Circuit Element Variations

- All electronic components have manufacturing tolerances.
  - Resistors can be purchased with ± 10%, ± 5%, and ± 1% tolerance. (IC resistors are often ± 10%.)
  - Capacitors can have asymmetrical tolerances such as +20%/-50%.
  - Power supply voltages typically vary from 1% to 10%.
- Device parameters will also vary with temperature and age.
- Circuits must be designed to accommodate these variations.
- We will use worst-case and Monte Carlo (statistical) analysis to examine the effects of component parameter variations.

Tolerance Modeling

- For symmetrical parameter variations

$$P_{NOM}(1 - \varepsilon) \le P \le P_{NOM}(1 + \varepsilon)$$

- For example, a 10K resistor with ±5% percent tolerance could take on the following range of values:

$$10k(1 - 0.05) \le R \le 10k(1 + 0.05)$$

$$9{,}500\ \Omega \le R \le 10{,}500\ \Omega$$

Circuit Analysis with Tolerances

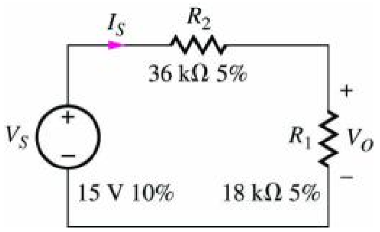
- Worst-case analysis
  - Parameters are manipulated to produce the worst-case min and max values of desired quantities.
  - This can lead to overdesign since the worst-case combination of parameters is rare.
  - It may be less expensive to discard a rare failure than to design for 100% yield.
- Monte-Carlo analysis
  - Parameters are randomly varied to generate a set of statistics for desired outputs.
  - The design can be optimized so that failures due to parameter variation are less frequent than failures due to other mechanisms.
  - In this way, the design difficulty is better managed than a worst-case approach.

Worst Case Analysis Example

Problem: Find the nominal and worst-case values for output voltage and source current.

Solution:

- **Known Information and Given Data**: Circuit topology and values in figure.
- **Unknowns**: V_o^{nom}, V_o^{min}, V_o^{max}, I_S^{nom}, I_S^{min}, I_S^{max}.
- **Approach**: Find nominal values and then select R1, R2, and Vs values to generate extreme cases of the unknowns.
- **Assumptions**: None.
- **Analysis**: Next slides…

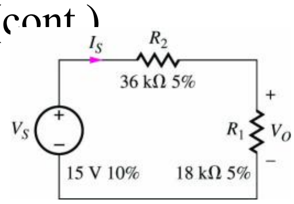


Nominal voltage solution:

$$V_o^{nom} = V_S^{nom} \frac{R_1^{nom}}{R_1^{nom} + R_2^{nom}}$$

$$= 15V \frac{18k\Omega}{18k\Omega + 36k\Omega} = 5V$$

Worst-Case Analysis Example (cont.)



Nominal Source current:

$$I_S^{nom} = \frac{V_S^{nom}}{R_1^{nom} + R_2^{nom}} = \frac{15V}{18k\Omega + 36k\Omega} = 278\mu A$$

Rewrite V_o to help us determine how to find the worst-case values.

$$V_o = V_S \frac{R_1}{R_1 + R_2} = \frac{V_S}{1 + \frac{R_2}{R_1}}$$

Vo is maximized for max Vs, R1 and min R2.

Vo is minimized for min Vs, R1, and max R2.

$$V_o^{max} = \frac{15V(1.1)}{1 + \frac{36K(0.95)}{18K(1.05)}} = 5.87V \qquad V_o^{min} = \frac{15V(0.95)}{1 + \frac{36K(1.05)}{18K(0.95)}} = 4.20V$$

Worst-Case Analysis Example (cont.)

Worst-case source currents:

$$I_S^{max} = \frac{V_S^{max}}{R_1^{min} + R_2^{min}} = \frac{15V(1.1)}{18k\Omega(0.95) + 36k\Omega(0.95)} = 322\mu A$$

$$I_S^{min} = \frac{V_S^{min}}{R_1^{max} + R_2^{max}} = \frac{15V(0.9)}{18k\Omega(1.05) + 36k\Omega(1.05)} = 238\mu A$$

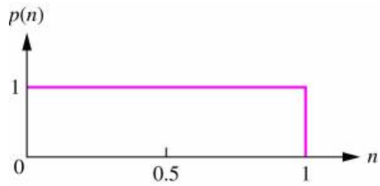
Check of Results: The worst-case values range from 14-17 percent above and below the nominal values. The sum of the three element tolerances is 20 percent, so our calculated values appear to be reasonable.

Monte Carlo Analysis

- Parameters are varied randomly and output statistics are gathered.
- We use programs like MATLAB, Mathcad, or a spreadsheet to complete a statistically significant set of calculations.
- For example, with Excel®, a resistor with 5% tolerance can be expressed as:

$$R = R_{nom}(1 + 2\varepsilon(RAND() - 0.5))$$

The RAND() functions returns random numbers uniformly distributed between 0 and 1.

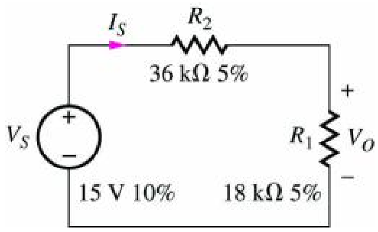


Monte Carlo Analysis Example

Problem: Perform a Monte Carlo analysis and find the mean, standard deviation, min, and max for Vo, Is, and power delivered from the source.

Solution:

- **Known Information and Given Data**: Circuit topology and values in figure.
- **Unknowns**: The mean, standard deviation, min, and max for Vo, Is, and Ps.
- **Approach**: Use a spreadsheet to evaluate the circuit equations with random parameters.
- **Assumptions**: None.
- **Analysis**: Next slides…



Monte Carlo parameter definitions:

$$V_s = 15(1 + 0.2(RAND() - 0.5))$$
$$R_1 = 18,000(1 + 0.1(RAND() - 0.5))$$
$$R_2 = 36,000(1 + 0.1(RAND() - 0.5))$$

Monte Carlo Analysis Example (cont.)

Nominal Source current:

$$I_S^{nom} = \frac{V_S^{nom}}{R_1^{nom} + R_2^{nom}} = \frac{15V}{18k\Omega + 36k\Omega} = 278\mu A$$

Rewrite V_o to help us determine how to find the worst-case values.

$$V_o = V_S \frac{R_1}{R_1 + R_2} = \frac{V_S}{1 + \frac{R_2}{R_1}}$$

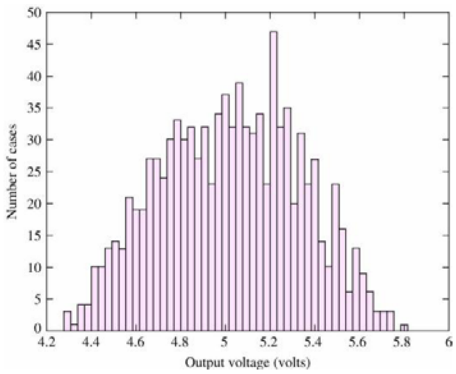
Vo is maximized for max Vs, R1 and min R2.

Vo is minimized for min Vs, R1, and max R2.

$$V_o^{\max} = \frac{15V(1.1)}{1 + \frac{36K(0.95)}{18K(1.05)}} = 5.87V$$

$$V_o^{\min} = \frac{15V(0.95)}{1 + \frac{36K(1.05)}{18K(0.95)}} = 4.20V$$

Monte Carlo Analysis Example (cont.)



Histogram of output voltage from 1000 case Monte Carlo simulation.

See table 5.1 for complete results.

Temperature Coefficients

- Most circuit parameters are temperature sensitive.

  $P=P_{nom}(1+\alpha_1\Delta T+ \alpha_2\Delta T^2)$ where $\Delta T=T-T_{nom}$

  P_{nom} is defined at T_{nom}
- Most versions of SPICE allow for the specification of TNOM, T, TC1(α_1), TC2(α_2).
- SPICE temperature model for resistor:

  $R(T)=R(TNOM)*[1+TC1*(T-TNOM)+TC2*(T-TNOM)^2]$
- Many other components have similar models.