Fundamentals of digital signal processing

Sound modeling

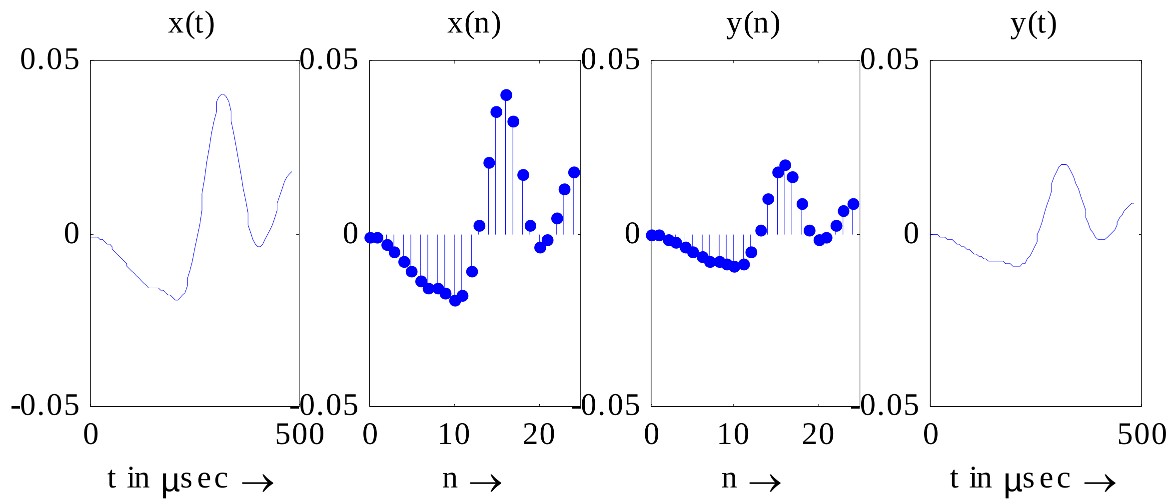
- sound
- symbols

- aims
  - analysis
  - synthesis
  - processing

- classification:
  - signal models
  - source models
  - abstract models

Digital signals

- analog
- sampling
- processing
- reconstruction

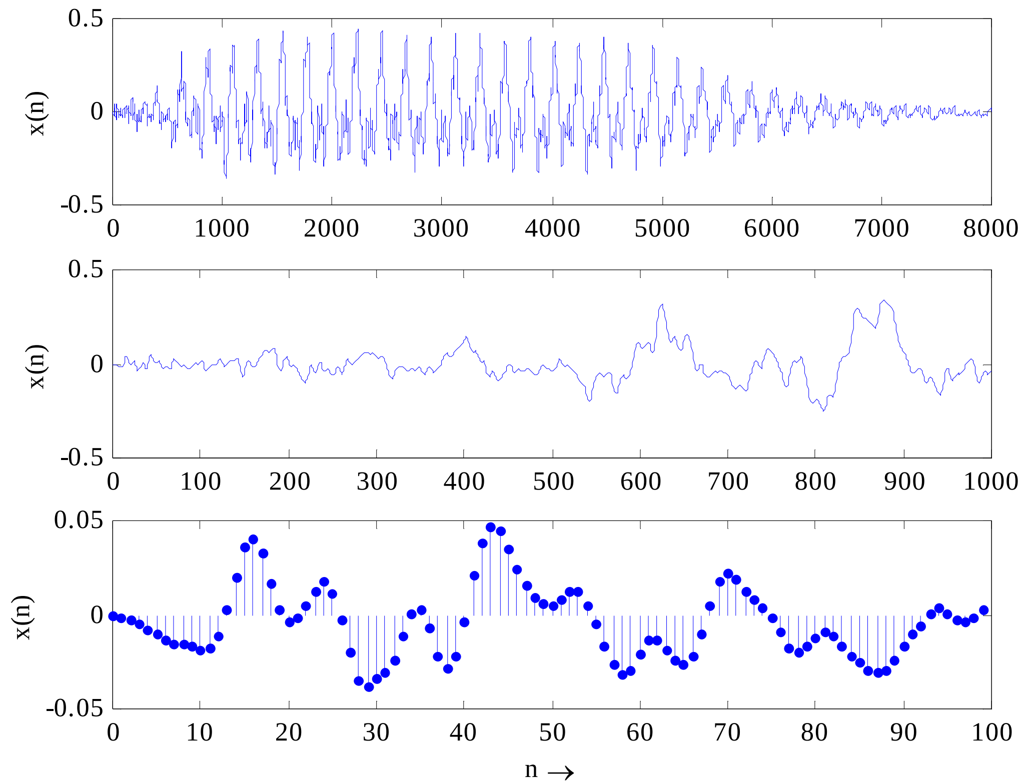


- sampling interval T
- sampling frequency $f_s = 1/T$

Digital signals: time representations

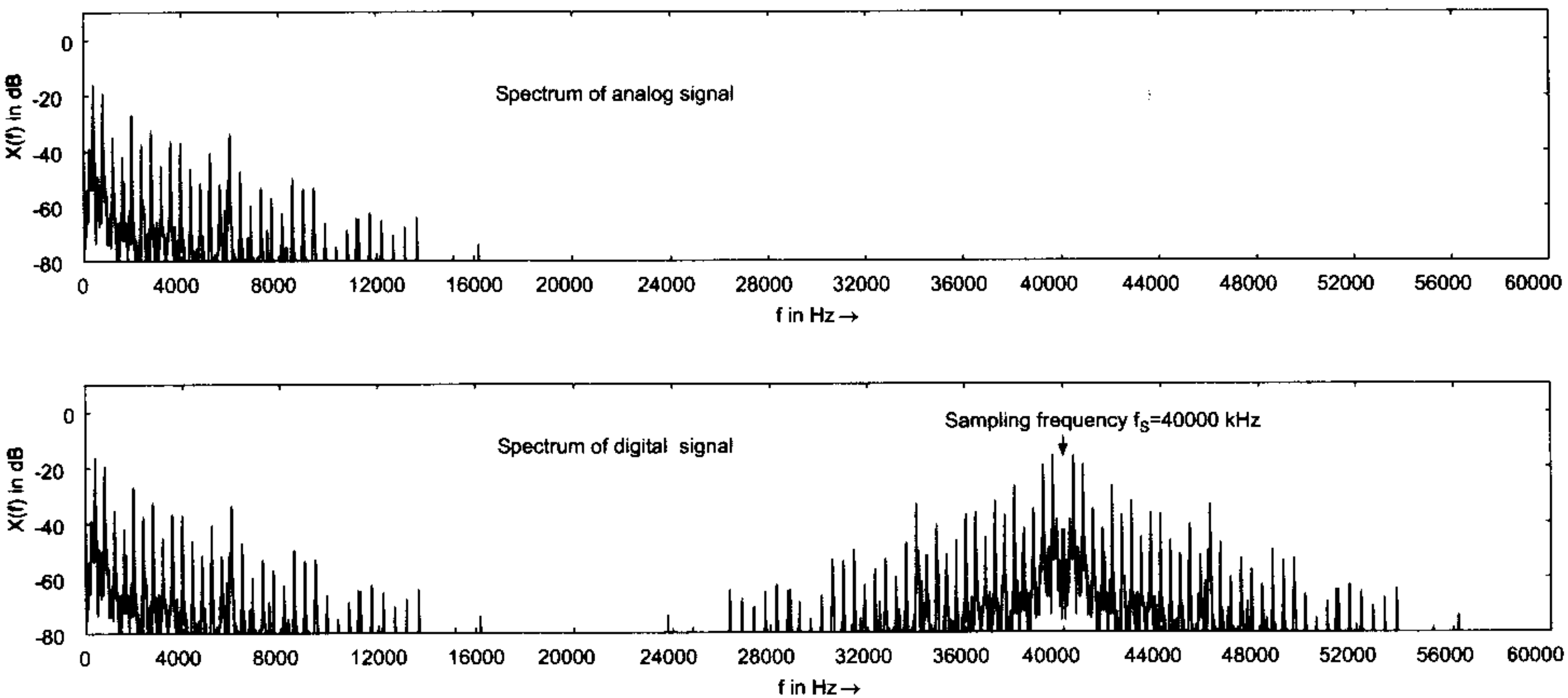
- 8000 samples
- 100 samples
- line with dots

- vertical quantization
  - integer
    e.g. -32768 .. 32767
  - normalized
    e.g. -1 .. (1-Q)
    Q = quantization step



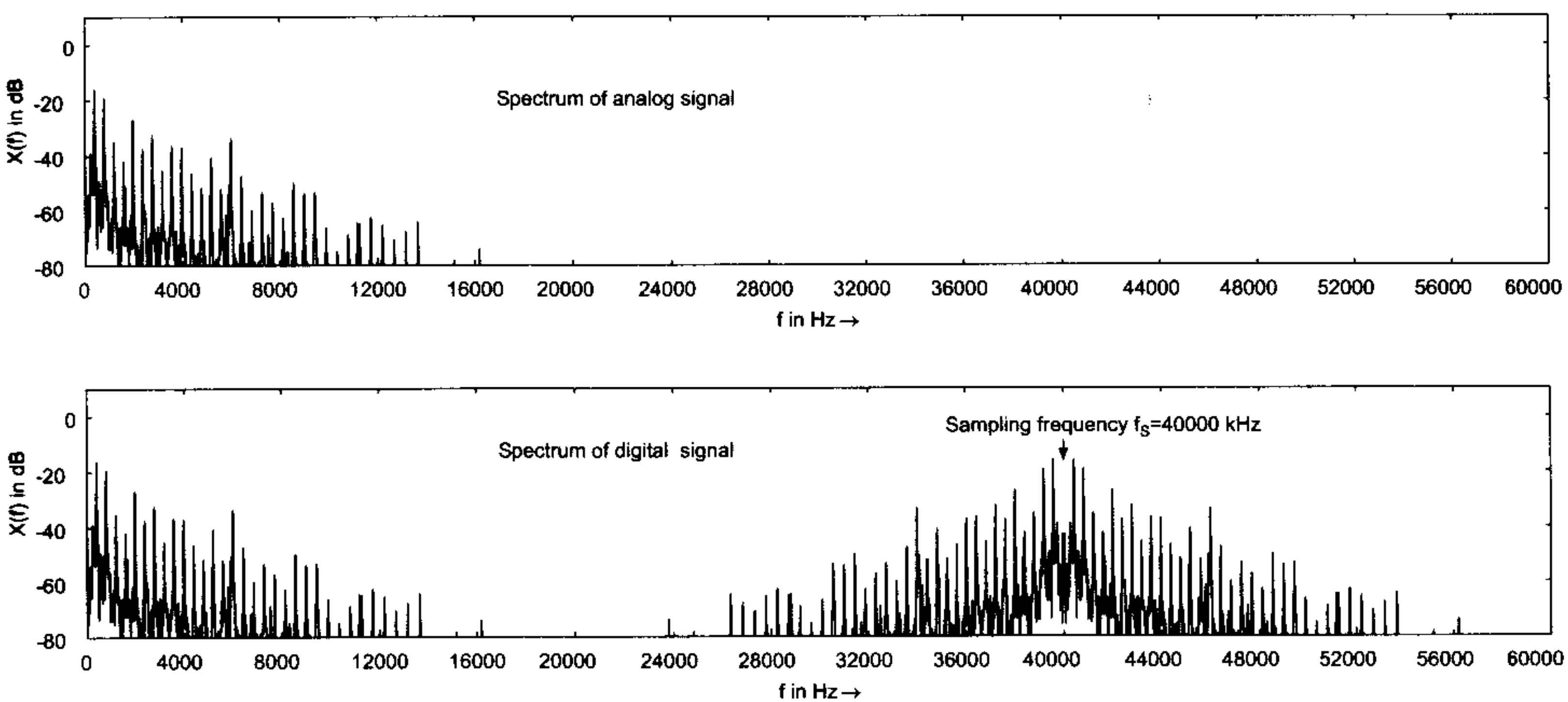
Spectrum: analog vs. digital signal

- sampling leads to a replication of the baseband spectrum



Spectrum: analog vs. digital signal

- Sampling leads to a replication of the analog signal spectrum
- Reconstruction of the analog signal:
  - low pass filtering the digital signal



Discrete Fourier Transform

$$X(k) = \mathrm{DFT}[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad k = 0, 1, \ldots, N-1.$$

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X=abs(fft(x,N))/N;
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- Magnitude

$$|X(k)| = \sqrt{X_R^2(k) + X_I^2(k)} \quad k = 0, 1, \ldots, N-1$$

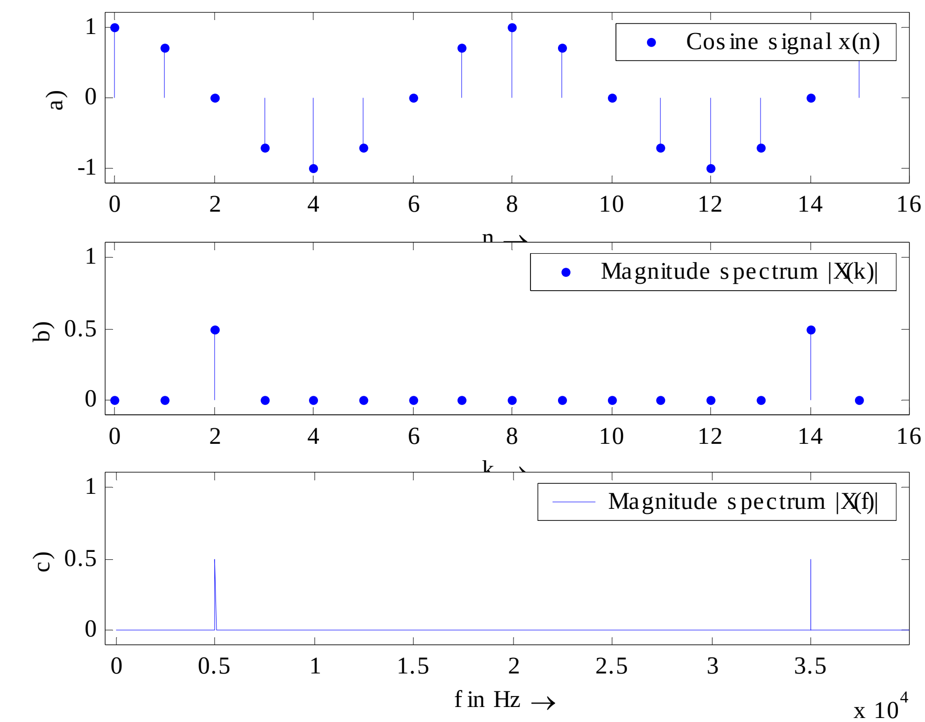
- Phase

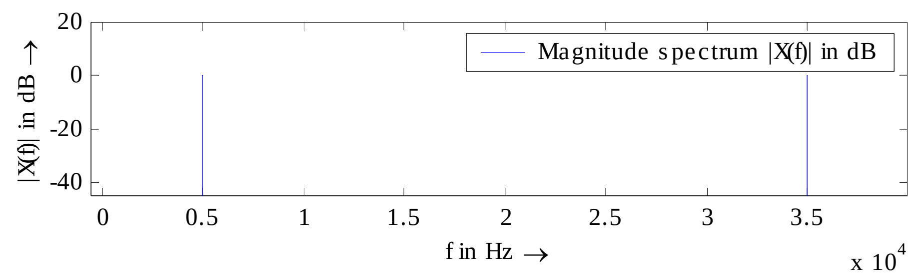
$$\varphi(k) = \arctan \frac{X_I(k)}{X_R(k)} \quad k = 0, 1, \ldots, N-1$$

Discrete Fourier Transform (example)

- FFT with 16 points
  - cosine (16 points)
  - magnitude (16 points)
    - normalization:
      0 dB for sinusoid ±1
  - magnitude
    (frequency points)
    - kf_s / N
    - step f_s/N
  - magnitude dB vs. Hz

$$20 \log_{10}\left(\frac{X(k)}{N/2}\right)$$





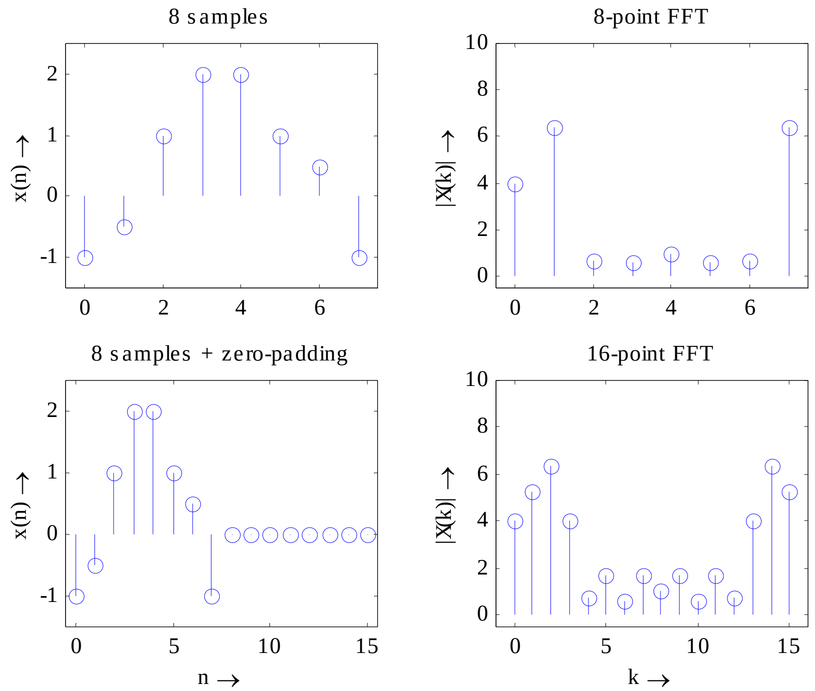
Inverse Discrete Fourier Transform (DFT)

$$x(n) = \text{IDFT}[X(k)] = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N}$$

- if *X(k) = X*(N-k)*
  then IDFT gives N discrete-time real values *x(n)*

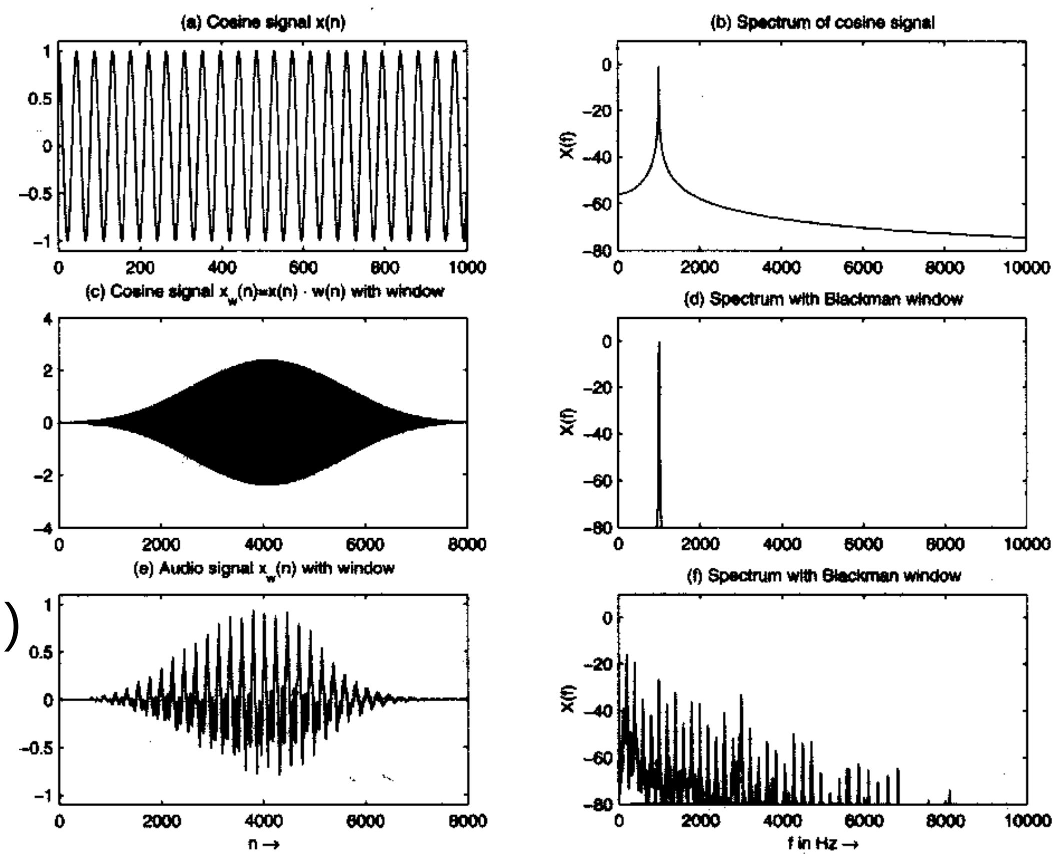
Frequency resolution

- Zero padding: to increase frequency resolution



Window functions

- to reduce leakage:
  - weight audio samples by a window



- Hamming window
  $w_H(n) = 0.54 – 0.46 \cos(2\pi n/N)$

$$w_H(n) \;=\; 0.54 - 0.46\cos(2\pi n/N)$$

- Blackman window
  $w_B(n) = 0.42 – 0.5\cos(2\pi n/N) + 0.08(4\pi n/N)$

Window

Reduction of the leakage effect by window functions:

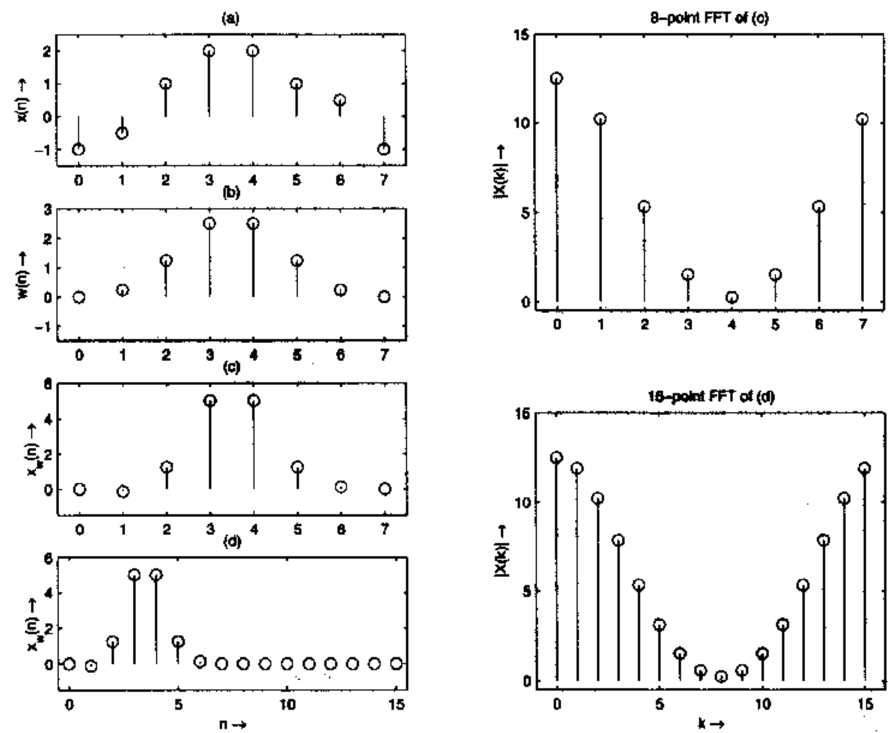
(a) the original signal,

(b) the Blackman window function of length *N* =8,

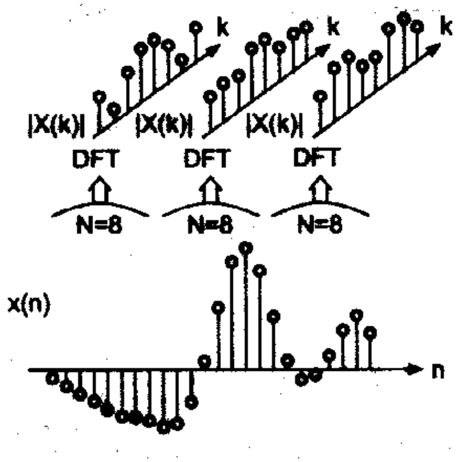
(c) product *x(n)w(n)* with $0 \leq n \leq N\text{-}i$,

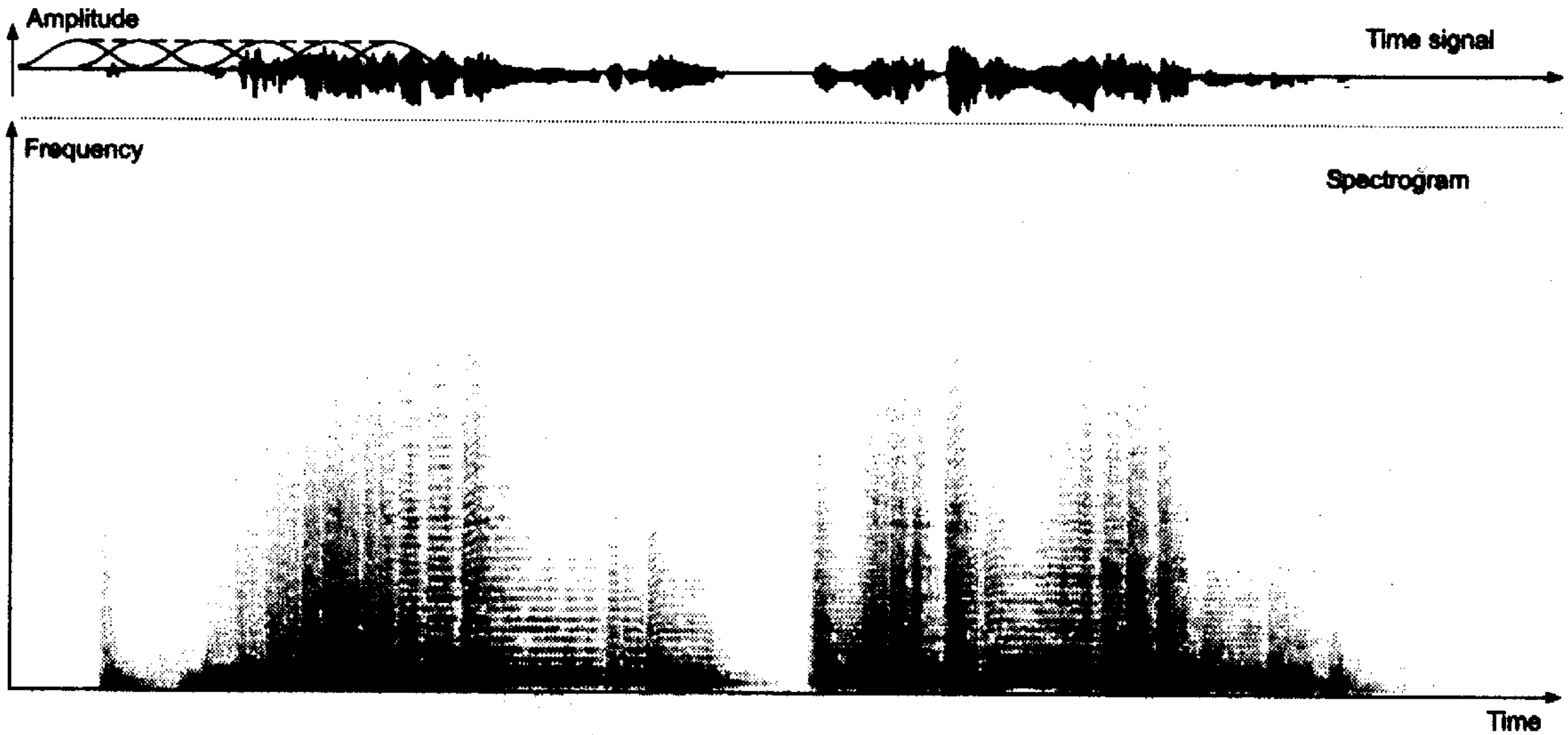
(d) zero-padding applied to *z(n) w(n)* up to length *N* = 16

The corresponding spectra are shown on the right side.

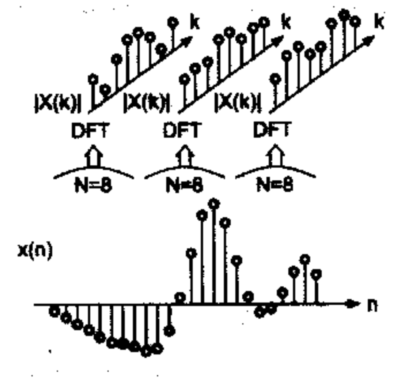


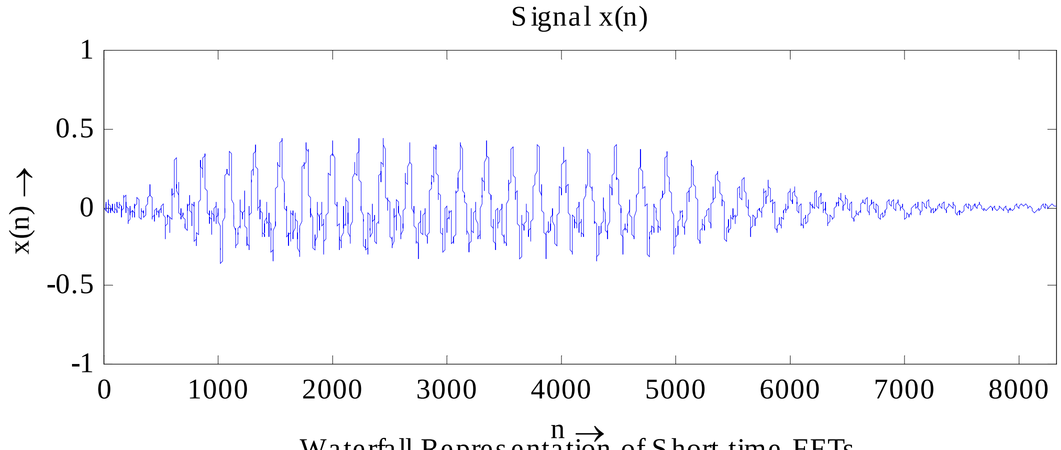
Spectrograms

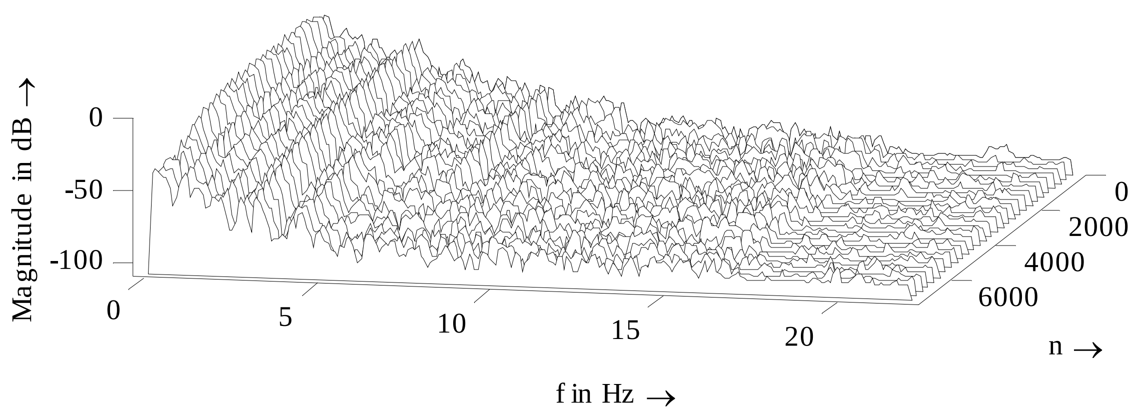




Waterfall representation







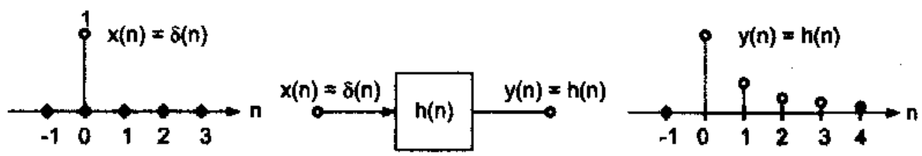
Digital systems

Definitions

- Unit impulse

$$\delta(n) = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0, \end{cases}$$

- Impulse reponse *h(n)* = output to a unit impulse
  h(n) describes the digital sistem



- Discrete convolution:
  *y(n)=x(n)*h(n)*

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k) = x(n) * h(n),$$

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y=conv(x,h)
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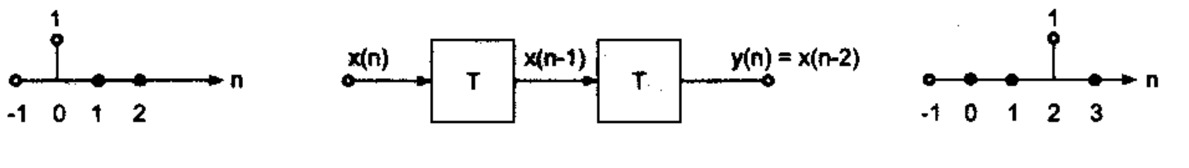
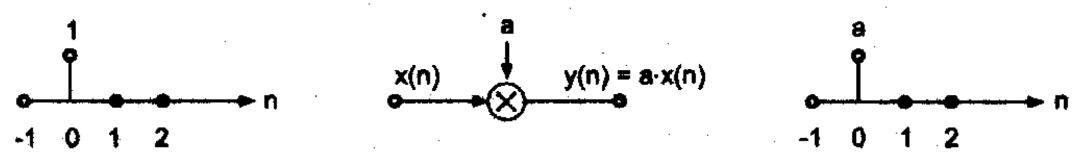
Algorithms and signal graphs

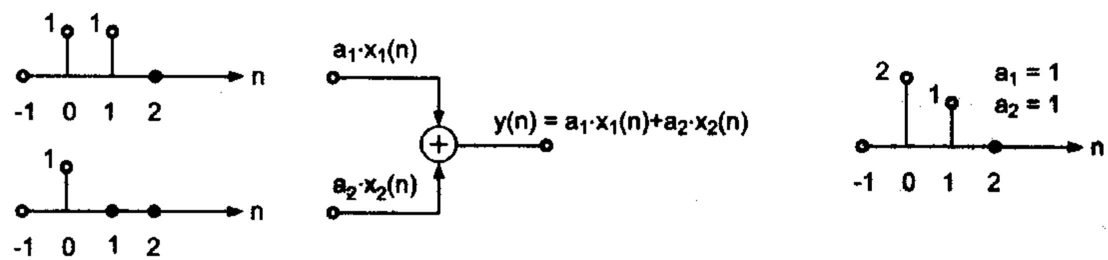
- Delay
  - e.g. *y(n) = x(n-2)*



- Weighting factor
  - e.g. *y(n) = a x(n)*

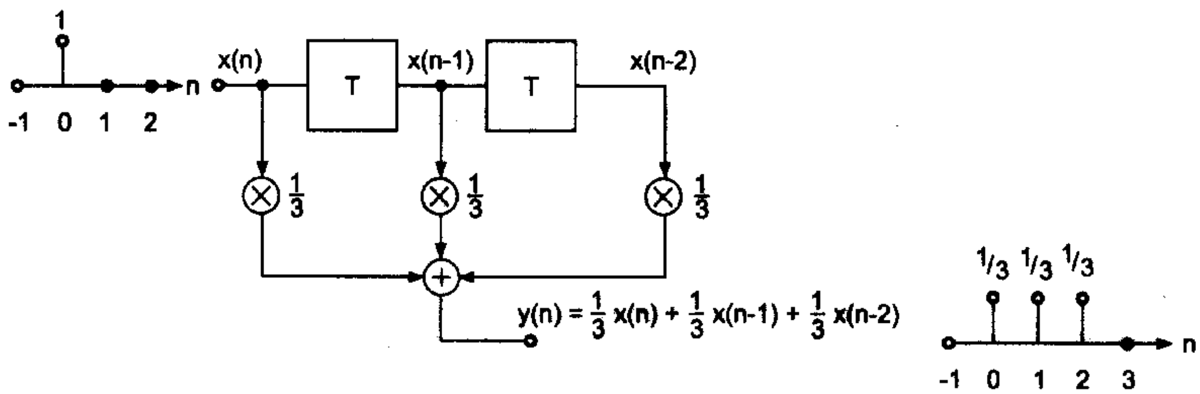


- Addition
  - e.g.
    y(n) = a1 x(n) + a2 x(n)



Simple digital system

- weighted sum over several input samples



Transforms

- Frequency domain desciption of the digital system

- Z transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$z \leftrightarrow e^{j\Omega}$$

- Discrete time Fourier transform

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\Omega n},$$

$$\text{with} \quad \Omega = \omega T = 2\pi f / f_s$$

- Transfer function *H(z)*: Z transform of *h(n)*

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) \cdot z^{-n}$$

- Frequency response: Discrete time Fourier transform of *h(n)*

$$H(f) = \sum_{n=-\infty}^{\infty} h(n) \cdot e^{-j2\pi f / f_s n}$$

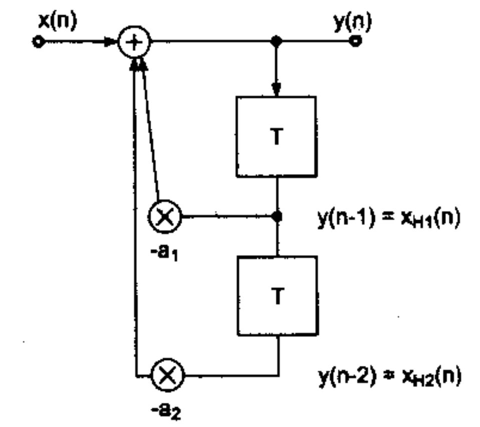
Causal and stable systems

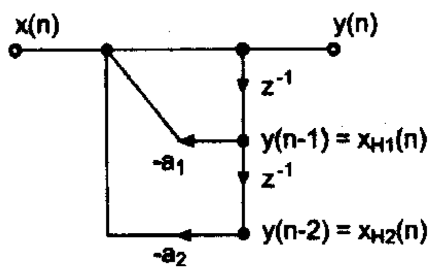
- Causality: a discrete-time system is causal
  - if the output signal *y(n)* = 0 for n <0   for a given input signal *u(n)* = 0 for n <0.
  - This means that the system cannot react to an input before the input is applied to the system
- Stability: a digital system is stable if $$\sum_{n=-\infty}^{\infty} |h(n)| < M_2 < \infty$$
  - stability implies that transfer function *H(z)* and frequency response are related by $z \leftrightarrow e^{j\Omega}$

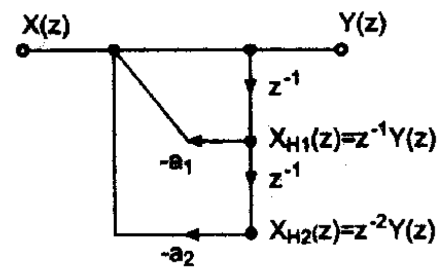
Signal	Z-transform	Discrete-time Fourier transform
$x(n)$	$X(z)$	$X(e^{j\Omega})$
$x(n-M)$	$z^{-M} \cdot X(z)$	$e^{-j\Omega M} \cdot X(e^{j\Omega})$
$\delta(n)$	1	1
$\delta(n-M)$	z^{-M}	$e^{-j\Omega M}$
$x(n) \cdot e^{-j\Omega_0 n}$	$X(e^{-j\Omega_0} \cdot z)$	$X(e^{j(\Omega-\Omega_0)})$

IIR systems

- = system with infinte impulse response
  - e.g. second order IIR system







  - Difference equation

$$y(n) = x(n) - a_1 y(n-1) - a_2 y(n-2).$$

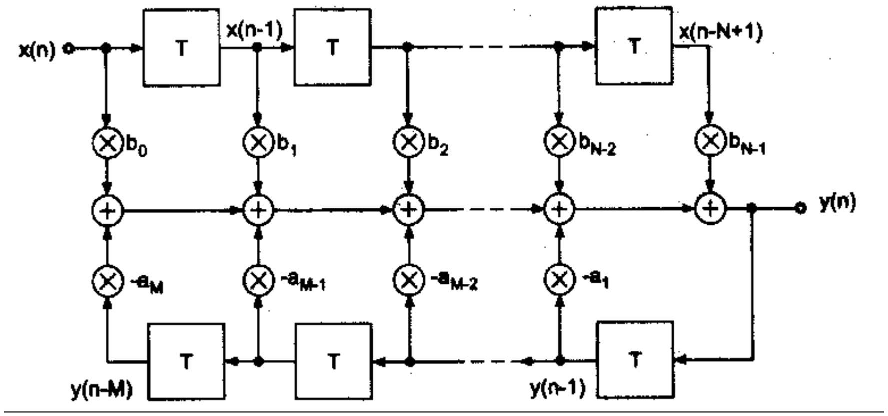
$$Y(z) = X(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z)$$

  - Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

IIR systems

- = system with infinte impulse response *h(n)*



- Difference equation

$$y(n) = -\sum_{k=1}^{M} a_k\, y(n-k) + \sum_{k=0}^{N-1} b_k\, x(n-k)$$

- Z transform of diff. eq.

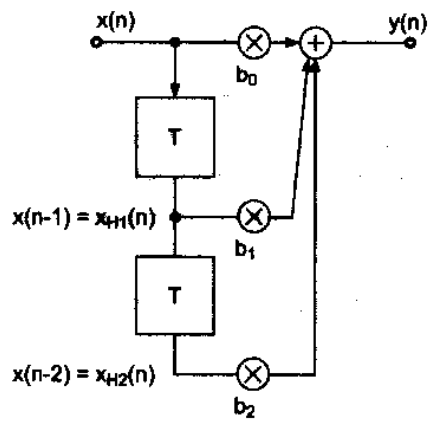
$$Y(z) = -\sum_{k=1}^{M} a_k\, z^{-k} Y(z) + \sum_{k=0}^{N-1} b_k\, z^{-k} X(z)$$

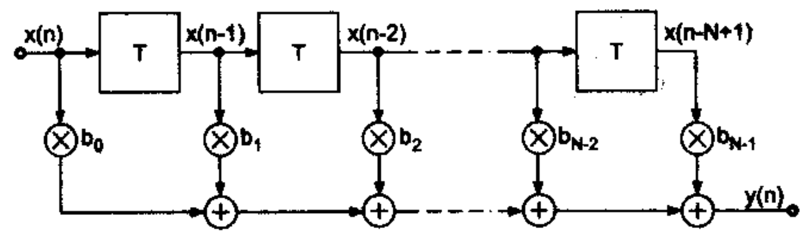
- Transfer function

$$H(z) = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{1 + \sum_{k=1}^{M} a_k z^{-k}}.$$

FIR systems

- = system with finite impulse response *h(n)*
  - e.g. second order FIR system





- Difference equation

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

- Z transform of diff. eq.

$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$$

- Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2}$$

Fir example

- computation of frequency response
  - impulse response
  - magnitude resp.
  - pole/zero plot
  - phase resp.