From Smith's Predictor to Model-based Predictive Control

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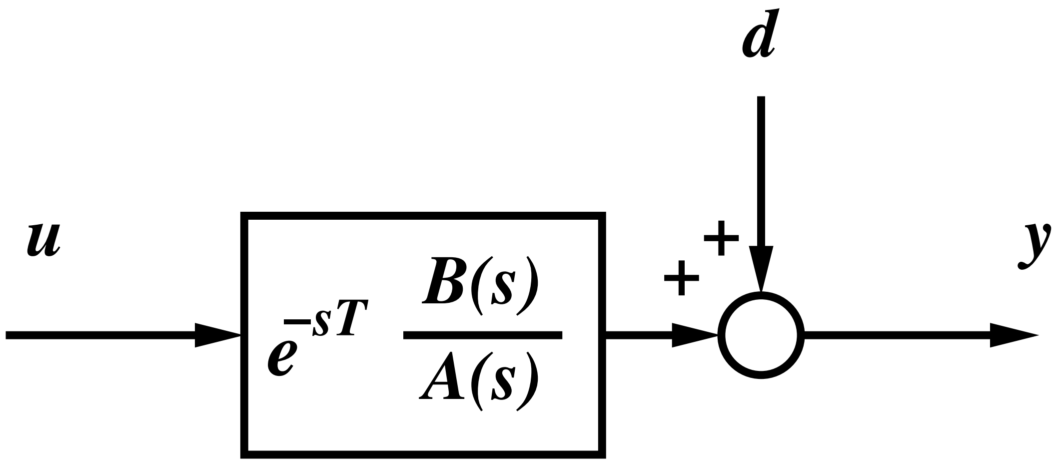
Outline

- Classical predictive control
  - A simple system with time delay
  - Smith's predictor
  - Åström's predictor
  - Time delay emulation
- Model-based predictive control
  - Intermittent control
- *References [n] in notes*

A System with Delay

$$\begin{cases} \dot{X}(t) & = AX(t) + BU(t-T) \\ Y(t) & = CX(t) + D(t) \end{cases} \quad (1)$$

$$y(s) = e^{-sT}\frac{B(s)}{A(s)}u(s) + d(s) \quad (2)$$



SISO, LTI.

Delayed input

State-space (1)

Transfer-function (2)

T Delay value

e^{-sT} Time delay

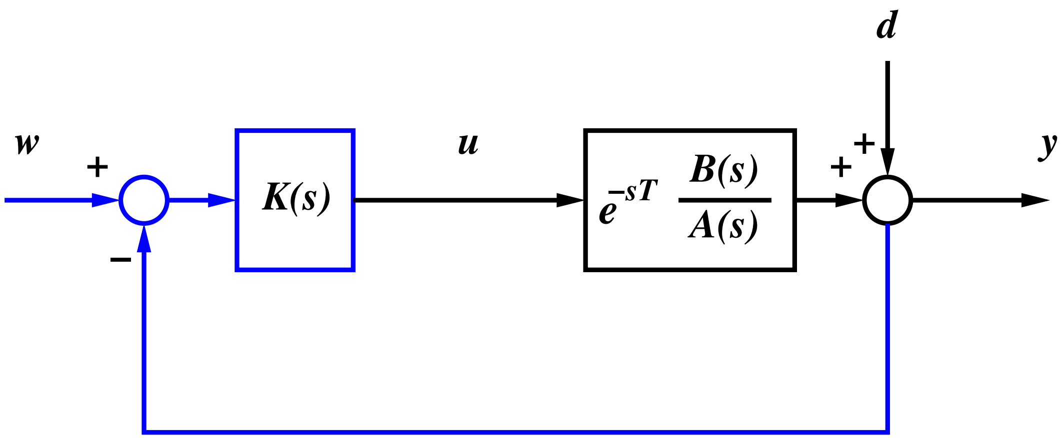
$e^{-sT}\frac{B(s)}{A(s)}$ system

d disturbance

u control signal

y system output

Non-predictive Control[1]

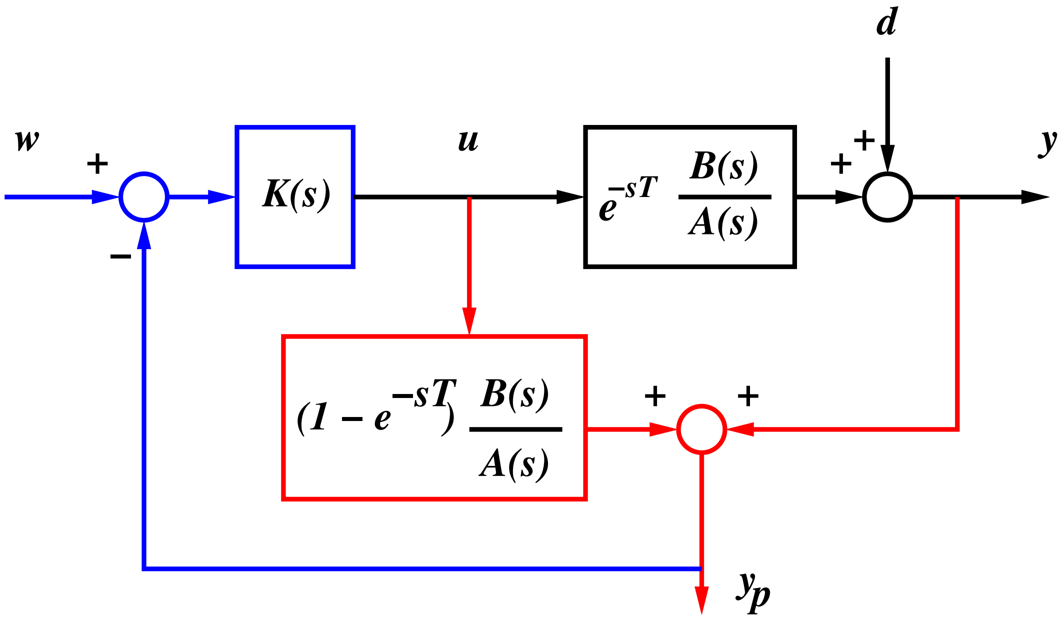


$$u = K(s)[w - y] \tag{3}$$

$$y = e^{-sT} \frac{K(s)B(s)}{A(s) + e^{-sT}K(s)B(s)} w + \frac{A(s)}{A(s) + e^{-sT}K(s)B(s)} d \tag{4}$$

- $K(s)$ Control compensator
- Feedback control (3)
- Closed-loop system (4)
- e^{-sT} inevitable in numerator
- **Problem** e^{-sT} in denominator
- hard to design $K(s)$

Smith's Predictor [2, 3]

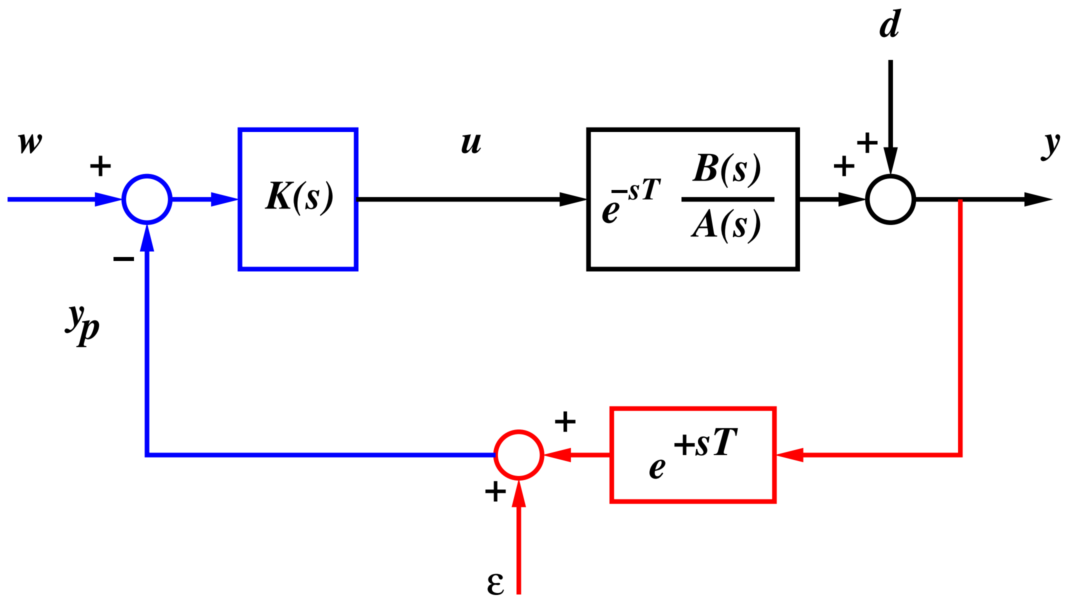


e^{-sT}	Time delay
$e^{-sT}\frac{B(s)}{A(s)}$	system
T	Delay value
$K(s)$	Controller
d	disturbance
u	control signal
w	Setpoint
y	system output
y_p	prediction
ε	prediction error

$$y_p = y + (1 - e^{-sT})\frac{B(s)}{A(s)}u \qquad (5)$$

$$= e^{sT}(y + \varepsilon); \qquad \varepsilon = \left(1 - e^{-sT}\right) d \qquad (6)$$

Smith's Predictor: equivalent diagram

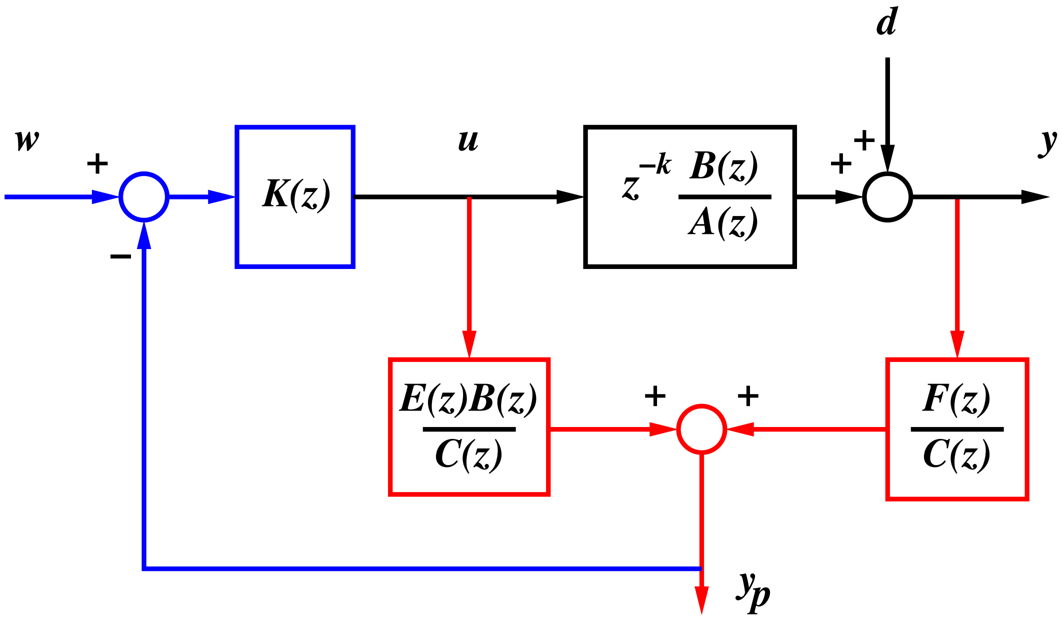


- **+** Delay removed from denominator
- **-** Initial conditions $A(s)$ ignored
- **-** So no good if system *unstable*
- **-** Properties of d not used

$$y = e^{-sT} \frac{K(s)B(s)}{A(s)+K(s)B(s)} (w-e) \tag{7}$$

$$+ \frac{A(s)}{A(s)+K(s)B(s)} d \tag{8}$$

Åström's Predictor[4]



z^{-k} time delay

k integer delay. $k = \frac{T}{h}$

Discrete-time

Stochastic (9)

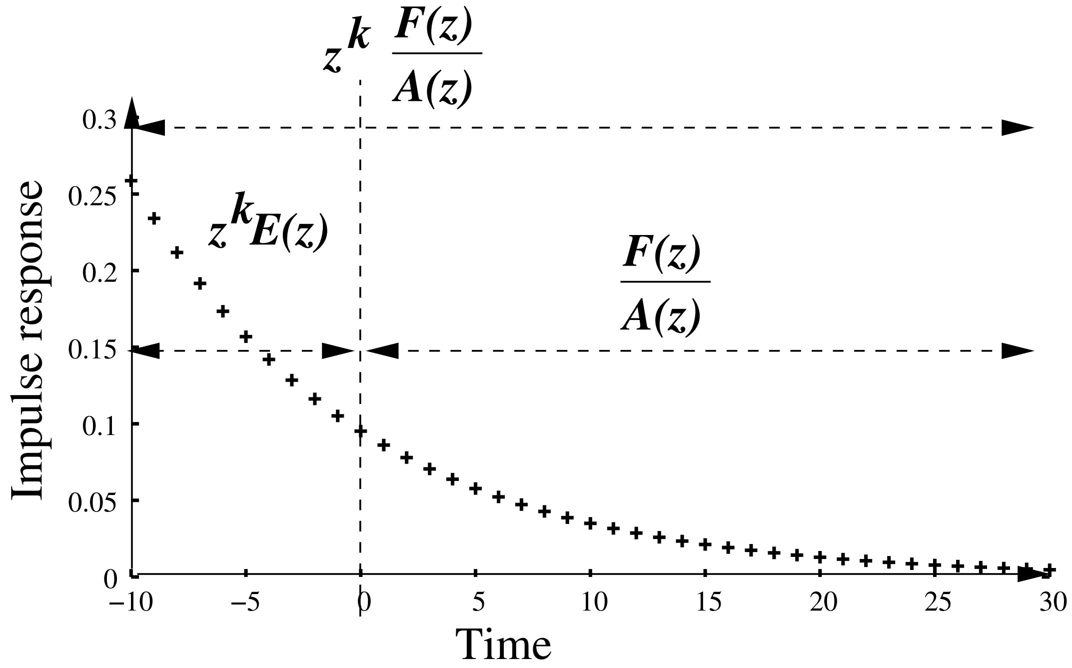
Minimum-variance $K(s) \to \infty$

Realisability decomposition (10)

$$d = \frac{C(z)}{A(z)} \xi \tag{9}$$

$$\frac{C(z)}{A(z)} = E(z) + z^{-k} \frac{F(z)}{A(z)} \tag{10}$$

Realisability decomposition



Algebraic long division (16)

"Future" $z^k E(z)$

"Past" $\frac{F(z)}{A(z)}$

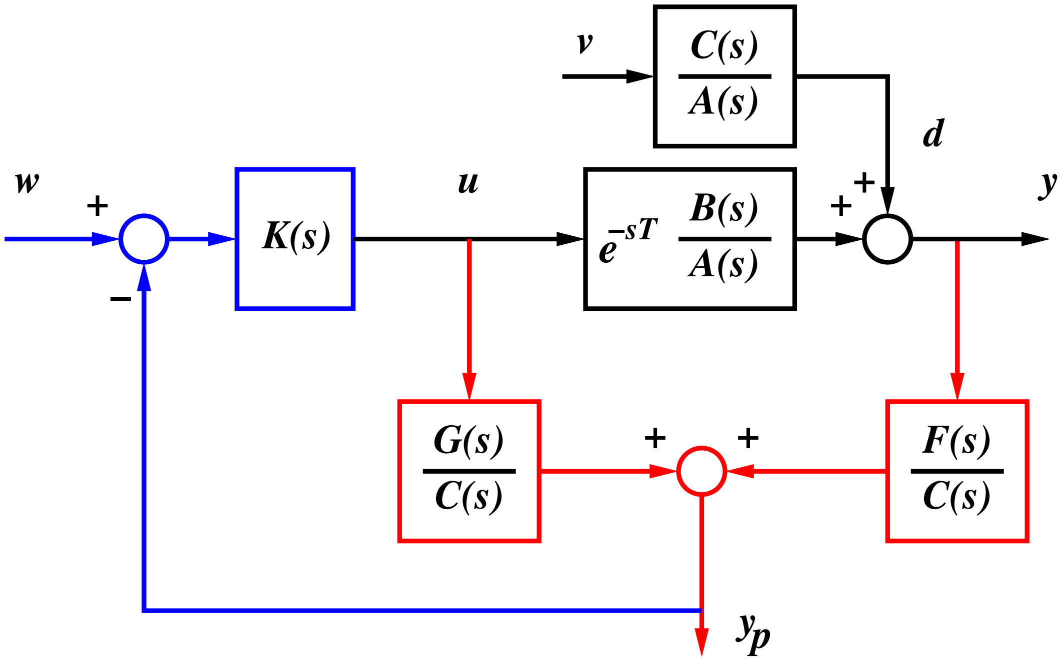
Extensions

- self-tuning[5, 6]
- generalised MV[7, 8, 9]

$$d = \frac{C(z)}{A(z)}\xi \tag{11}$$

$$\frac{C(z)}{A(z)} = E(z) + z^{-k}\frac{F(z)}{A(z)} \tag{12}$$

Emulator-based control[10, 11]



Continuous-time

Non-stochastic v

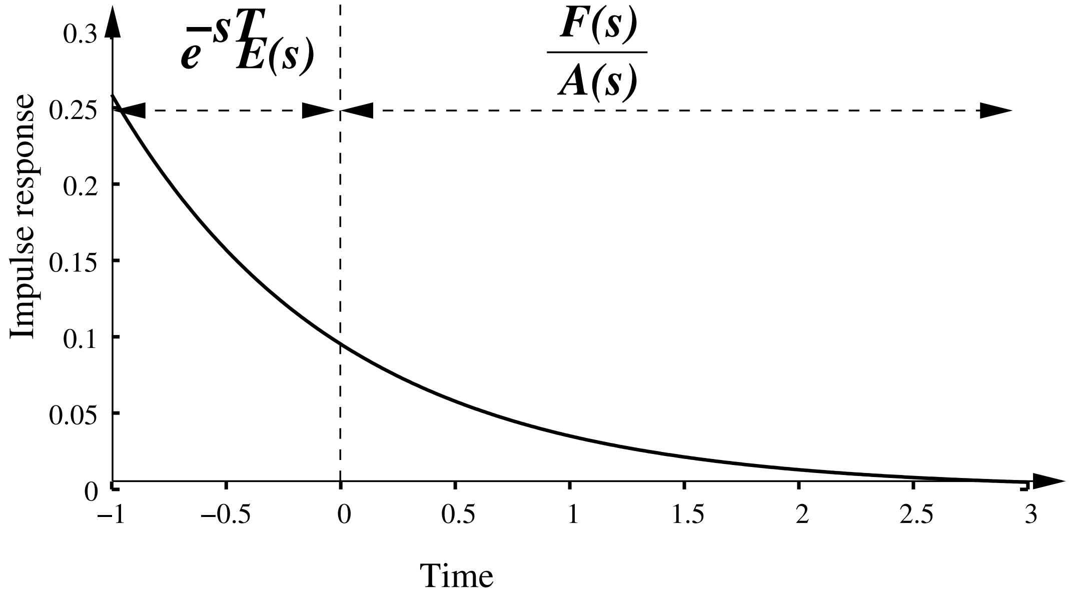
Minimum-variance $K(s) \to \infty$

Realisability decomposition

$$y_p = \frac{F(s)}{C(s)}y + \frac{G(s)}{C(s)}u \qquad (13)$$

$$= e^{sT}y + e; \qquad e = e^{sT}Ev \qquad (14)$$

Realisability decomposition



"Future" $e^{sT}E(s)$

"Past" $\frac{F(s)}{A(s))}$

self-tuning [10, 11, 12, 13]

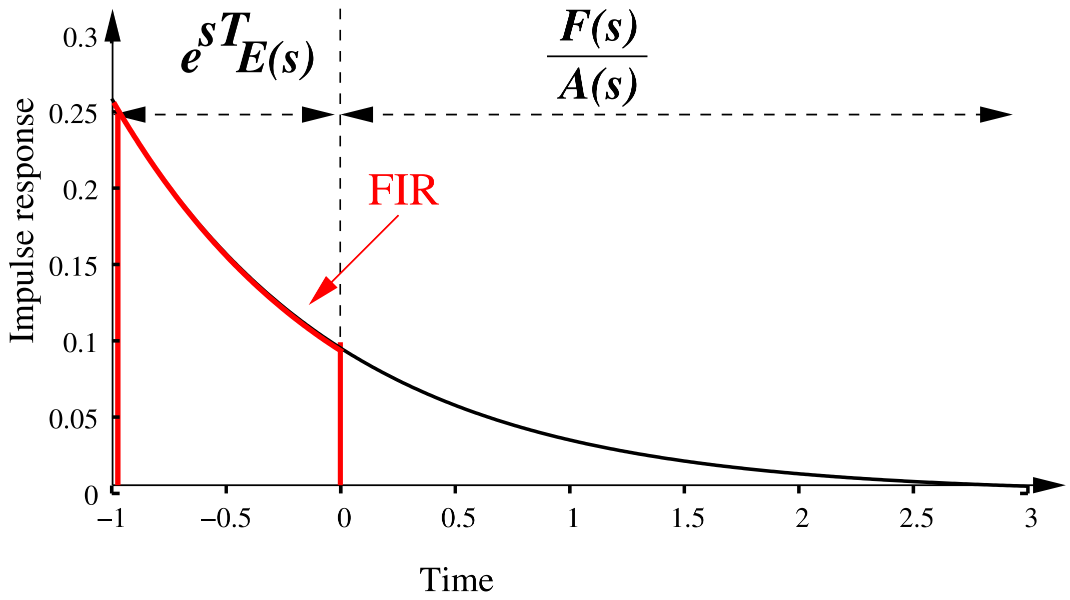
cf "Model predictive control"[14, 15, 16]

$E(s)$ FIR transfer function

$$d = \frac{C(z)}{A(z)}\xi \tag{15}$$

$$\frac{C(z)}{A(z)} = E(z) + z^{-k}\frac{F(z)}{A(z)} \tag{16}$$

Continuous-time Finite Impulse Responses

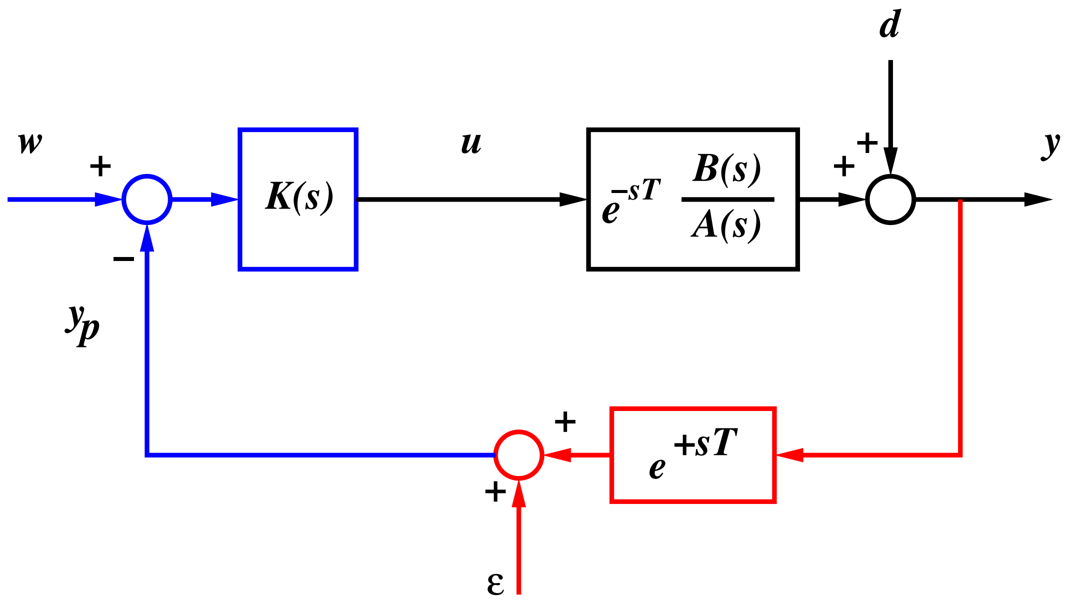


- Eg: $A(s) = s + a$
- Eg: $C = 1$
- Pole of E(s) has zero residue
- Implementation issues

$$E(s) = \frac{1 - e^{-(s+a)T}}{s+a} \tag{17}$$

$$\frac{F(s)}{A(s)} = \frac{e^{-at}}{s+a} \tag{18}$$

Emulator: equivalent diagram



$$\varepsilon = e^{sT} E(s) v = e^{sT} E(s) \frac{A(s)}{C(s)} d \qquad (19)$$

+ Delay removed from denominator
+ Initial conditions $C(s)$, not $A(s)$
+ So OK even if system *unstable*

$C(s)$ design parameter

Summary

- Emulator-based Predictor
  - removes delay e^{-sT} from denominator
  - accounts for initial conditions
  - sensitivity analysis? [3]
- Extensions: can emulate

  e^{st} Prediction

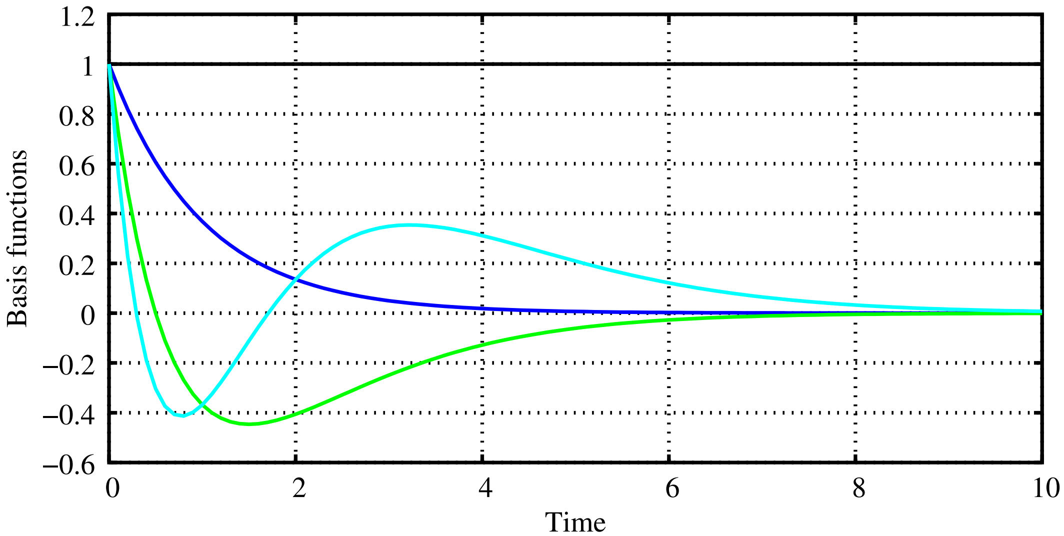
  $P(s)$ Improper transfer function

  $\frac{1}{B^{\star}(s)}$ Unstable transfer function
- Self-tuning Control [10, 11, 17]
- Cannot predict further ahead than T

Model-based Predictive Control

- Background
  - Long history [16, 18, 19, 20]
  - Related to Generalised Predictive Control[21, 22, 23]
  - Related to "Open-loop feedback optimal" control[24, 25]
  - Mostly discrete time[18]
  - Continuous time possible [26, 27, 28]
  - Predicts ahead further than the time delay
  - Trajectory based
- Current research on Intermittent Predictive Control
  - Overcomes delay due to optimisation
  - Physiological interpretation

Parameterising the control signal[29]



$u^\star(t,\tau)$ Control signal

$U^\star(\tau)$ Basis funs

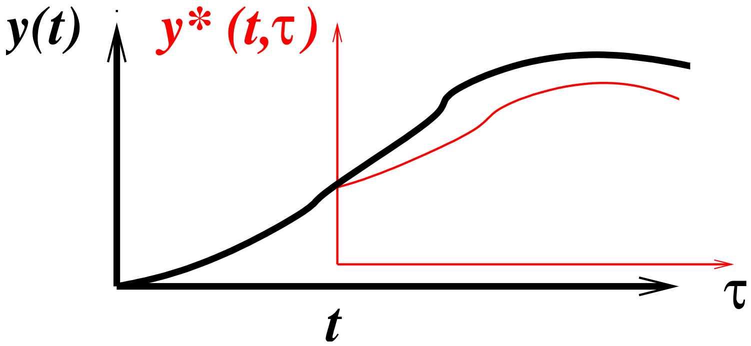
$U(t)$ Parameters to be optimised

t Actual time

τ Time-to-go

$$u^\star(t,\tau) = U^\star(\tau)U(t) \tag{20}$$

Moving Horizons



$$\begin{cases} \frac{d}{dt}x(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \\ x(0) &= x_0 \end{cases} \quad (21)$$

$$\begin{cases} \frac{d}{d\tau}x^\star(t,\tau) &= Ax^\star(t,\tau) + Bu^\star(t,\tau) \\ y^\star(t,\tau) &= Cx^\star(t,\tau) \\ x^\star(t,0) &= x_0^\star \end{cases} \quad (22)$$

- * Moving horizon
- **21** Fixed axes
- **22** Moving axes
- $x_0^\star = x(t)$
- $u(t) = u^\star(t,0)$
- **Optimise** in *moving* axes
- **Control** applied in *fixed* axes

Optimisation

$$J(U(t)) = \frac{1}{2}\int_{\tau_1}^{\tau_2} \left\| y^\star(t,\tau) - w^\star(t,\tau) \right\| d\tau \quad (23)$$

$$+ \frac{1}{2}\int_{\tau_1}^{\tau_2} \left\| u^\star(t,\tau) \right\| d\tau \quad (24)$$

$$+ \left\| (x^\star(t,\tau_2) - x_w(\tau)) \right\|_P \quad (25)$$

$$u^\star(t,\tau_{uk}) \leq \bar{u}^\star(t,\tau_{uk}) \quad (26)$$

$$y^\star(t,\tau_{yk}) \leq \bar{y}^\star(t,\tau_{yk}) \quad (27)$$

23 Output cost

24 Input cost

25 Terminal cost

26 Input constraint

27 Output constraint

QP to determine $U(t)$

Intermittency[30, 31, 32, 33]

- In predictive control

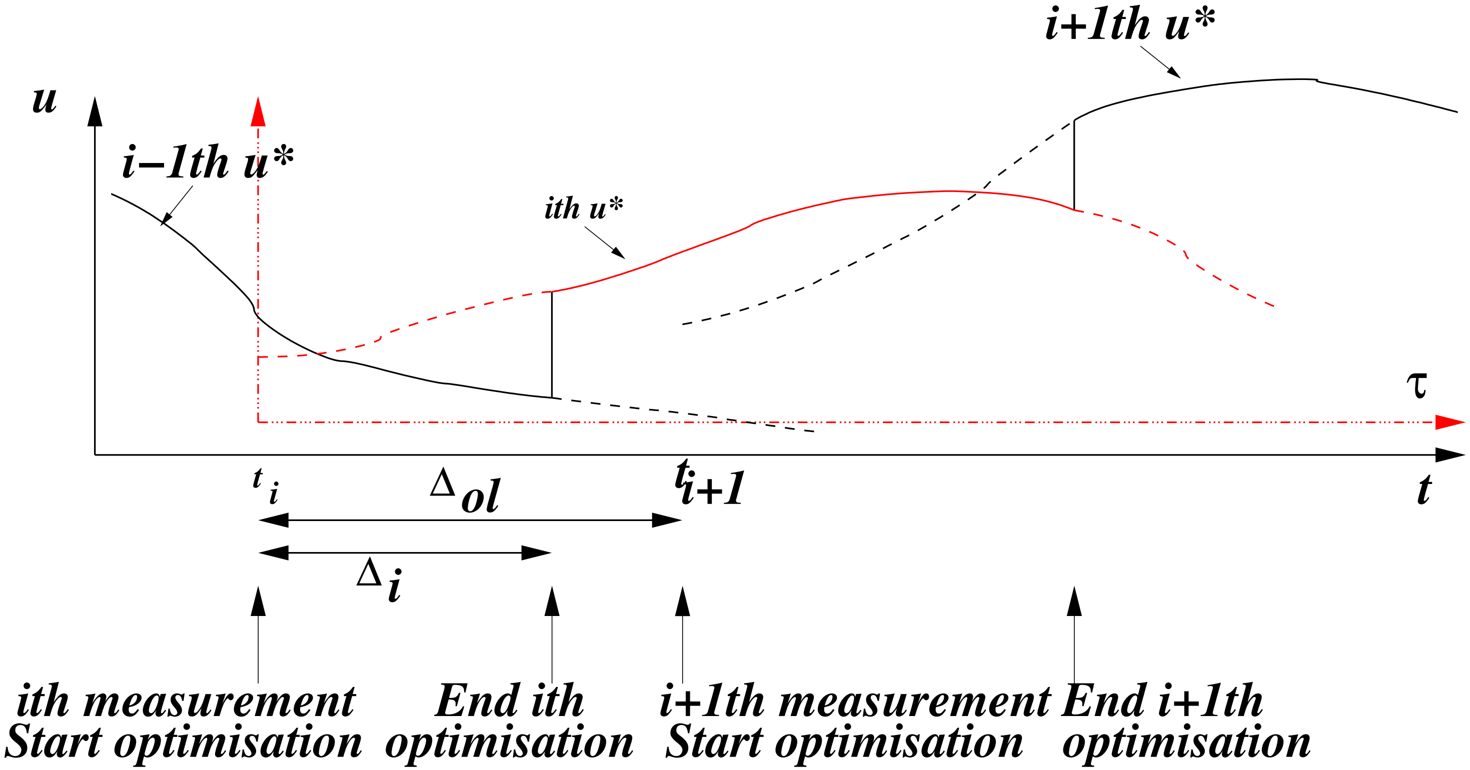
> "Continuous-time predictive control algorithms have the apparently fatal drawback that optimisation must be completed within an infinitesimal time. However, this problem can be overcome using intermittent control" [30]

- In physiological control

> "A finite interval of time is required by the CNS [central nervous system] to preplan the desired perceptual consequences of a movement ... This behaviour introduces *intermittency* into the planning of movements." [31]

- Neither continuous-time nor discrete-time

Intermittent Control

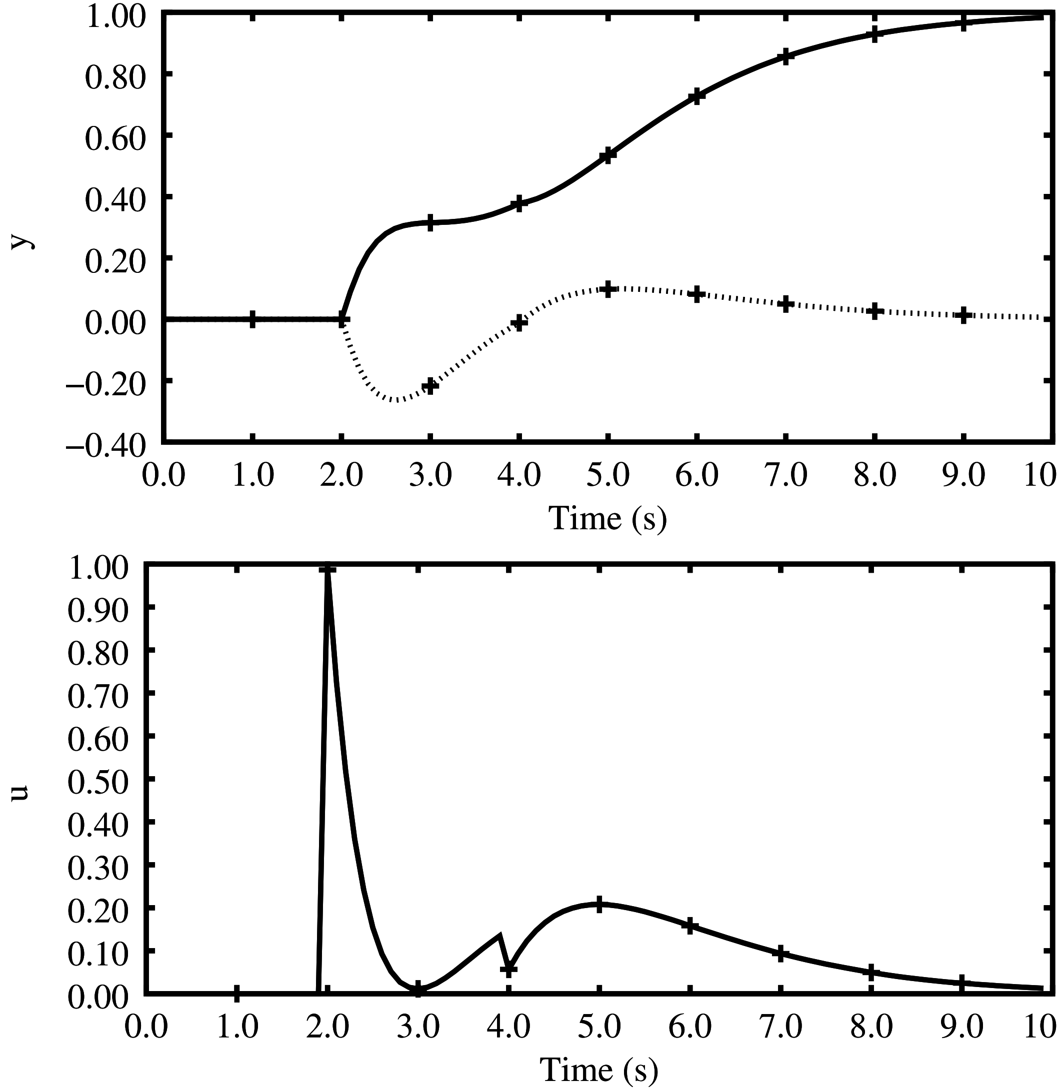


$$u(t) = u(t_i + \tau_i) = \begin{cases} u^\star_{i-1}(\tau_{i-1}) & \tau_i < \Delta_i \\ u^\star_i(\tau_i) & \tau_i \geq \Delta_i \end{cases} \tag{28}$$

A Physical System

- Lego Mindstorms Cart-Pendulum System
- legOS posix-compliant real-time kernel
  - compute $u(t)$
- Laptop Optimisation
  - compute $U(t)$
  - estimate state $X(t)$
  - estimate parameters θ
- IR connection to laptop
  - send $U(t)$.

Simulations



w**:** Unit step

y**:** System output Angle and position

u**:** System input

$\Delta_{ol}(=1.0)$**:** Open-loop interval

estimate state $X(t)$

estimate parameters θ

computational delay

Summary

- Model-based predictive control
  - Continuous-time setup
  - Basis function approach
  - Moving axes optimisation
- Intermittent control
  - Framework for MPC
  - Combines best of continuous-time & discrete-time
  - Physiological control systems
  - Engineering applications ...

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