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Cost Efficient Routing Strategies over Virtual Coordinates for Wireless Sensor Networks

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Abstract— **In this paper we focus on routing strategies for Wireless Sensor Networks over Hop Count (HC) virtual coordinates. We consider the problem of optimally delivering data packets by means of multi–hop forwarding techniques where we assume that each node in the network, upon the execution of a proper distribution algorithm, can obtain a hop count number, i.e., the minimum number of transmissions needed to get to the *sink* (destination) node on the shortest path. We exploit HCs in place of commonly considered geographical coordinates as a valuable indication of the direction towards the sink. Within this framework, we present localized *greedy routing* schemes and compare them against globally optimal solutions, where the objective is to minimize a properly defined cost function. Further, we present novel routing algorithms where the statistical knowledge of the minimum costs of second order (two hops away) neighboring nodes is used as an aid to drive the forwarding process. These *statistically enhanced* schemes are found to outperform both hop count greedy approaches and geographical routing of up to one order of magnitude in terms of goodness of the selected path.**

I. Introduction

In the last few years, advances in the hardware for wireless networking, micro–fabrication and embedded microprocessors technologies have made it possible to manufacture a generation of large scale sensor networks of very small nodes in a cost–effective way. Wireless Sensor Networks (WSNs) are applicable to a large range of identification and field measurement problems for both civilian and military applications (an overview of sensor networks related problems and application fields can be found in [1]). Two basic and very important functionalities of WSNs are data forwarding and area coverage capability. Nodes in the sensor field are equipped with sensors that enable them to gather information from the surrounding environment. This information is then forwarded through the sensor field to reach special devices, usually referred to as *sink nodes* (SNs), where the information gathered by the sensor field is collected, processed, and eventually sent to a central database. Sensor nodes usually have very limited computing, memory and energy resources and for these reasons they cannot perform complex operations.

One of the most challenging problems in such a complex and resource constrained scenario is to provide energy efficient solutions for data forwarding, so as to prolong network lifetime. Further, due to the energy/computational constraints characterizing micro–sensors, it is desirable to obtain this goal by means of very simple algorithms. In WSNs, classical pro–active [2] or reactive [3][4][5] routing algorithms proposed for ad hoc networks are not a good solution as they need a substantial exchange of information among nodes to update routing tables. In WSNs, due to energy constraints, this approach is not usable. Instead, it is better to route packets by means of *localized* schemes [6][7][8], where the next hop is decided, at every node, based on a local view of the network and on a possibly limited amount of global information [6]. A typical approach in this sense is given by geographical routing schemes [7], where the next hop is selected based on node geographical coordinates. On the one hand, geographical algorithms route packets in a best effort manner and only based on node distances. For this reason they do not give any guarantee on the global quality (cost) of the selected path. On the other hand, they have the property of being very scalable and of exchanging a very limited amount of routing information between nodes. In [8], the authors focus on optimal advancements for geographical routing in faded channels, by also studying the impact of link layer ARQ. In the present paper we propose new routing solutions that, while retaining the good qualities of geographical routing, are also able to minimize a properly defined cost function so that the selected path will be in some sense optimal. The concepts driving the design of these schemes are similar to the ones in geographical routing. However, two novelties are introduced. First of all, node coordinates are replaced by virtual coordinates, by substituting geographical positions with node hop counts (HCs). Further, we devise optimal localized routing algorithms over HC virtual coordinates. In particular, HCs are used as an indication for the direction to follow to get to SN, while both local information (nodes within range) and statistical knowledge regarding the cost of out–of–sight nodes are exploited in making forwarding decisions.

The paper is structured as follows. In Section II we introduce WSN and cost models. In Section III we briefly introduce the concept of hop count. In Section IV we propose two greedy routing schemes based on the HC concept, whereas in Section V we devise novel next hop selection rules which take into account statistical information of neighbors located two hops away. In Section VI we compare both greedy and statistically enhanced HC schemes against optimal solutions and geographical routing. Finally, Section VII concludes the paper.

II. Sensor network model

We model the network as a weighted graph $\mathcal{G} = (\mathcal{M}, \mathcal{A})$, where $\mathcal{M}$ is the set of nodes and $\mathcal{A}$ is the set of "arcs" or links between nodes. Among the $m = |\mathcal{M}|$ nodes in $\mathcal{M}$, we consider a special device called *sink node* (SN) with the function of gathering, storing and processing network messages. The function

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of the remaining $m-1$ nodes is instead to generate traffic and to forward packets towards SN using a multi-hop routing technique. The set $\mathcal{A}$ is a set of ordered pairs (i, j), where $i, j \in \mathcal{M}$. The pair (i, j) is referred here to as the directed arc (or link) connecting node i to node j. It is worth noting that we are keeping the communication/connectivity model as general as possible, without making any specific assumption on physical channel model, modulation and coding techniques. Therefore, any further specification on radio propagation characteristics, fading, etc. will not affect the validity of the model. Our analysis is based upon the concept of *neighboring sets*, i.e., upon sets of nodes within coverage of a given node and at a given time instant. We stress that neighboring sets may dynamically vary between subsequent forwarding actions, depending on the network configuration, fading phenomena, connectivity model and, among other factors, node sleeping cycles [7][9]. It would therefore be infeasible to derive these sets once for all, e.g., at the beginning of network operations, whereas it is reasonable to obtain neighboring sets *on–demand*, when the forwarding decision is actually made. Each arc $(i, j) \in \mathcal{A}$ is characterized by a cost c_{ij}, which is strictly dependent on the resources that are needed to transmit a message from node i to node j. Such a function could be related, for instance, to the energy required to transmit a single information bit, but many other factors can also be taken into consideration, e.g., the congestion level at node j (which may be represented by the state of its queue), the node failure probability or the residual energy of the nodes, if they have to rely on a limited energy reserve. In our investigation, we do not propose a specific model to determine such costs, so as to keep our framework as general as possible, but refer to related papers where the issue of cost computation is addressed. We refer to the hop count (HC) of a given node $i \in \mathcal{M}$, HC(i), as the the minimum number of transmissions needed to get to the *sink* on the shortest path. Furthermore, we define $\mathcal{N}_i$ as the set containing all the neighboring nodes of node i. We further define $\mathcal{N}_i(n)$, $\mathcal{N}_i(n-1)$ and $\mathcal{N}_i(n+1)$, $n \in \mathbb{N}^+$ as the sets of neighbors of node i with hop count (HC) equal to n, $n-1$ and $n+1$, respectively, where $\mathcal{N}_i = \mathcal{N}_i(n-1) \cup \mathcal{N}_i(n) \cup \mathcal{N}_i(n+1)$. A *path* from node i to node d is defined as an ordered list of nodes, i.e., $\mathcal{P} = \{i, r_1, r_2, \ldots, r_{n-1}, d\}$, where nodes i and d are referred to as the *source* and the *destination* node, respectively. The intermediate nodes r_j, $j \in \{1, 2, \ldots, n-1\}$ in $\mathcal{P}$ are referred to as relay nodes. The cost $C(\mathcal{P})$ of the entire path $\mathcal{P}$ connecting i to d is found as the sum of the costs associated with every link in the path[1]

$$C(\mathcal{P}) = c_{ir_1} + \sum_{j=1}^{n-2} c_{r_j r_{j+1}} + c_{r_{n-1}d} \qquad (1)$$

In the present work, we assume that the cost c_{ij} of the link between i and j does not depend on node i, that is, $c_{ij} = c_j$, $\forall\, j \in \mathcal{M}$. This assumption is made here to simplify the notation that will be used in the following analytical treatment. In fact, with minor modifications the analysis can be re-written for the general case where costs depend on both endpoints i and j. Also, we observe that such a simplification of the cost model is reasonable, for instance, in scenarios where all nodes transmit with the same constant power and/or when the objective is to minimize any metric that is independent of the *transmission power*. An example is given by the need to avoid congestion, where queue states could be used at each node to calculate congestion costs. In addition, we may relate costs to the residual energy at every device. Let E_j and E be the residual and initial energy of node j, respectively. The cost of transmitting to node j could therefore be written as $c_j = 1 - E_j/E$, where $0 \le E_j \le E$ and $c_j \in [0, 1]$. In this way, the smaller the amount of still available energy from a node's reserve, the higher the cost to communicate with it. In the sequel, without loss of generality we will consider normalized costs in $[0, 1]$. Observe that the cost model is itself very important, as different models influence the solutions found by cost–based routing algorithms. Details on how link costs can be actually derived can be found in [8][11][12][13][14]: in general, the link cost c_{ij} is written as $c_{ij} = f(f_1(\gamma_{ij}), f_2(a_j), f_3(E_j))$, where γ_{ij}, a_j and E_j are the SINR associated with link (i, j) the node j's advancement towards SN and its residual energy, respectively. Moreover, function $f_1(\cdot)$ accounts for modulation, coding and possibly ARQ, $f_2(\cdot)$ accounts for the advancement towards SN provided by node j and $f_3(\cdot)$ for its residual energy. These functions can be jointly considered through function $f(\cdot)$ that is often modeled as a weighted sum [13] or a product [12][14]. Both [8] and [13] focus on optimal forwarding in faded radio channels, by calculating optimal forwarding distances and related cost expressions. The algorithms presented in this paper can be used, with minor modifications, with any of these cost models.

[1]Under the hypothesis of additive cost function, see [10].

III. Setting up a Minimum Hop Count Field

In order to avoid the need to know the exact or the estimated geographical position of network nodes, the following gradient algorithm is adopted (similar to the one in [15]). The algorithm has to be re–executed only when network nodes change their positions. In the case of sensor networks characterized by fixed nodes, it has to be executed only once in the deployment start–up phase:

1) The sink node initially broadcasts a hop count packet (*hc_pkt*) with Hop Count (HC) value 1; all the sensors that receive this packet store this value.
2) Each node that receives a *hc_pkt*, say with hop count i, broadcasts a new *hc_pkt* with hop count $i + 1$. The procedure is repeated recursively until all nodes in the network have received and forwarded a *hc_pkt*.
3) If a node receives more than one *hc_pkt*, the one containing the *lowest hop count value* is considered to select the hop count value (HC) for the current node.

The scenario we have in mind is one where nodes may be stationary and are densely deployed and they can turn on and off providing a random topology. We also consider that each sensor can only transmit using a fixed power, i.e., no power control is accounted for.

IV. Greedy Routing Algorithms

Next, we report two HC based greedy routing algorithms. These scheme will be compared, in Section VI, against optimal routing rules. The first greedy routing scheme (**scheme 1**) is reported in Alg. 1, and is based on the same rationale as in [7],

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d ← sink node;
i ← current node;
repeat
    N_i(n−1) = {j ∈ N_i s.t. HC(j) < HC(i)};
    i* = argmin_{j∈N_i(n−1)} c_j;
    Break ties arbitrarily. i ← i*;
until i = d;
```

Algorithm 1: Scheme 1: greedy shortest path routing algorithm.

with the difference that here the next hop is selected depending on HCs rather than using geographical coordinates. When a data packet is to be transmitted at a given node, say node i with $\mathrm{HC}(i) = n$, set $\mathcal{N}_i(n-1)$ is obtained from $\mathcal{N}_i$ by picking all nodes $j \in \mathcal{N}_i$ for which $\mathrm{HC}(j) < \mathrm{HC}(i)$ and the forwarding process is driven towards the lowest cost node in $\mathcal{N}_i(n-1)$. In this way, we force a selection among those nodes leading to a positive (maximum) hop count advancement towards the sink. Therefore, scheme 1 selects shortest paths to SN. The second scheme (**scheme 2**) routes packets towards the lowest cost node in the set $\mathcal{N}_i(n-1) \cup \mathcal{N}_i(n)$. This routing scheme aims at finding the *lowest local cost node* leading to a HC advancement towards the sink greater than or equal to zero. It shall be observed that scheme 2 aims at selecting optimal local solutions, i.e., to select the minimum cost node contained in the local view of the local forwarder. Scheme 2 may be obtained from Alg. 1 by substituting $\mathcal{N}_i(n-1)$ with $\mathcal{N}_i(n-1) \cup \mathcal{N}_i(n)$ and $<$ with a $\leq$.

V. Statistically Assisted Routing Algorithm

A. Routing as a Sequential Decision Problem

We formulate the routing problem as a sequential decision process, where at every decision stage a node has to take a specific action, i.e., to select the best relay node for the current transmission. In this work, we are interested in *on–line* routing algorithms, where such forwarding decisions are made based on local knowledge and on statistical measures related to the second-order (two-hops away) neighbors of the current node. Here, the *local knowledge* is the cost information for the nodes within radio range. Throughout the paper, we will assume this information as available through an ideal MAC and at no cost. In fact, our main goal is to devise a forwarding criterion to guarantee low cost paths; how this criterion can be combined with practical MAC schemes is the objective of our current and future research and is not within the scope of the present paper. Next, we define some quantities to proceed with our analysis. First, we assume that the currently occupied node is node $i \in \mathcal{M}$, that its hop count is $\mathrm{HC}(i) = n$ and that the forwarding process is at stage $t \in \mathbb{N}$, where time evolves one unit every decision stage (i.e., every forwarding action). The problem to be solved is to decide which is the best relay among the nodes in sets $\mathcal{N}_i(n)$ and $\mathcal{N}_i(n-1)$. Nodes in set $\mathcal{N}_i(n+1)$ are not considered as they very unlikely lead to satisfactory solutions.[2] We refer to $j_{n-1}^t \in \mathcal{N}_i(n-1)$, $j_n^t \in \mathcal{N}_i(n)$ and to c_{n-1}^t, c_n^t as the minimum cost nodes[3] in sets $\mathcal{N}_i(n)$ and $\mathcal{N}_i(n-1)$ and their associated costs, respectively. We further define *forwarding cycle* as the sequence of steps between the forwarding stage where a node with hop count n is reached for the first time and the stage where a neighbor with hop count $n-1$ is eventually selected as relay. That is, a cycle is the number of stages it takes the decision maker (packet) to advance one hop towards SN. The objective of our routing algorithms is to minimize the total cost of the final selected path, which is computed as in Eq. (1). Observe that the whole path can be decomposed as a disjoint sequence of n forwarding cycles[4] and its total cost equals the summation of the costs incurred in each cycle. Moreover, assuming that forwarding cycles are statistically independent, the minimization of the expected cost associated with the entire path is obtained by minimizing the expectation of each term taken separately. In such a case, devising a scheme to minimize the cost of the full path is equivalent to devising an algorithm to minimize the expected cost of forwarding cycles. These schemes are presented in the following Section V-B.

B. Optimal Routing Rules

Consider $t \geq 0$ to be the current forwarding step and consider node i, with $\mathrm{HC}(i) = n$, to be the currently occupied node. Moreover, let $j_{n-1}^t \in \mathcal{N}_i(n-1)$ and $j_n^t \in \mathcal{N}_i(n)$ and c_{n-1}^t and c_n^t, be the minimum cost nodes and their costs, respectively. The decision maker at time t has to choose a forwarding action, i.e., whether the packet has to be forwarded to node j_{n-1}^t or j_n^t. We define the action set and the decision maker's current state as $\mathcal{A}_t = \{a_{n-1}^t = j_{n-1}^t, a_n^t = j_n^t\}$ and $X_t = (c_{n-1}^t, c_n^t)$, respectively. Now, we suppose that if action $a(t) \in \mathcal{A}_t$ is taken when in state X_t, at time $t \geq 0$, a cost $C(X_t, a(t)) \geq 0$ is incurred. Furthermore, let time $t = 0$ be the stage associated with the beginning of a new cycle. For any policy π, the total expected cost incurred over time when $X_0 = X$ is the initial state is defined as [16]

$$V_\pi(X) = E_\pi \left[\sum_{n=0}^{+\infty} C(X_n, a(n)) | X_0 = X \right] \quad (3)$$

Observe that $V_\pi(X)$ is well defined, though possibly infinite as $C(X_t, a_t) \geq 0$. Moreover, let $V(X) = \inf_\pi V_\pi(X)$ be the minimum expected cost under any policy. We say that a policy π^* is optimal if $V_{\pi^*}(X) = V(X)$, $\forall\ X$. In order to seek for an optimal policy, we start citing the main theoretical result for negative dynamic programming [16], i.e., that the optimal policy is determined by the optimality equation as follows

$$V(X_t) = \min_{a(t) \in \mathcal{A}_t} \left[C(X_t, a(t)) + \int_{\mathcal{D}_X} V(X_{t+1})\, \mathrm{d}F(X_{t+1}) \right] \quad (4)$$

where X_t and X_{t+1} are the current and the next state, respectively, $C(X_t, a(t))$ is the cost incurred at the current decision stage, where we consider $C(X_t, a_{n-1}^t) = c_{n-1}^t$ and $C(X_t, a_n^t) = c_n^t$, the term $\int_{\mathcal{D}_X} V(X_{t+1})\, \mathrm{d}F(X_{t+1})$ accounts for the average cost incurred in future decisions, $\mathcal{D}_X$ is the domain set of X_{t+1} and $F(X_{t+1})$ is the cdf governing the state for the next forwarding step. Our forwarding process can be modeled as an optimal stopping problem, where at

[2] This was verified through simulation as well as by previous work [7].

[3] In case there should be multiple nodes with the same minimum cost in one of these two sets, we break ties arbitrarily as they are, by definition, equivalent.

[4] If the starting node $i \in \mathcal{M}$ has $\mathrm{HC}(i) = n$.

$$\begin{aligned}\mathcal{B}_1 &= \left\{X_t : C(X_t, a_{n-1}) \le C(X_t, a_n) + \int_{\mathcal{D}_X} C(X_{t+1}, a_{n-1})\, \mathrm{d}F(X_{t+1})\right\} \\ &= \left\{X_t : c_{n-1}^t - c_n^t \le \mathcal{E}\right\} \end{aligned} \quad (2)$$

the generic stage t the decision maker can decide to either continue or stop. In the former case, a cost $C(X_t, a_n^t) = c_n^t$ is paid and the cycle is continued, whereas in the latter the cycle is ended with a final cost $C(X_t, a_{n-1}^t) = c_{n-1}^t$ and the integral $\int_{\mathcal{D}_X} V(X_{t+1})\, \mathrm{d}F(X_{t+1})$ is zero because once the cycle has ended all the future costs are zero. $V(X_t)$ (Eq. (4)) can therefore be written as

$$\min\left[C(X_t, a_{n-1}), C(X_t, a_n) + \int_{\mathcal{D}_X} V(X_{t+1})\, \mathrm{d}F(X_{t+1})\right] \quad (5)$$

Now, let us focus on set $\mathcal{B}_1$ in Eq. (2) on the top of the page, where

$$\mathcal{E} = E[c_{n-1}^{t+1}] = \int_0^1 c_{n-1}^{t+1}\, \mathrm{d}F_{min}(c_{n-1}^{t+1}) \quad (6)$$

is the expected minimum cost among nodes with hop count $n-1$ at stage $t+1$ and $F_{min}(\cdot)$ is the minimum cost cdf. $\mathcal{B}_1$ contains all states for which stopping is (on average) at least as good as continuing exactly for one more period and then stopping. The policy that stops the first time the process enters set $\mathcal{B}_1$ is called *one–stage look–ahead policy*. In the sequel, we will focus on this policy first. The one–stage optimal policy tells us to stop at the stage in which set $\mathcal{B}_1$ is entered for the first time, i.e., at time t we should select node j_{n-1}^t and end the cycle only if $c_{n-1}^t - c_n^t \le \mathcal{E}$.

Before discussing about the optimality of the above one–stage policy we need to introduce some quantities related to the decision process. At every stage $t \ge 0$, the decision maker is asked to make a decision in the set $\mathcal{A}_t = \{a_{n-1}^t, a_n^t\}$. If decision a_{n-1}^t is made node j_{n-1}^t is selected and the cycle ends with a total cost $C_{tot}(t)$, where

$$C_{tot}(t) = C_{par}(t) + c_{n-1}^t \quad (7)$$

where $C_{par}(t)$ is obtained as

$$C_{par}(t) = \begin{cases} 0 & t < 1 \\ \sum_{i=0}^{t-1} c_n^i & t \ge 1 \end{cases} \quad (8)$$

Instead, if decision a_n^t is made, the cycle is continued towards node j_n^t with an accumulated partial cost $C_{par}(t+1)$. Observe that when $C_{par}(t+1) \ge C_{tot}(t)$ there is no point in further searching for a better path and the cycle should end. The minimum cost of the paths encountered by the decision maker up to and including time t is evaluated as

$$C_{tot}^{min}(t) = \min_{0 \le k \le t}\left\{C_{tot}(k)\right\} \quad (9)$$

A graphical representation of the decision tree and related quantities is reported in Fig. 1. In the following, we prove that the one–stage policy dictated by set $\mathcal{B}_1$ above is not globally optimal.

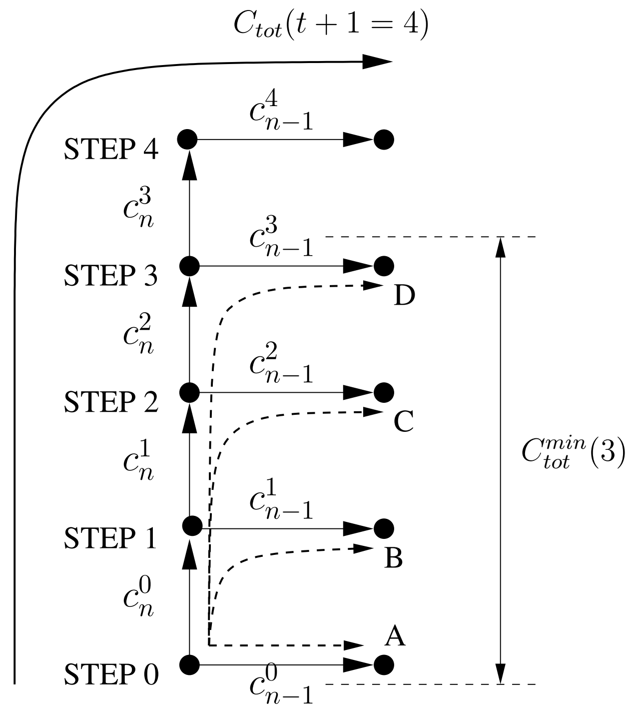

Fig. 1. Decision tree after 4 steps. The current stage is step $t = 3$, whereas c_{n-1}^4 is the unknown cost of the minimum cost node with hop count $n-1$ at the next step. A, B, C and D are all the possible paths encountered so far. $C_{tot}^{min}(3) = \min\{\mathrm{Cost}(A), \mathrm{Cost}(B), \mathrm{Cost}(C), \mathrm{Cost}(D)\}$.

Theorem 1 *The one–stage look-ahead forwarding policy defined by set $\mathcal{B}_1$ is not globally optimal.*

Proof: Let P1 be the one–stage policy dictated by set $\mathcal{B}_1$ above and let t_1 be the generic time at which set $\mathcal{B}_1$ is entered for the first time. Further, let P2 be a second policy that stops according to set $\mathcal{B}_1$ as long as $C_{tot}(t) = C_{tot}^{min}(t)$ and that stops with probability one when $C_{tot}(t) > C_{tot}^{min}(t)$. Note that, as P1 stops at time t_1 the following inequalities must hold: $c_n^0 - c_{n-1}^0 > \mathcal{E}$, $c_n^1 - c_{n-1}^1 > \mathcal{E}$, $\dots$, $c_n^{t_1-1} - c_{n-1}^{t_1-1} > \mathcal{E}$, $c_n^{t_1} - c_{n-1}^{t_1} \le \mathcal{E}$. Observe that for $t_1 \ge 1$ the previous constraints are not enough to guarantee that $C_{tot}(t_2)$ is equal to the minimum cost $C_{tot}^{min}(t_2)$ for every step $t_2 < t_1$. Therefore, on the average there is a non zero probability that P2 will stop at $t_2 < t_1$ given that t_1 is the stopping time for policy P1, and since this is true for every $t_1 \ge 1$ and the average cost of stopping at $t_2 < t_1$ is strictly lower than the average cost of stopping at t_1, the expected total cost of policy P2 is strictly lower than the expected total cost of policy P1. The theorem follows as another policy (P2) exists with a strictly lower expected cost than policy P1. □

In order to seek for an optimal policy to minimize the total cost of a cycle, let us consider the following extended version of the forwarding problem. At the generic step t, the decision maker must choose an action in the set $\mathcal{A}_t = (a_{n-1}^t, a_n^t)$, where


```
d ← sink node
s ← source node
C_tot^min ← +∞
C_acc ← 0
T ← ∅
repeat
    i ← current node
    node j_{n-1} s.t. c_{n-1} = min_{z∈N_i(n-1)} {c_z}
    node j_n s.t. c_n = min_{z∈N_i(n)} {c_z}
    C_tot^min ← min{C_tot^min, C_acc + c_{n-1}}

    if (C_tot^min − (C_acc + c_n) ≤ ℰ) or (j_n ∈ T) then
        - select next hop j ← j_{n-1}
        - C_acc ← 0
        - C_tot^min ← +∞
    else
        - select next hop j ← j_n
        - C_acc ← C_acc + c_n
    end

    Update tabu list T:
    begin
        - T ← T ∪ j
        - Delete nodes with age ≥ tabulen from T
    end
until i = d;
```

Algorithm 2: Multi–stage Statistically–Assisted greedy Routing Algorithm (SARA). A tabu list T is used to prevent ping-ponging between nodes at the same hop count distance.

a^t_{n-1} is equivalent to stopping thereby choosing node j^t_{n-1}, whereas action a^t_n means to continue the path search towards the current minimum cost node with hop count n (j^t_n). At every decision stage t, the decision maker knows the previously encountered costs $\{c^0_{n-1}, c^0_n, c^1_{n-1}, c^1_n, \dots, c^t_{n-1}, c^t_n\}$ and can therefore evaluate the minimum cost of all paths encountered so far $C^{min}_{tot}(t)$ (Eq. (9)). We now assign the cost $C^{min}_{tot}(t)$ to action a^t_{n-1}; this is the cost incurred in stopping at stage t. The decision maker should therefore stop as $C^{min}_{tot}(t) = C^{min}_{tot}(t+1)$ (*stopping rule*), where $C^{min}_{tot}(t+1) = \min(C^{min}_{tot}(t), C_{tot}(t+1))$. This is equivalent to stopping when continuing the forwarding process for one more stage ($t \to t+1$) does not lead to any improvement in terms of the cost of the solution found. It is worth observing that the *stopping rule* above tries to drive the forwarding process towards those paths for which $C^{min}_{tot}(\cdot)$ is strictly decreasing, i.e., to seek for the lowest cost solution. From what discussed above, it easily follows that the *stopping rule* is verified iff $C^{min}_{tot}(t) \le C_{tot}(t+1)$, ($C_{tot}(t+1) = C_{par}(t+1) + c^{t+1}_{n-1}$) and that the corresponding one–stage policy obeys the following stopping set

$$\mathcal{B}_2 = \left\{ X_t : C^{min}_{tot}(t) - C_{par}(t+1) \le \mathcal{E} \right\} \quad (10)$$

where, in this case $X_t = (c^0_{n-1}, c^0_n, c^1_{n-1}, c^1_n, \dots, c^t_{n-1}, c^t_n)$, while $\mathcal{E}$, $C_{par}(t+1)$ and $C^{min}_{tot}(t)$ are defined in Eqs. (6), (8) and (9), respectively. The one–stage policy dictated by the previous set states that it is convenient to stop at time t whenever the expected cost of stopping at time $t+1$ is greater than or equal to the minimum cost of all paths encountered through time 0 to t. In the following, we prove that the on–line forwarding strategy dictated by the stopping set $\mathcal{B}_2$ is globally optimal.

Theorem 2 *The one–stage look-ahead forwarding policy defined by set $\mathcal{B}_2$ is globally optimal.*

Proof: The result follows by showing that $\mathcal{B}_2$ is a closed set of states (see [16], Theorem 2.2, p. 54, or [17], p. 164). In particular, $\mathcal{B}_2$ is closed if $X_t \in \mathcal{B}_2$ implies that $X_\tau \in \mathcal{B}_2$, $\forall\, \tau > t$. The set rule can be re-written as $C_{par}(t+1) + \mathcal{E} \ge C^{min}_{tot}(t)$. Now, assume that this rule is verified at the generic time t with X_t, then it is verified for X_τ, $\forall\, \tau > t$ since $C^{min}_{tot}(t)$ and $C_{par}(t+1)$ are non increasing and non decreasing in t, respectively. □

The theorem above tells us the globally optimal behavior that the decision maker has to follow to minimize the expected cost of a cycle. We stress that the optimal on–line policy dictated by set $\mathcal{B}_2$ is the one leading to the minimum long term expected cost (Eq. (3)), among all on–line policies exploiting first (c^t_n, c^t_{n-1}) and second ($\mathcal{E}$, i.e., two-hops away nodes) order cost information. This is the scheme that in Section VI will be tested against the optimal cost solution obtained by means of an off–line shortest path tree algorithm [18]. This Multi–Stage algorithm is named **SARA** (*Statistically–Assisted greedy Routing Algorithm*); in Alg. 2, we report its full version, where a tabu list T is used to avoid loops and ping–ponging between nodes at the same hop count distance. The algorithm is a straightforward implementation of Eq. (10) (set $\mathcal{B}_2$).

VI. Results

As a reference model for the performance evaluation we consider a random topology network with normalized node density $\lambda_n = \lambda \pi R^2$, where λ is the average number of nodes per unit area, whereas R is the constant node transmission range. We consider a unit disk connectivity model, i.e., two nodes can communicate iff their distance is lower than or equal to R. However, we stress that the schemes presented here can work for any topology setting as λ and connectivity model just translate into different neighboring sets. In this sense, λ_n can be seen as the average number of nodes actually awake within coverage. In the following results, we consider a normalized density $\lambda_n = 15$, $R = 1$, nodes are randomly and uniformly deployed on a square area of $16R \times 16R$ and we focus on the performance of a source node (originator of the message) with hop count equal to 8. Moreover, every node is assumed to have a good estimate of the expected minimum cost $\mathcal{E}$ (Eq. (6))[5], in its second order neighborhood. This is assumed here with the aim of understanding how far we can get with statistically assisted policies. The performance reported in this section is therefore valid from a theoretical point of view and gives us an indication of the maximum achievable gains, i.e., under ideal MAC and with a perfect estimate of $\mathcal{E}$. As will be shown next, the performance gain is good and thereby encouraging

[5]As a reference model, in this section node costs are considered to be uniformly distributed in [0,1].

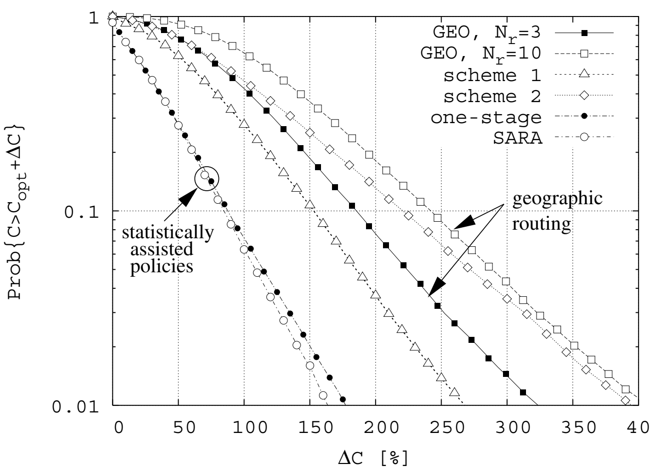

Fig. 2. Complementary distribution function of the cost difference between greedy routing and the (off–line) non–dominated optimal cost solution.

to proceed with further research. In addition to the hop count based schemes discussed above, we consider an idealized geographical routing algorithm where we subdivide the relaying area into a number N_r of priority regions, according to the related advancement towards the destination [7]. Moreover, we pick the relay node as the lowest cost node within the highest priority and non–empty region, i.e., the lowest cost node among those leading to the maximum advancement towards the destination. This is an idealized version of geographical forwarding for two reasons: we consider an ideal MAC and we always select the best (lowest cost) node leading to the maximum advancement. In the sequel, this scheme is compared with the previously proposed hop count routing strategies. In Fig. 2, we report the performance of the geographical routing scheme (GEO) against the on–line routing schemes presented in Sections IV and V. In particular, we plot the probability that the cost obtained with on–line algorithms will exceed the optimal cost solution by a given percentage ΔC of the total path cost. Clearly, geographical routing performs worse than both greedy (scheme 1 and scheme 2) and SARA HC algorithms, since maximizing geographical advancement is not a good strategy to select low cost paths. It shall also be observed that the node selection strategy considered here for the GEO scheme could be improved if advancements and costs are considered jointly. However, how to relate these two metrics to obtain good global solutions is not a simple task and is one of the objectives of our current research. Furthermore, it is worth observing that the exploitation of one–step ahead cost predictions makes SARA perform significantly better than both scheme 1 and scheme 2. If we consider scheme 1 (the *shortest path HC greedy scheme*) we note an advantage of about one order of magnitude when $\Delta C = 150\%$. In addition, Fig. 2 also reveals a tremendous improvement over pure geographical routing as N_r increases.

VII. Conclusions

In this work, we introduced and discussed hop count based routing schemes for wireless sensor networks. In these algorithms, the hop count information is taken into account to drive the forwarding process towards low cost paths. In particular, we have proposed SARA, a HC routing policy where statistical estimates of the cost regarding out–of–sight nodes are used as an aid to drive the forwarding process towards good solutions. Results show improvements of up to one order of magnitude with respect to both HC greedy forwarding and geographical routing. We observe that the achievable gains obtained in this work can be seen as upper bounds as they were obtained without accounting for estimation errors and considering an ideal MAC layer, i.e., the cost information regarding in range nodes is obtained at no communication cost. The next steps are therefore to propose a properly designed MAC scheme to be coupled with the SARA routing strategy and test the achievable gains with respect to geographical solutions. Further, we still need to devise good estimators for $\mathcal{E}$ and to verify the dependence on the estimation error.

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