Circuit Element Variations

- All electronic components have manufacturing tolerances.
  - Resistors can be purchased with $\pm 10\%$, $\pm 5\%$, and $\pm 1\%$ tolerance. (IC resistors are often $\pm 10\%$.)
  - Capacitors can have asymmetrical tolerances such as $+20\%/-50\%$.
  - Power supply voltages typically vary from $1\%$ to $10\%$.
- Device parameters will also vary with temperature and age.
- Circuits must be designed to accommodate these variations.
- We will use worst-case and Monte Carlo (statistical) analysis to examine the effects of component parameter variations.

Tolerance Modeling

- For symmetrical parameter variations
  \[ P_{\text{ NOM}}(1 - \epsilon) \leq P \leq P_{\text{ NOM}}(1 + \epsilon) \]
- For example, a 10K resistor with $\pm 5\%$ percent tolerance could take on the following range of values:
  \[ 10k(1 - 0.05) \leq R \leq 10k(1 + 0.05) \]
  \[ 9,500 \, \Omega \leq R \leq 10,500 \, \Omega \]
Circuit Analysis with Tolerances

- **Worst-case analysis**
  - Parameters are manipulated to produce the worst-case min and max values of desired quantities.
  - This can lead to overdesign since the worst-case combination of parameters is rare.
  - It may be less expensive to discard a rare failure than to design for 100% yield.

- **Monte-Carlo analysis**
  - Parameters are randomly varied to generate a set of statistics for desired outputs.
  - The design can be optimized so that failures due to parameter variation are less frequent than failures due to other mechanisms.
  - In this way, the design difficulty is better managed than a worst-case approach.

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**Worst Case Analysis Example**

**Problem:** Find the nominal and worst-case values for output voltage and source current.

**Solution:**

- **Known Information and Given Data:** Circuit topology and values in figure.
- **Unknowns:** \( V_{o}^{\text{nom}}, V_{o}^{\text{min}}, V_{o}^{\text{max}}, I_{S}^{\text{nom}}, I_{S}^{\text{min}}, I_{S}^{\text{max}} \).
- **Approach:** Find nominal values and then select \( R_1 \), \( R_2 \), and \( V_S \) values to generate extreme cases of the unknowns.
- **Assumptions:** None.
- **Analysis:** Next slides...

**Nominal voltage solution:**

\[
V_{o}^{\text{nom}} = \frac{V_{S}^{\text{nom}} R_1^{\text{nom}}}{R_1^{\text{nom}} + R_2^{\text{nom}}}
\]

\[
= 15V \frac{18\Omega}{18\Omega + 36\Omega} = 5V
\]
Worst-Case Analysis Example (cont.)

Nominal Source current:

\[ I_{S}^{\text{nom}} = \frac{V_{S}^{\text{nom}}}{R_{1}^{\text{nom}} + R_{2}^{\text{nom}}} = \frac{15V}{18k\Omega + 36k\Omega} = 278\mu A \]

Rewrite \( V_o \) to help us determine how to find the worst-case values.

\[ V_o = V_S \frac{R_1}{R_1 + R_2} = \frac{V_S}{1 + \frac{R_2}{R_1}} \]

Vo is maximized for max \( V_S, R_1 \) and min \( R_2 \).

Vo is minimized for min \( V_S, R_1, \) and max \( R_2 \).

\[ V_o^{\max} = \frac{15V(1.1)}{1 + \frac{36K(0.95)}{18K(1.05)}} = 5.87V \]

\[ V_o^{\min} = \frac{15V(0.95)}{1 + \frac{36K(1.05)}{18K(0.95)}} = 4.20V \]

Check of Results: The worst-case values range from 14-17 percent above and below the nominal values. The sum of the three element tolerances is 20 percent, so our calculated values appear to be reasonable.

Worst-case source currents:

\[ I_{S}^{\max} = \frac{I_{S}^{\max}}{R_{1}^{\min} + R_{2}^{\min}} = \frac{15V(1.1)}{18k\Omega(0.95) + 36k\Omega(0.95)} = 322\mu A \]

\[ I_{S}^{\min} = \frac{I_{S}^{\min}}{R_{1}^{\max} + R_{2}^{\max}} = \frac{15V(0.9)}{18k\Omega(1.05) + 36k\Omega(1.05)} = 238\mu A \]
Monte Carlo Analysis

- Parameters are varied randomly and output statistics are gathered.
- We use programs like MATLAB, Mathcad, or a spreadsheet to complete a statistically significant set of calculations.
- For example, with Excel®, a resistor with 5% tolerance can be expressed as:

\[ R = R_{\text{nom}} (1 + 2\varepsilon(R\text{AND}() - 0.5)) \]

The R\text{AND}() functions returns random numbers uniformly distributed between 0 and 1.

Monte Carlo Analysis Example

**Problem:** Perform a Monte Carlo analysis and find the mean, standard deviation, min, and max for \( V_o \), \( I_s \), and power delivered from the source.

**Solution:**
- **Known Information and Given Data:** Circuit topology and values in figure.
- **Unknowns:** The mean, standard deviation, min, and max for \( V_o \), \( I_s \), and \( P_s \).
- **Approach:** Use a spreadsheet to evaluate the circuit equations with random parameters.
- **Assumptions:** None.
- **Analysis:** Next slides...

Monte Carlo parameter definitions:
\[
\begin{align*}
V_s &= 15(1 + 0.2(\text{RAND}() - 0.5)) \\
R_1 &= 18,000(1 + 0.1(\text{RAND}() - 0.5)) \\
R_2 &= 36,000(1 + 0.1(\text{RAND}() - 0.5))
\end{align*}
\]
Monte Carlo Analysis Example (cont.)

Nominal Source current:
\[
I_S^{\text{nom}} = \frac{V_S^{\text{nom}}}{R_1^{\text{nom}} + R_2^{\text{nom}}} = \frac{15V}{18k\Omega + 36k\Omega} = 278\mu A
\]

Rewrite \( V_o \) to help us determine how to find the worst-case values.

\[
V_o = V_S \cdot \frac{R_1}{R_1 + R_2} = \frac{V_S}{1 + \frac{R_2}{R_1}}
\]

Vo is maximized for max \( V_s \), \( R_1 \) and min \( R_2 \).
Vo is minimized for min \( V_s \), \( R_1 \), and max \( R_2 \).

\[
V_o^{\text{max}} = \frac{15V(1.1)}{1 + \frac{36K(0.95)}{18K(1.05)}} = 5.87V \\
V_o^{\text{min}} = \frac{15V(0.95)}{1 + \frac{36K(1.05)}{18K(0.95)}} = 4.20V
\]

Histogram of output voltage from 1000 case Monte Carlo simulation.

See table 5.1 for complete results.
Temperature Coefficients

- Most circuit parameters are temperature sensitive.
  \[ P = P_{\text{nom}}(1 + \alpha_1 \Delta T + \alpha_2 \Delta T^2) \]
  where \( \Delta T = T - T_{\text{nom}} \)
  \( P_{\text{nom}} \) is defined at \( T_{\text{nom}} \)
- Most versions of SPICE allow for the specification of \( T_{\text{nom}}, T, TC1(\alpha_1), TC2(\alpha_2) \).
- SPICE temperature model for resistor:
  \[ R(T) = R(T_{\text{nom}})[1 + TC1(T-T_{\text{nom}}) + TC2(T-T_{\text{nom}})^2] \]
- Many other components have similar models.