FAST-RESPONSE HIGH-QUALITY RECTIFIER WITH SLIDING-MODE CONTROL

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Abstract. **A PWM rectifier including an uncontrolled rectifier and a _uk converter stage driven by a sliding-mode controller is described.**

Similarly to other high-quality rectifiers, this solution allows low-distorted and in-phase line current. Moreover, due to the sliding-mode control, fast and stable response is achieved, in spite of the large output filter. Control complexity is the same of standard current-mode controls.

Converter analysis, design criteria, and experimental results are reported.

INTRODUCTION

High Quality Rectifiers (HQR) provide dc voltage regulation together with high input power factor and small line current distortion.

Single-phase topologies based on standard dc/dc converter stages are increasingly popular *[1,2]*, due also to the availability of application-oriented control IC's, either for continuous-conduction mode (CCM) or discontinuous-conduction mode (DCM). Detailed analysis and design criteria of these circuits are available in the literature *[3,4]*.

An inherent limitation of single-phase HQR's is the slow response. In fact, sinusoidal input current means large input power fluctuations, at twice the line frequency, resulting in an output voltage ripple which cannot be corrected by the control, otherwise input current waveform is affected. A big output filter capacitor must therefore be adopted in order to limit the voltage ripple, resulting in a bandwidth limitation of the voltage control loop. Accordingly, these converters are normally used as pre-regulators, their output performances being below the requirements of standard power supplies.

In practice, this bandwidth limitation can be overcome if some input current distortion is accepted (within the limits posed by the standards, e.g. IEC 555-2 and IEEE 519), provided that this results in smaller power fluctuation. From this point of view, sliding-mode control is very powerful, since it can provide an optimal trade-off between the needs for increasing response speed and reducing input current distortion and output voltage ripple.

Following this approach, this paper describes a single-phase HQR based on a _uk stage driven by a second-order sliding-mode controller. This solution maintains all advantages of _uk converters (single switch; full utilization of the flux swing in the transformer core; full-range output regulation; small high-frequency ripple on the input and output current), while ensuring high control robustness and good static and dynamic performances (typical of sliding-mode control). Moreover, control complexity is the same of standard current-mode controllers.

CONVERTER SCHEME AND PRINCIPLE OF OPERATION UNDER SLIDING-MODE CONTROL

Fig.1 shows the basic converter scheme with fourth-order sliding-mode control.

The power stage includes an uncontrolled rectifier followed by a _uk stage. As known, this converter can be operated with all state variables continuous (CSV mode), or with some of them discontinuous (DSV mode), resulting in different control-to-output characteristics *[4,5]*.

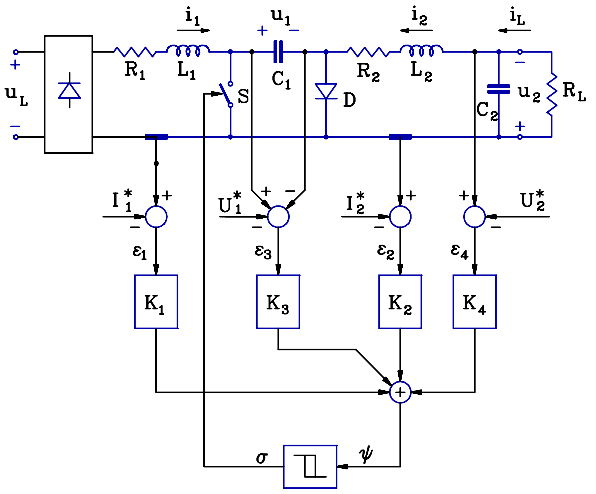

Fig.1 - Basic HQR scheme with sliding-mode control

Achieving a sinusoidal line current in the CSV mode calls for duty-cycle modulation, as for any boost-like structure *[6,7]*. Instead, operating the circuit in the DSV mode (either Discontinuous Inductor Current Mode, DICM, or Discontinuous Capacitor Voltage Mode, DCVM), causes sinusoidal absorption even without duty-cycle modulation *[4,8]*.

Accordingly, most low-power PFC's behave in DSV mode, which allows simpler control (the output voltage control loop drives directly the switch, according to a PWM technique) while a high input power factor is "naturally" ensured.

Instead, CSV operation, which allows better circuit exploitation, calls for a control loop in order to enforce sinusoidal absorption. This is normally accomplished by implementing an inner current loop driven by the voltage loop. The reference signal of this current loop is sinusoidal, with an amplitude which is adjusted by the voltage loop so as to obtain the desired dc output. This control technique is simple and reliable but, as mentioned before, the voltage loop must have a crossover frequency below the line frequency, in order to avoid input current distortion while ensuring stability *[9]*.

Better performances can be obtained by sliding-mode control, which acts simultaneously on all state variables, as shown in the basic scheme of Fig.1.

In principle, sliding-mode control keeps near zero, according to a hysteretic technique, the sliding variable:

$$\psi = \sum_{i=1}^{4} K_i \cdot \varepsilon_i \tag{1}$$

which is defined as the weighted sum (K_i are suitable gains) of all state variable errors ε_i. These error terms are given by:

$$\varepsilon_i = v_i - V_i^* \tag{2}$$

where v_i is the generic state variable (i_1, i_2, u_1, u_2) and V^*_i is the corresponding reference value.

Clearly, this kind of control is not practical, since all state variables must be sensed and a suitable reference must be provided for each of them.

However, it offers several benefits, deriving from the property to act on all state variables simultaneously. First, let N to be the system order (in our case N=4), the system response has order N-1. In fact, under sliding mode we have:

$$\psi \approx 0 \tag{3}$$

which means that only N-1 state variables are independent, the N-th being constrained by (3). This order reduction makes simpler both system analysis and control design. Second, dynamic response is very fast, since all control loops act concurrently. Third, stability (even for large input and output variations) and robustness (against load, supply and parameter variations) are those typical of hysteretic controls. Fourth, the system response depends only slightly on the actual converter parameters.

These advantages have already been proved for _uk dc/dc converters *[10]*. They hold also for ac/dc converters, as it will be shown later.

SIMPLIFIED CONTROL SCHEME

In *[10]* it has been demonstrated that excellent performances can be obtained even with reduced-order controllers, like that shown in Fig.2, where only input current i_1 and output voltage u_o are sensed, as for standard current-mode controls.

In this scheme, including an insulation transformer, reference values (i_1^*, U_o^*) are required for both state variables. However, while U_o^* is an input variable for the control, i_1^* must be evaluated as a function of voltage error ε_u. For this purpose, similarly to the case of current-mode control, an error amplifier R_i is used to determine reference amplitude I_1^*, which is then multiplied by rectified voltage signal u_R to obtain current reference i_1^*.

Given i_1^* and U_o^*, an hysteretic control is then performed to keep near zero the sliding variable by proper operation of the converter switch. Accordingly, the system trajectories lie on a *sliding hyperplane*, which is defined, in the state space, by the equation:

$$\psi = K_i \cdot \varepsilon_i + K_u \cdot \varepsilon_u = 0 \tag{4}$$

As discussed in *[10]*, selection of coefficients K_i and K_u must be done in order to satisfy three requirements:

- *existence condition* of the sliding mode, which requires that the state trajectories are directed toward the sliding plane for both possible statuses of the converter switch;
- *hitting condition*, which requires that the system trajectories encounter the sliding plane irrespective of their starting point in the state space;
- *stability* of the system trajectories on the sliding plane.

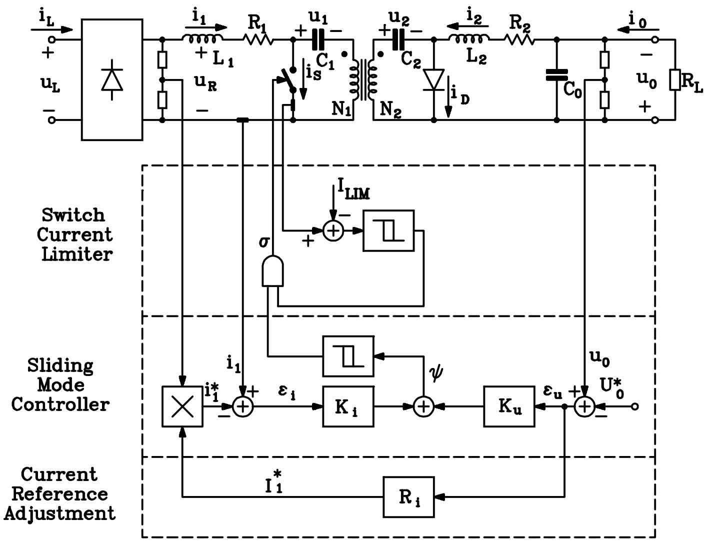

Fig.2 - Actual converter scheme

If coefficients K_i and K_u are chosen properly, we gain additional advantages. First, in the steady state the controller tends to maintain the minimum errors ε_i and ε_u which are consistent with energy balance conditions. The value of the coefficients also determines the relative amplitudes of voltage and current errors. Second, under dynamic conditions the system moves along state trajectories which can be made stable irrespective of the circuit parameters. Third, the speed of response can be optimized, by a careful selection of system and control parameters *[10]*.

It must be emphasized that these benefits are generally paid with large variations of the switching frequency. However, also this limitation can be overcome by a modulation of the hysteresis band, as it will be shown later.

Additional functions can be implemented to provide switch or inductor current limitation. For instance, the current limiter shown in Fig.2 overrides sliding mode control when the switch current exceeds threshold I_{LIM}. If this happens, control maintains the switch current at the value I_{LIM}.

Note lastly that the scheme of Fig.2 resembles that of current-mode control, with the only addition of a hysteretic control on variable ψ. However, as mentioned before, resulting performances are quite different.

CIRCUIT EQUATIONS

Converter operation can be described by two sets of linear differential equations, which are valid when the switch is closed and when it is open, respectively. As compared to the case of dc/dc converters, some modifications must be introduced to take into account the time-varying input voltage. Moreover, in order to keep the same notation used in *[10]*, all elements on the transformer secondary side are moved to the primary side (and indicated by superscript ′) according to the relations:

$$L'_2 = n^2 L_2 \quad R'_2 = n^2 R_2 \quad R'_L = n^2 R_L \qquad (5)$$
$$C'_2 = \frac{C_2}{n^2} \qquad C'_0 = \frac{C_0}{n^2} \qquad i'_0 = \frac{i_0}{n} \qquad u'_0 = n\, u_0$$

where $\boldsymbol{n}$ is the transformer turn ratio N_1/N_2. We also define the equivalent energy transfer capacitor:

$$C_e = \frac{C_1 C'_2}{C_1 + C'_2} \qquad (6)$$

Assuming now the following base quantities:

$$Z_b = \sqrt{\frac{L'_2}{C_e}} \qquad \text{base impedance} \qquad (7.a)$$

$$U_b = \hat{U}_R \qquad \text{base voltage} \qquad (7.b)$$

$$\omega_b = \frac{1}{\sqrt{L'_2 C_e}} \qquad \text{base angular frequency} \qquad (7.c)$$

$$I_b = \frac{U_b}{Z_b} \qquad \text{base current} \qquad (7.d)$$

and the adimensional coefficients:

$$\alpha_L = \frac{L_1}{L'_2}\ ; \quad \alpha_C = \frac{C'_0}{C_e};\quad \alpha_U = \frac{u_R(t)}{\hat{U}_R} = |\sin(\omega_l\, t)| \qquad (8)$$

where ω_l is the angular frequency of the supply, the converter equations can be rewritten in the form:

$$\underline{\dot{v}}_N = \underline{A}\,\underline{v}_N + \underline{B}\,\sigma + \underline{F} \qquad (9)$$

where σ is the switch status, and $\underline{v}_N$ 1 and $\underline{\dot{v}}_N$ 2 are the vectors of the normalized status variables (i_{1N}, i_{2N}', u_{eN}, u_{0N}') and their derivatives, respectively. Matrices $\underline{A}$, $\underline{B}$, $\underline{F}$ are given by:

$$
\underline{A} = \begin{bmatrix} \frac{-R_{1N}}{\alpha_L} & 0 & \frac{-1}{\alpha_L} & 0 \\ 0 & -R'_{2N} & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha_C} & 0 & \frac{-1}{R'_{LN}\,\alpha_C} \end{bmatrix}
$$

$$
\underline{B} = \begin{bmatrix} \frac{u_{eN}}{\alpha_L} \\ u_{eN} \\ -i_{1N} - i'_{2N} \\ 0 \end{bmatrix} \qquad \underline{F} = \begin{bmatrix} \frac{\alpha_U}{\alpha_L} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}
$$

where subscript N refers to normalized quantities.

SELECTION OF CONTROL PARAMETERS

According to the general sliding-mode control theory, we will refer to normalized state variable errors x_i:

$$
\underline{x} = \underline{v}_N - \underline{V}_N^{\,*} \tag{11.a}
$$

where $\underline{V}_N^*$ is the vector of normalized dc references:

$$
\underline{V}_N^{\,*} = \begin{bmatrix} I_{1N}^* \\ I'^{\,*}_{2N} \\ U_{eN}^* \\ U'^{\,*}_{0N} \end{bmatrix} = \begin{bmatrix} \frac{2\,U'^{*\,2}_{0N}}{R_{LN}} \cdot \alpha_U \\ \frac{2\,U'^{*}_{0N}}{R_{LN}} \cdot \alpha_U^{\,2} \\ \alpha_U + U'^{*}_{oN} \\ U'^{*}_{oN} \end{bmatrix} \tag{11.b}
$$

Note that, according to this definition, most elements of $\underline{V}_N^*$ exhibit low-frequency variations corresponding to the instantaneous input voltage variations. This is consistent with the assumption of *quasi-stationary behavior*, which implies that the input voltage varies so slowly, as compared to the circuit response, that all circuit variables follow, in the low-frequency domain, the supply variations. In other words, we assume that an operation cycle at the line frequency is made up of a sequence of steady-state conditions, each one corresponding to a different value of the input voltage.

Substituting (11.a) and (11.b) in (9) we obtain:

$$
\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\sigma + \underline{G} \tag{12}
$$

where:

$$
\underline{G} = \underline{A}\,\underline{V}_N^{\,*} + \underline{F} \tag{13}
$$

In the general theory *[11,12]*, sliding function ψ is defined by a linear combination of all state variable errors. Using a normalized notation we can therefore write:

$$
\psi = K_{1N}\,x_1 + K_{2N}\,x_2 + K_{3N}\,x_3 + K_{4N}\,x_4 = \underline{K}_N^T \cdot \underline{x} \tag{14}
$$

where $\underline{K}_N^T$ is the transpose of the vector of normalized sliding coefficients. In the case of the second-order controller shown in Fig.2, coefficients K_{2N} and K_{3N} are zero, while coefficients K_{1N} and K_{4N} are related to K_i and K_u by:

$$
K_i = K_{1N}\,\frac{Z_b}{U_b} \tag{15.a}
$$

$$
K_u = K_{4N}\,\frac{n}{U_b} \tag{15.b}
$$

Note that, since in the sliding mode $\psi \approx 0$, only the ratio between K_i and K_u is important. This allows easy control tuning by simply adjusting a gain.

As mentioned before, control coefficients must be chosen so as to satisfy existence and hitting conditions, and to ensure stability.

Hitting condition. As demonstrated in *[11,12]* the hitting condition is satisfied if:

$$
\underline{K}_N^T\,\underline{A}_4 \le 0 \quad - \quad \frac{-K_{4N}}{R'_{LN} \cdot \alpha_C} \le 0 \tag{16}
$$

where $\underline{A}_4$ is the fourth column of matrix $\underline{A}$.

This relation shows that K_{4N} must be positive.

Existence condition. As shown in *[10-12]*, the existence condition can be expressed by the following inequalities:

$$
\frac{\partial \psi}{\partial t} = \underline{K}_N^T\,\underline{A}\,\underline{x} + \underline{K}_N^T\,\underline{G} < 0 \qquad 0 < \psi < \xi \tag{17.a}
$$

$$
\frac{\partial \psi}{\partial t} = \underline{K}_N^T\,\underline{A}\,\underline{x} + \underline{K}_N^T\,\underline{B} + \underline{K}_N^T\,\underline{G} > 0 \quad -\xi < \psi < 0 \tag{17.b}
$$

where ξ is an arbitrary small positive quantity.

Since vector $\underline{V}_N^*$ varies during the supply period, selection of K_{iN} so as to satisfy these inequalities is possible, but calls for complicated calculations, even in the simplified assumption that state variable errors are suitably smaller than references $\underline{V}_N^*$.

Stability condition. As done in *[10]*, the *equivalent input approach [11]* can be applied, in which switch status σ is assumed to be a continuous variable. A system non-linear equations results, which is linearized around the working point. The system eigenvalues are then computed as a function of coefficients K_{iN}. Only those values of K_{iN} corresponding to eigenvalues with negative real part and suitable damping factor are accepted.

Obviously, this procedure must be repeated for different operating points, corresponding to different values of the input voltage values and different load conditions, in order to determine suitable values of the control coefficients. As one may expect, for low values of the input voltage the range of acceptable coefficients reduces.

CONVERTER DESIGN

Designing the converter for operation under sliding-mode control has the advantage, over solutions using other control techniques, that the circuit elements can be sized only on the basis of power/energy requirements.

Power section design

Output capacitor (C_0). This element must absorb low-frequency power fluctuations while satisfying voltage ripple specifications. Accordingly, it is selected by the relation:

$$C_0 = \frac{I_0}{\omega_l\, \Delta u_0} \tag{18}$$

where Δu_0 is the desired peak-to-peak output voltage ripple.

Energy transfer capacitors (C_1,C_2). These are crucial items. In fact, since these capacitors charge at a voltage which is the sum of the input and output voltages, increasing their size causes a considerable increase of the energy stored in the circuit, affecting the circuit response. In addition, these must be fast capacitors, since they must withstand very high current derivatives. Thus, these elements must be as small as possible.

Instead, large transfer capacitors are normally used, in order to decouple the input (boost) and output (buck) stages, which is a condition to achieve stability with standard control techniques. In our case, however, stability is ensured by the control, irrespective of the transfer capacitors size, so that the transfer capacitors can be selected in order to suit voltage ripple requirements only, according to the relation:

$$C_1 = \frac{2\, M\, I_0}{f\ \hat{U}_R\, \Delta u_1} \cdot \frac{1}{1 + n\, M} \tag{19.a}$$

$$C_2 = \frac{2\, I_0}{f\ \Delta u_2} \cdot \frac{n\, M}{1 + n\, M} \tag{19.b}$$

where f is the rated switching frequency, Δu_1 and Δu_2 are the specified peak-to-peak voltage ripples, and:

$$M = \frac{U_0}{\hat{U}_R} \tag{20}$$

is the *voltage conversion ratio*.

Another constrain derives from the interaction between transfer capacitors and transformer magnetizing inductance (L_μ). The corresponding resonance frequency must be suitably higher than the line frequency, in order to avoid low frequency oscillations, which affect line current waveform and may cause control problems. Thus the condition:

$$\frac{1}{\sqrt{L_\mu \left(C_1 + \dfrac{C_2}{n^2}\right)}} >> \omega_l \tag{21}$$

must be satisfied.

Input and output inductors (L_1,L_2). The corresponding current ripples are related to the operating mode, either CCM or DCM. The condition to ensure CCM operation is *[3]*:

$$K_a > K'_{crit\,max} \tag{22.a}$$

where:

$$K_a = \frac{2\ f}{R_L} \cdot \frac{L_1\ L'_2}{L_1 + L'_2};\quad K'_{crit\,max} = \frac{1}{2\, M^2} \tag{22.b}$$

while from ripple considerations we can write:

$$L_1 = \frac{\hat{U}_R}{f\ \Delta i_1} \cdot \frac{n\, M}{1 + n\, M} \tag{23.a}$$

$$L_2 = \frac{U_0}{f\ \Delta i_2} \cdot \frac{1}{1 + n\, M} \tag{23.b}$$

where Δi_1 and Δi_2 are the desired peak-to-peak current ripples.

Insulation transformer. Design is not critical, except for the leakage inductances, which cause overvoltages and energy loss each time the switch is turned off. This effect can however be limited by a suitable *clamping circuit*.

The number of primary turns is given by:

$$N_1 = \frac{\hat{U}_R}{2\, f\ B_{max}\, S_{fe}} \cdot \frac{n\, M}{1 + n\, M} \tag{24}$$

where B_{max} and S_{fe} are the rated flux density and the cross section of the iron core, respectively.

Control section design

Sliding mode coefficients (K_i, K_u). They must be designed according to the procedure discussed in the previous section.

Output voltage loop regulator R_i. It must adjust current reference i_1^* within suitable times. In fact, sliding mode control reacts immediately to any transient condition (it does not include delays in the loop), ensuring a fast response at the expense of the energy stored in the circuit: the controlled state variables can therefore show a temporary error, which disappears as soon as R_i responds.

As known, below the line frequency the power stage can be modeled by a single pole transfer function *[9]*. Thus, for regulator R_i a suitable transfer function is:

$$R_i(s) = K_R \frac{(1 + s\ \tau_z)}{s(1 + s\ \tau_p)} \tag{25}$$

and the parameters must be chosen to have a crossover frequency well below the line frequency. In particular, pole $1/\tau_p$ is introduced to remove the 100 Hz ripple on the error voltage signal.

Switching frequency stabilization. Observe that the ripple of current i_1 is given by:

$$\Delta i_1 = \frac{u_R\ \delta}{f\ L_1} \tag{26}$$

where δ is the duty-cycle. Since in the steady state Δi_1 is proportional to current error ε_i, which is a part of sliding function ψ, we can get some stabilization of the switching frequency by making the hysteresis band proportional to u_R.

EXPERIMENTAL RESULTS

A converter prototype was built having the following parameters:

$U_L = 100\ V_{peak}$, 50 Hz	$U_o = 15\ V \pm 3\%$
$I_o = 2\ A$	$N_1/N_2 = 10$
$L_1 = 4\ mH$	$L_2 = 40\ \mu H$
$C_1 = 0.22\ \mu F$	$C_2 = 100\ \mu F$
$C_o = 0.01\ F$	

The hysteresis band was selected to provide a switching frequency near 60 kHz.

Typical waveforms describing the converter operation are shown in Fig.3 and Fig.4.

Fig.3 refers to the case of $K_i/K_u = 4.6$, which means that the current error term is dominant in the sliding function ψ. In this case, sliding-mode control performs like standard current-mode controls, the voltage accuracy being primarily determined by the slow external voltage loop. Accordingly, a good ac current waveform results (Fig.3a), proportional to the supply voltage. The low distortion is confirmed by Fig.3b, showing the harmonic spectrum of the input current. Fig.3c shows the converter response to load variations (from light-load to full-load and viceversa). The response time is mainly determined by the output stage characteristics and the overshoot is about 10% of the rated voltage. Instead, in steady-state conditions (e.g., between the two load transitions), the output voltage ripple drops to 3% of rated voltage.

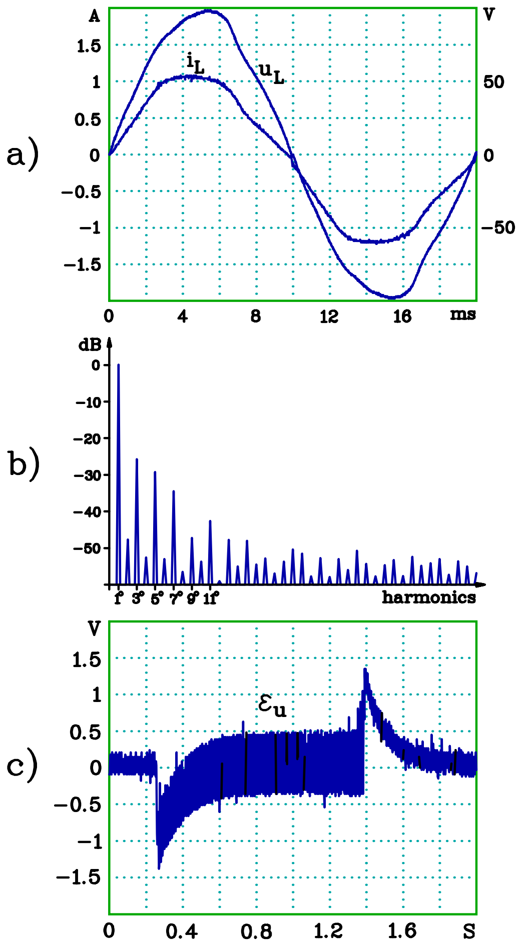

Fig.3 - Experimental waveforms with K_i/K_u=4.6
a) input voltage and current; b) input current spectrum; c) response to step-load variation

Fig.4 shows similar waveforms for $K_i/K_u = 1.1$. In this case, the voltage error term influences heavily the sliding function. Correspondingly, the voltage ripple is reduced to 2%, but a penalty is paid on the input current waveform (Fig.4a and Fig.4b). The dynamic behavior improves, resulting in faster response and lower overshoots. In fact, voltage error terms are rapidly compensated by the sliding-mode controller, before the intervention of the external loop.

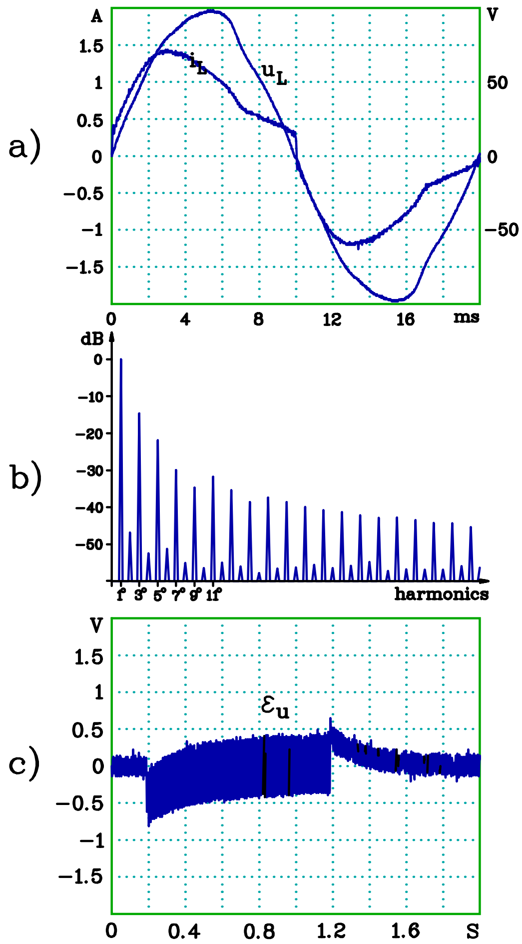

Fig.4 - Experimental waveforms with K_i/K_u=1.1
a) input voltage and current; b) input current spectrum; c) response to step-load variation

CONCLUSIONS

A high-quality single-phase rectifier including a _uk stage driven by a sliding-mode controller has been presented.

This solution maintains the advantages of _uk converters (limited input and output current ripple, good transformer utilization, step-up and step-down conversion ratio, etc.), with the additional benefit of small transfer capacitors.

In spite of the simple control circuitry, which resembles that of current-mode controls, the converter provides excellent stability, robustness and dynamic response, taking advantage of the potentiality of sliding-mode control.

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