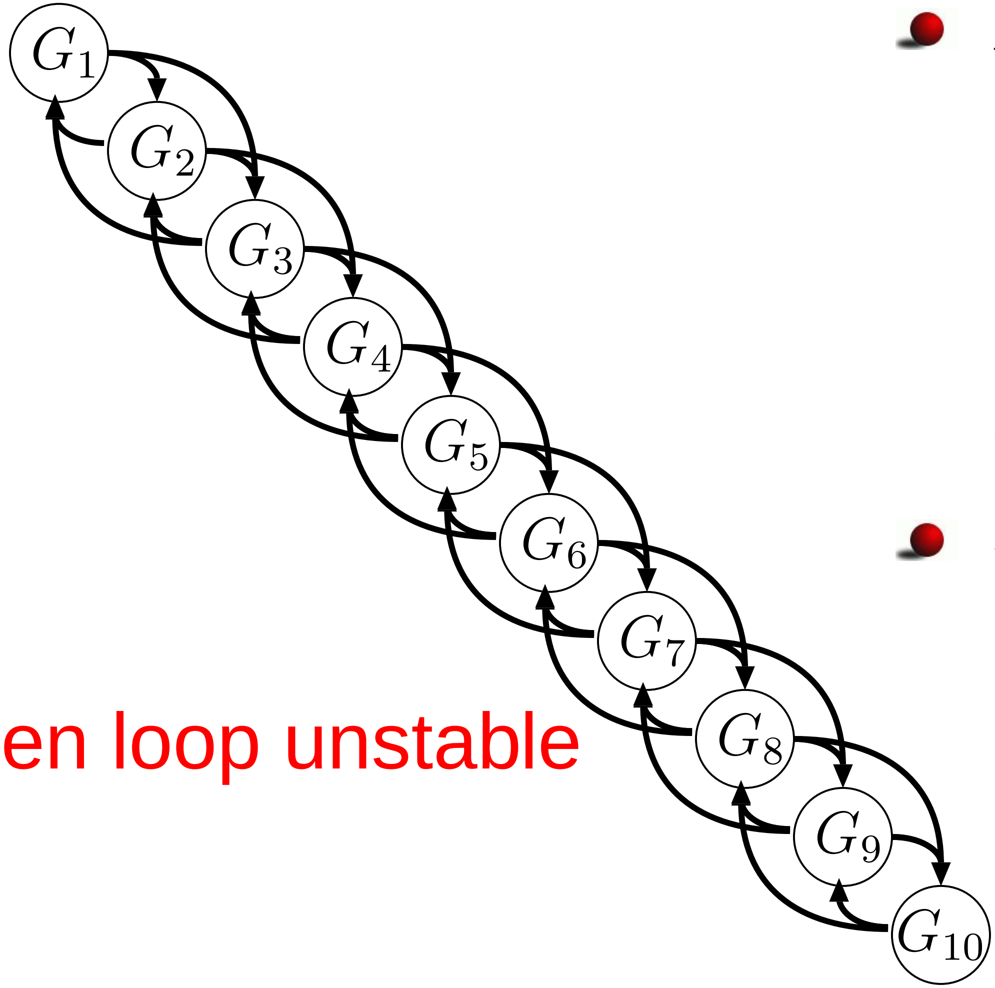

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randomly generated

select four nodes ($N_c = 4$)

Numerical Example 2 (Control Node Selection)

Results ($_{10}C_4 = 210$)

Centralized Approach: 37.80 [sec] ($\approx$0.18$\times$210)

Proposed Approach: 1.43 [sec] ($\approx$0.14$\times$10)

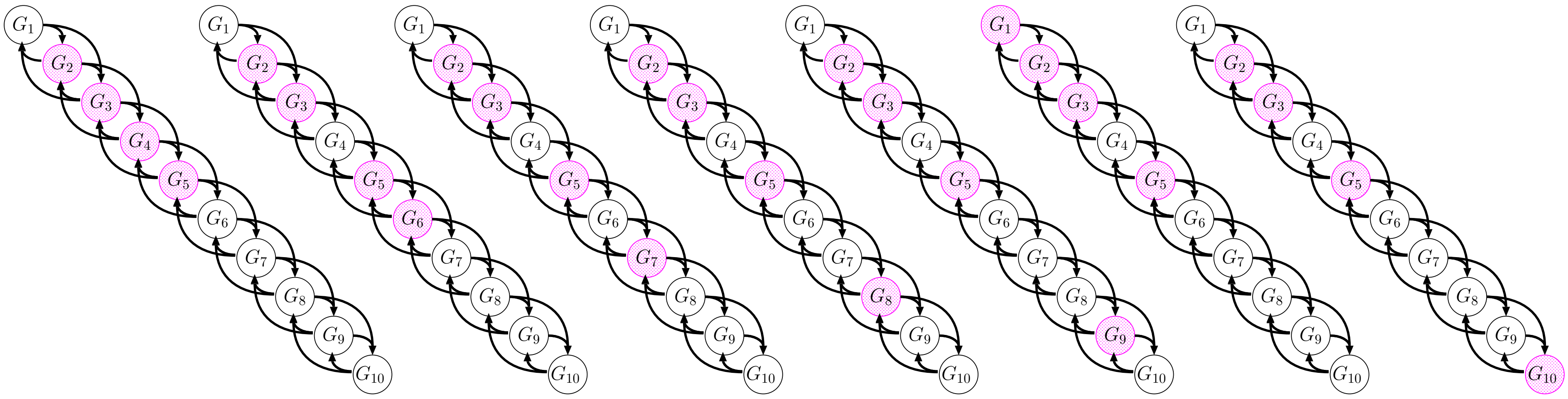
Numerical Example 2 (Control Node Selection)

Results (${}_{10}C_4 = 210$)

Centralized Approach: 37.80 [sec] (≈0.18×210)

Proposed Approach: 1.43 [sec] (≈0.14×10)

Selected Nodes (7 patterns)



$\lambda_{\rm F}:$ -0.1673 -0.1332 -0.1016 -0.0829 -0.0757 -0.0732 -0.0653

- nodes 2, 3, 5 are important

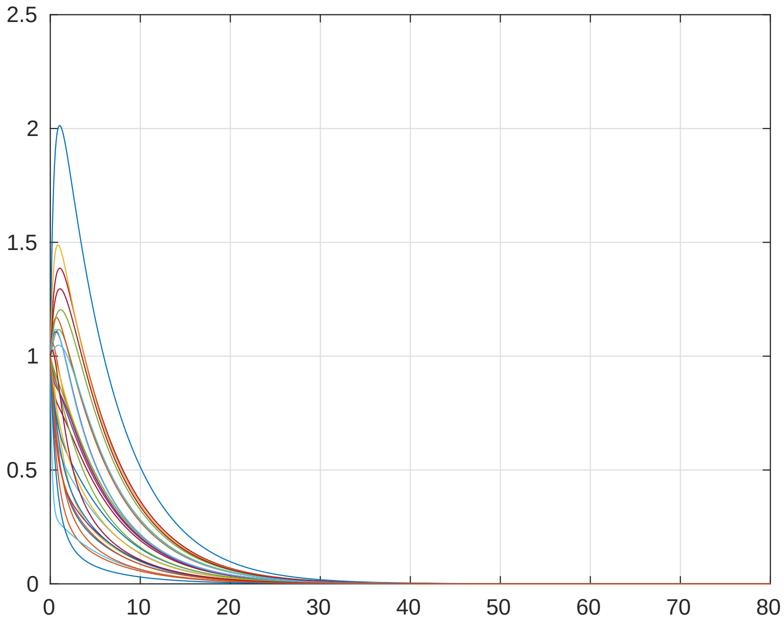
Numerical Example 2 (Control Node Selection)

Results (${}_{10}C_4 = 210$)

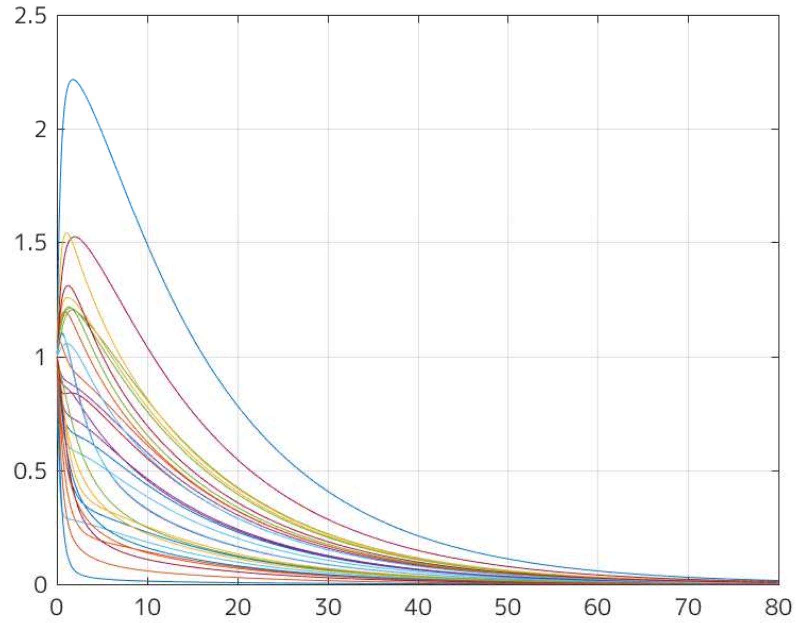
Centralized Approach: 37.80 [sec] ($\approx 0.18 \times 210$)

Proposed Approach: 1.43 [sec] ($\approx 0.14 \times 10$)

Initial Response with $x_i(0) = \mathbf{1}_3$



$\mathcal{N}_c = \{2, 3, 4, 5\}$



$\mathcal{N}_c = \{2, 3, 5, 10\}$

Conclusion

- Copositive Lyapunov Functions
- Weighted L_1-induced Norm
- Stability of Interconnected Positive Systems
  - characterized by weighted L_1-induced norm

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strong analysis results using positivity

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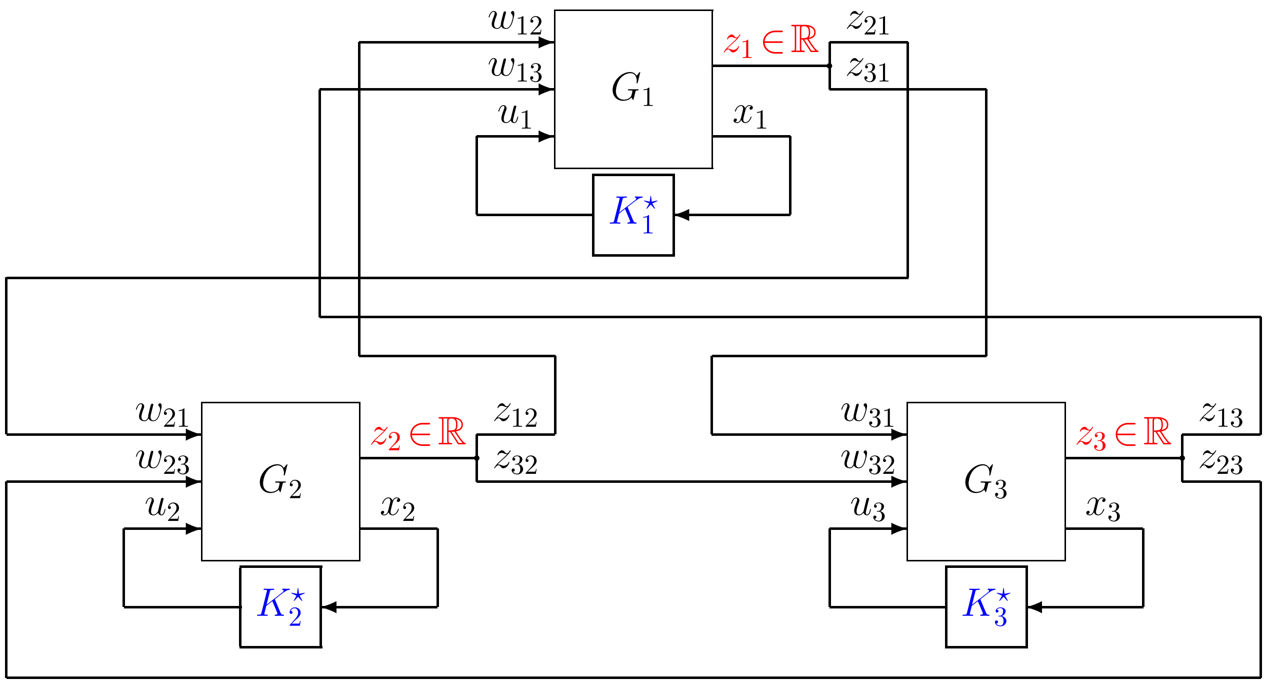
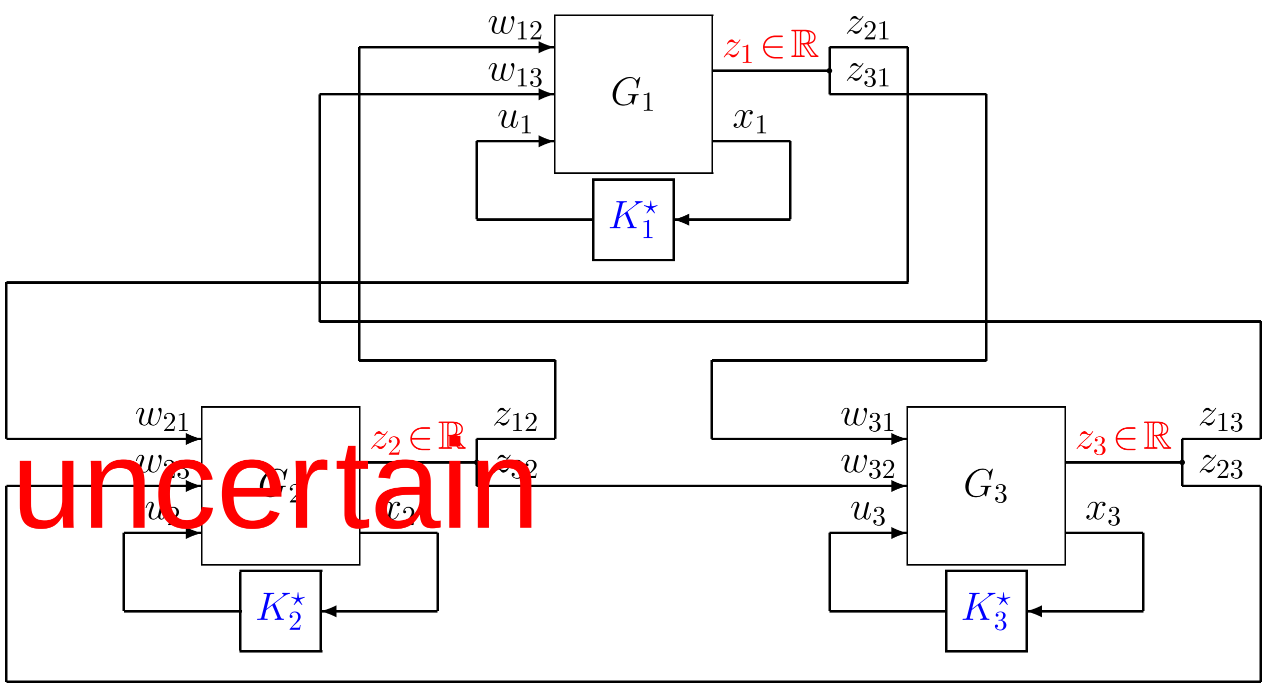


  - synthesis of $K_i^\star$ purely locally by LP
  - optimality property of $K^\star = \{K_1^\star, \cdots, K_N^\star\}$
    - ➤ minimizes $\lambda_{\mathrm{F}}(A_K)$

Conclusion

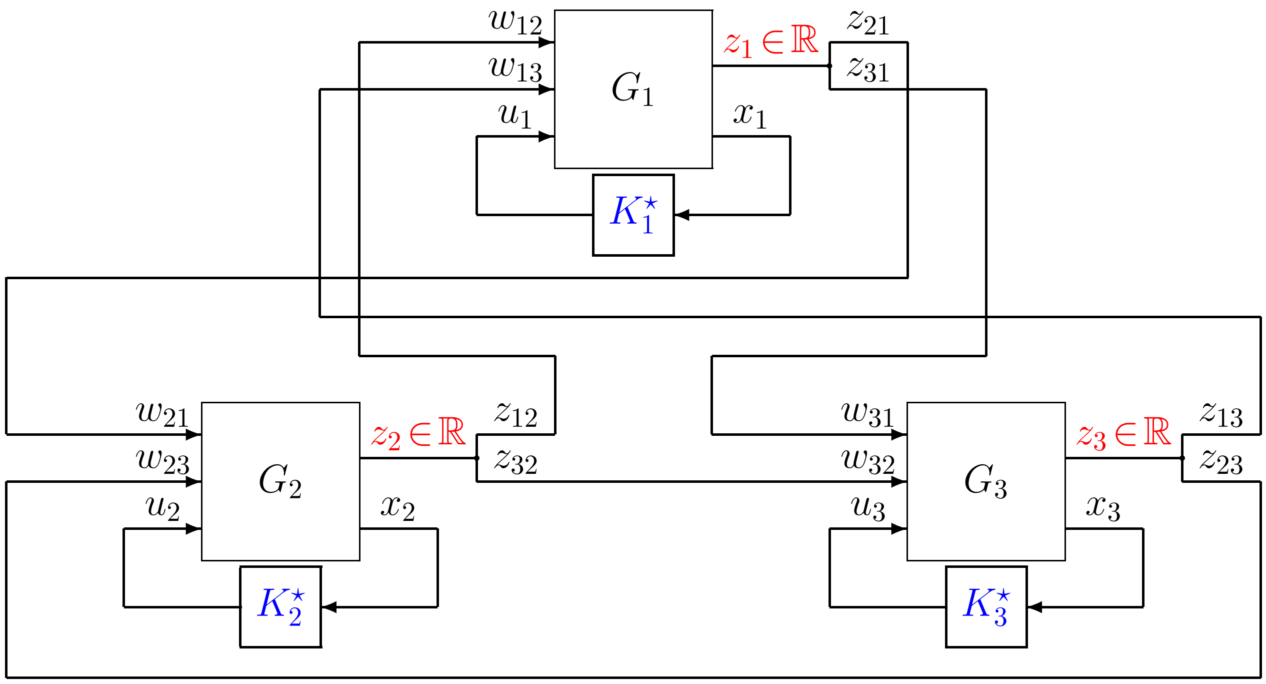
- Decentralized Stabilizing SF Synthesis



- $K_1^\star$ and $K_3^\star$ remains to be optimal even if G_2 is totally uncertain
- centralized approach infeasible

Conclusion

- Decentralized Stabilizing SF Synthesis



strong synthesis results
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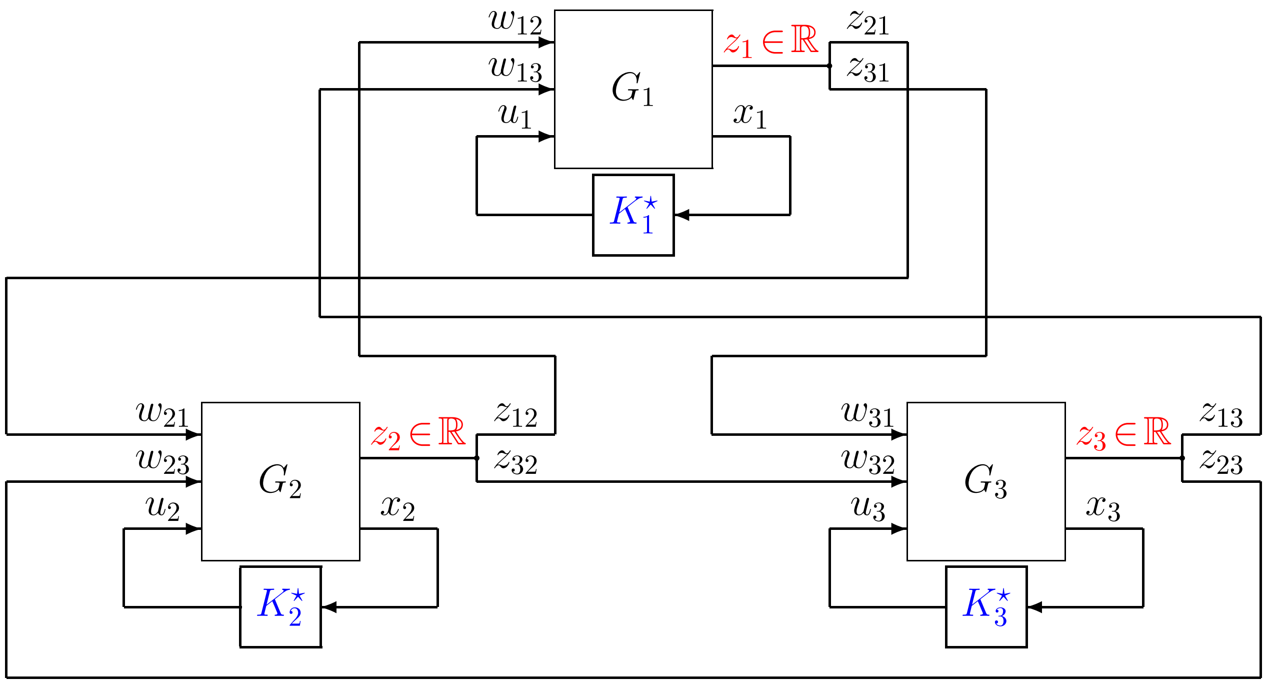
Conclusion

- Decentralized Stabilizing SF Synthesis



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