Fountain Codes and their Application to Broadcasting in Underwater Networks: Performance Modeling and Relevant Tradeoffs

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ABSTRACT

Our aim in this paper is to study the performance of broadcasting algorithms for underwater acoustic sensor networks (UWASNs). The targeted scenario is very simple; we consider a source (the initiator of the broadcast transmission) and a number of nodes randomly placed within a given geographical area. For an efficient broadcast transmission we advocate the use of a hybrid ARQ scheme, where Fountain Codes (FC) are exploited to enhance the efficiency of the data dissemination process in the face of poor and possibly unknown channel conditions. FC codes, being rateless, are in fact able to adapt to diverse error rates and correct packet losses on the fly through the transmission of additional redundancy packets. The main contribution of this paper is a mathematical model to characterize the performance of fountain codes as applied to broadcasting in underwater networks. Our analysis allows us to find performance metrics such as transmission delay, reliability (e.g., percentage of covered users) and power consumption. Relevant tradeoffs are highlighted and quantified; in particular the implications of transmission power on covered distance, rate and delay are discussed. Even though we do not propose a practical broadcasting protocol here, the results and tradeoffs we obtain are essential to a proper design of practical schemes.

Categories and Subject Descriptors

C.2.1 [**Computer–Communication Networks**]: Network Architecture and Design; C.2.2 [**Computer–Communication Networks**]: Network Protocols

General Terms

Algorithms, Design, Performance

Keywords

Underwater acoustic networks, broadcast, fountain codes, hybrid ARQ, modeling, performance investigation.

1. INTRODUCTION

Underwater ad hoc networking is becoming a popular research theme in the scientific community. On one hand, underwater scenarios offer many important applications such as oceanographic data collection, pollution monitoring, offshore exploration, disaster prevention, etc., to name a few. On the other hand, however, many challenges are still to be faced and solved in this harsh communication environment. One for all, the propagation of sound in the underwater medium incurs long delays and is only possible at limited bit rates (which sharply decrease with distance and frequency). Also, underwater channels feature peculiar attenuation phenomena such as the so called shadow zones, i.e., channel conditions fluctuate and may momentarily drop due to flows of water at different temperatures. All of this makes the design of underwater communication devices and related networking protocols a difficult task.

The focus of this paper is on underwater acoustic sensor networks (UWASNs) and, in particular, on broadcasting algorithms for single-hop networks as well as on their implications on practical schemes for multi-hop networks. Research on networking protocols for UWASNs is quite recent. Some papers dealt with the design of MAC protocols [1–3] and some others [4,5] discussed a few preliminary routing schemes. Much, however, still has to be done in terms of protocol design. In this paper we try to partially fill this gap through the study of broadcasting policies and related tradeoffs. In particular, our main interest is on the use of fountain codes as a means of smoothing out and coping with the sources of errors in underwater channels. We propose a broadcasting scheme based on fountain codes and identify some important system parameters whose choice affects the performance of the scheme, allowing to trade off between different performance metrics such as delay, advancement per hop, and transmission power. We specifically focus on broadcasting because this is a fundamental network primitive which has received little attention in the context of underwater networks (to the best of our knowledge, [6] and [7] are the only available studies to date). Yet, broadcast is of primary importance for a number of actions, such as sending distress calls, spreading alarms, as well as for more network-related services, such as wireless node reprogramming.

Along these lines, in this paper we look into techniques for reliable broadcasting in underwater networks. More precisely, we model the performance of hybrid ARQ schemes based on rateless fountain codes [8] through some metrics of interest, such as the delay incurred in the dissemination of a set of packets (i.e., the message) to multiple sensor nodes over single-hop networks, an energy efficiency index, defined as the number of packet transmissions per correctly delivered packet per node, and a reliability index, defined as the percentage of users which eventually receive the message. The single-hop metrics can then be used to derive a first estimate of the behavior of the protocol over multi-hop topologies. Our model is based on the assumption that the nodes are randomly placed within a given geographical area according to a Poisson distribution. The performance that we obtain here can be considered as an upper bound for the performance that we would get from a practical scheme. We, in fact, neglect the impact of MAC and multi-user interference. The obtained tradeoffs are nevertheless important for the proper setting of transmission power and coding/retransmission parameters, as they still apply to more general settings. As an example, an important and by no means trivial tradeoff is related to the optimal transmission power and redundancy level (coding) that should be selected, in order to optimize the various metrics discussed above. One would in fact expect too high a transmission power to lead to some performance degradation. In fact, a high power is only justified by the need to reach farther users with a single transmission. However, these users would experience in turn a very small sustainable bit-rate, due to the unfavorable propagation effects that translate into a smaller transmission bandwidth [9]. Moreover, as the broadcast rate is unique and dictated by the less capable receivers, the farthest users will also fix the communication rate for the overall broadcast system. In this case, we would incur unacceptably long delays, especially over multiple hops. We precisely and quantitatively study this fact in the sequel.

The paper is structured as follows. Section 2 gives a review of the related work in the field of network protocol design for UWASNs. Section 3 introduces the fountain codes used in our HARQ strategy. Section 4 presents our analytical framework. Section 5 illustrates relevant results and tradeoffs, as per our discussion above. Finally, Section 6 draws the conclusions of our work.

2. RELATED WORK

The use of acoustics for underwater communication has received increased interest in recent years. While the main use of acoustic waves is still sonar detection and ranging, as well as telemetry [10], relatively recent efforts have proven that reliable links can be set up in water, using signal processing techniques which provide good communication efficiency or rate [11–14].

There are still many open issues in building underwater acoustic networks [15]. Most of the research work done so far focused on the design of MAC protocols. For example, [1–3, 16–19] deal with the design and/or performance evaluation of many deterministic as well as random access schemes. Some work focuses on tradoffs that arise in clustered topologies [20] or propose protocols that are partly deterministic and partly random [21].

Optimal packet sizes for ARQ protocols as well as different variants of Stop and Wait ARQ were also investigated in [22, 23]. The tradeoffs discussed in that paper are key to the design of any reliable network protocol in UWANs.

Research on more complex network protocols for, e.g., routing or broadcasting is very recent. We describe below the few research efforts in this sense. We observe that most of the literature is focused on the adaptation of terrestrial radio protocols to the underwater environment. Segmented Data Reliable Transport (SDRT) [4] employs FEC to guarantee error protection. Each node encodes and forwards data continuously using a simplified version of Tornado codes, until some positive feedback is received. Each receiver must decode the whole block of data before transmitting again. In [5], the authors deploy a framework for addressing delay-sensitive and -insensitive applications, involving Reed-Solomon packet coding and scheduling of packets according to their delay requirements. The focus of the investigation is on the impact of the long delays and stronger attenuation of the acoustic channel on packet routing. The variation of the available bandwidth with distance is considered in [24], where the authors find, by simulation, optimal transmission distances in terms of energy consumption over multihop paths.

The papers that specifically deal with broadcasting are, to the best of our knowledge, [6, 7]. The first one proposes a broadcast protocol based on Foward Error Correction (FEC) capabilities, combined with dual short- and long-distance trasmissions. These are accomplished by employing specific frequency bands where the signals are or are not expected to travel long distances, respectively. The second paper takes a different approach, by analyzing optimal hybrid ARQ policies for single-hop broadcasting.

In this paper we continue the above line of research on networking protocols for UWASNs by studying HARQ schemes for broadcasting in underwater channels. While preserving the idea behind [4–7] that FEC is a viable and profitable approach, in this work we adopt fountain codes as they involve potentially very lightweight encoding and decoding procedures, which are key for UWASNs. In addition, and most importantly, fountain codes are rateless, which means that the amount of redundancy to send is theoretically unlimited and can be decided on the fly. This makes these coding schemes robust to incorrect estimates of the channel error rate. In this paper we present a general model which allows to quantify the performance of fountain codes in underwater broadcasting schemes in terms of number of transmissions, reliability, delay and advancement per hop, and see how these change as a function of distance, number of nodes, and coding scheme parameters.

3. INTRODUCTION TO FOUNTAIN CODES

In this section, we summarize the main concepts underlying fountain encoding techniques, as presented in [7].

Digital Fountain Codes [8,25] are high performance sparse codes on bipartite graphs. These codes are rateless, i.e., the amount of redundancy is not fixed prior to transmission but can be decided on the fly as the error recovery algorithm evolves. These codes are known to be asymptotically near-optimal for every erasure channel and extremely efficient as the size of the message to transmit grows. FCs work on packet units by means of simple XOR operations, which allows for lightweight implementation of encoder and decoder. This makes these codes considerably faster than, e.g., traditional Reed Solomon codes. Consider a set of K (source) packets, each having the same length of S bits. The encoding procedure works as follows:

1. An encoded packet, t_n, is obtained by randomly picking a degree d_n from a given degree distribution $\rho(\cdot)$, whose characteristics depend on the set size K and on the targeted performance (e.g., coding complexity vs. overhead).

2. d_n distinct input packets are then picked uniformly at random from the K input packets and t_n is set equal to the bitwise sum, modulo 2, of these d_n packets. (This can be implemented by successively XORing the d_n packets.) At this point, an encoding vector is created for t_n, which contains an indication of which of the input packets were XORed together. This vector can be transmitted along with t_n or retrieved at the receiving side through the usage of suitable pseudo random number generators (in this case transmitter and receivers should use the same random seed; this last method is often preferred in practice).

Decoding can be achieved by solving the system $\boldsymbol{t} = \boldsymbol{G}\boldsymbol{s}$, where the matrix $\boldsymbol{G}$ is composed of the received encoding vectors, the vector $\boldsymbol{t}$ contains the received encoded packets, and $\boldsymbol{s}$ contains the K original packets to be retrieved. Recovery at each interested receiver requires the reception of K linearly independent coded packets; in this case in fact $\boldsymbol{G}$ has full rank and can be inverted. As encoded packets are randomly generated according to the degree distribution $\rho(\cdot)$, it is possible that some of the received packets are linearly dependent, so that in general a node may need to receive $K' \geq K$ packets before being able to decode. The performance of these codes, in terms of overhead $O = K' - K$, depends on the degree distribution, and can be kept small through a proper design of $\rho(\cdot)$. While efficient decoding methods and degree distributions can be designed when K is large [25], in case of smaller Ks reference [26] presents good degree distributions $\rho(\cdot)$ which allow to optimize for the complexity of the decoding process, the transmission overhead, or a combination of both.

Underwater communications are also a scenario where fountain codes are constrained to work with small K; this is in line with the typical message sizes in underwater channels (that are small due to the limited bandwidth). In this case, special degree distributions are to be designed such as in [26]. Here, however, we rather focus on the theoretical gains provided by fountain codes; thus we consider the *digital random fountain* as presented in [8]. Actual random codes are expected to perform very close to the digital random fountain for a good design of $\rho(\cdot)$ [26].

We now define two suitable functions which will be used in the following analysis. We model the dynamics of the fountain-based encoding/decoding system through a distribution $\Psi_K(x)$ with x integer. $\Psi_K(x)$ returns the probability of correct decoding at a generic user when x encoded packets have been collected. Note that $\Psi_K(x) = 0$ for $x < K$ as a full rank matrix (i.e., with rank equal to the number of input packets, K) can never be obtained in such a case. For our *digital random fountain*, $\Psi_K(x)$ can be computed exactly as it corresponds to the probability that a $K \times x$ (with $x \geq K$) random binary matrix (i.e., a matrix with elements in $GF(2)$) has full rank. This probability can be found as follows (this equation can be promptly derived from, e.g., the informal arguments in [8]):

$$\Psi_K(x) = \prod_{i=0}^{K-1} \left(1 - 2^{i-x}\right). \tag{1}$$

4. ANALYSIS OF BROADCASTING IN UNDERWATER NETWORKS

Let us assume to have K uncoded packets (hereafter "the message"). These packets are sent by a transmitter (the broadcast source) using fountain codes. The transmission takes place according to a hybrid ARQ procedure, which works in rounds as we now explain. At the first round, the sender transmits $K + \xi$ encoded packets, and then collects feedback from all intended receivers. The parameter $\xi > 0$ is fixed and corresponds to the amount of redundant packets sent at the generic transmission round. The feedback information is very limited, and is only used to tell whether the K packets in the data block have been successfully recovered or not by a particular receiver. We observe that sending ξ redundancy packets in the first round allows to decrease the probability that the receivers cannot decode the whole message (due to the randomness of the code), as well as to compensate for some channel errors. Further redundancy is transmitted in the subsequent rounds, according to the users' feedback. From the second round on, only ξ packets are sent per round. In this way, exactly $K + j\xi$ packets are sent by the end of round j, $j = 1, 2, \ldots, L$, where L is the maximum number of transmission rounds allowed (in other words, there may be up to $L-1$ *retransmission* rounds, where only incremental redundancy is sent). The feedback is always collected at the end of a round. The transmission stops whenever the maximum number of rounds L has been reached or all receivers have fully recovered the K data packets. The intended receivers for the broadcast message are labeled as $1, 2, \ldots, N_u$ and their number and positions are modeled as follows. Unless differently specified, we fix a maximum transmission distance R_n, i.e., the transmission power is selected such that all nodes with distance smaller than or equal to R_n (covering an area $\mathcal{A} = \pi {R_n}^2$) from the source experience a packet error probability of 0.25. The calculation of the packet error probability is outlined later in section 4.1. Once R_n (and thus the transmission power) has been fixed, N_u and the positions of the intended receivers within $\mathcal{A}$ are modeled assuming that the users are distributed according to a Poisson process of rate μ users per unit area. Finally, for a given distance $d \leq R_n$ between the source and a given receiver, the packet error probability $p(d)$ is found according to the underwater channel model in [6]. More details about the derivation of $p(d)$ are given in the following subsection.

4.1 Statistics of the number of rounds needed for full recovery at all receivers

Let $\Psi_K(N)$ be the probability that the K data packets can be recovered from N received codewords. This is the probability that the $K \times N$ matrix containing the encoding vectors (used to generate the received packets) has full rank.

Let us call $P_1^{>j}(p)$ the probability that more than j rounds are needed to recover the original K packets for a given user. This probability can be found as

$$P_1^{>j}(p) = \sum_{e=0}^{K+j\xi} B(K+j\xi, e, p)\left(1 - \Psi_K(K+j\xi-e)\right), \tag{2}$$

where $B(n,k,p) = \binom{n}{k} p^k (1-p)^{n-k}$, and $\Psi_K(x)$ is given by (1) in the previous section. We also define its complement as $P_1^{\leq j}(p) = 1 - P_1^{>j}(p)$. These probabilities depend on the probability of packet error, p, which is intrinsically a function of the distance d between the source and the receiver. In order to approximate p, we employ the attenuation and noise equations given in [9] as computed at the frequency $f_0(d)$ where the product of attenuation and noise is minimum for the given distance.[1] Since the determination of this optimal frequency value requires lengthy numerical integrations, we develop an approximation by following the procedure highlighted in [9] to approximate the relationship between bandwidth and distance. Basically, this requires to find the parameters ϕ and θ of a log-linear relation of the kind $\log_{10} \bar{f}_0(d) = \phi + \theta \log_{10} d$, so that the approximate curves fit the actual value of $f_0(d)$. Such a model is fairly accurate over a wide range of distances, but tends to diverge significantly from the actual values for small d. Since, in our derivation, we need to accurately model $f_0(d)$ down to very small d, we develop a piece-wise log-linear approximation by dividing the range of distances in regions where a single log-linear curve accurately fits the numerical results. Using an entirely analogous procedure, we also developed a piece-wise

[1] We call this frequency the "center frequency," even though the $3\,\mathrm{dB}$ bandwidth is not exactly centered in $f_0(d)$ [9].

log-linear approximation $\bar{B}(d)$ for the optimal transmission bandwidth $B(d)$ (chosen according to the empirical 3 dB definition) as a function of the distance d. Fig. 1 shows the goodness of our fitting method. The SNR of a certain transmission in the network as a function of distance can finally be found as

$$SNR(d) = \frac{P_{tx}/B(R_n)}{A\big(d, f_0(R_n)\big)\, N\big(f_0(R_n)\big)}\,, \tag{3}$$

where P_{tx} is the transmission power, $A\big(d, f_0(R_n)\big)$ is the attenuation incurred by transmitting at the frequency $f_0(R_n)$ from a distance d, and $N\big(f_0(R_n)\big)$ is the noise. Furthermore, note that both the transmission bandwidth B and the transmission frequency f_0 are distance-dependent as specified before, and thus are chosen so that a prescribed transmit distance R_n (and all nodes within this distance) can be covered by a transmission performed inside the band $B(R_n)$. Assuming the use of a BPSK modulation, the probability of error per bit, p_b is then found by using the SNR computed in Eq. (3) and the BPSK error equation, properly modified as in [21]. The packet error probability (PER) is finally found depending on the distance d of the receiver, as $p(d) = 1 - \big(1 - p_b(d)\big)^S$, where $p_b(d)$ is the actual bit error probability at the distance d, and S is the packet size in bits.

Since the users are uniformly located within the coverage area $\mathcal{A}$ of the source, the average recovery probability in at most j rounds, for a given user within this area, becomes

$$\bar{P}_1^{\le j} = \int_0^{R_n} P_1^{\le j}\big(p(\ell)\big)\frac{2\ell}{{R_n}^2}\mathrm{d}\ell\,, \tag{4}$$

where $2\ell/{R_n}^2$ is the probability density function (pdf) of the user's distance from the source node, given that this user is located within $\mathcal{A}$. We also wish to find the probability of recovery for all intended receivers $1, 2, \ldots, N_u$ in $\mathcal{A}$, $\bar{P}_{N_u}^{\le j}$. Since the users are independently located and experience independent errors, this probability can be found as

$$\bar{P}_{N_u}^{\le j} = \left(\bar{P}_1^{\le j}\right)^{N_u} = \left(\int_0^{R_n} P_1^{\le j}\big(p(\ell)\big)\frac{2\ell}{{R_n}^2}\mathrm{d}\ell\right)^{N_u}\,. \tag{5}$$

Next, we define two tail distributions which model the probability that more than j rounds are needed for full recovery at all users. The first is derived as the complement of (5):

$$\bar{P}_{N_u}^{>j} = 1 - \bar{P}_{N_u}^{\le j}\,, \tag{6}$$

and will be used for the average number of incomplete users in (10) and the average advancement in (14). The second tail distribution is required for computing the average numer of transmission rounds, and is defined as

$$\bar{Q}_{N_u}^{>j} = \begin{cases} \bar{P}_{N_u}^{>j}\,, & \text{if } j < L \\ 0\,, & \text{otherwise} \end{cases}\,. \tag{7}$$

This definition models the fact that no more than L transmission rounds are allowed.

To find the average number of rounds required during a broadcast, we first condition on N_u users:

$$\mathrm{E}[j|N_u] = \sum_{j=0}^{+\infty} \bar{Q}_{N_u}^{>j} = \sum_{j=0}^{L-1} \bar{P}_{N_u}^{>j}\,, \tag{8}$$

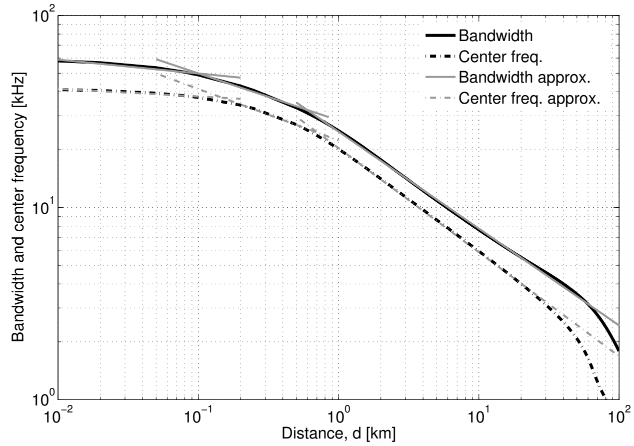

Figure 1: Piece-wise log-linear approximation of B and f_0 as a function of distance.

where the maximum number of rounds L is counted if the transmission is still unsuccessful after $L-1$ retransmissions. The average number of rounds is finally found by averaging over N_u as

$$\begin{aligned} \mathrm{E}[j] &= \sum_{N_u=0}^{+\infty} \mathrm{E}[j|N_u]\frac{(\mu\mathcal{A})^{N_u}e^{-\mu\mathcal{A}}}{N_u!} \\ &= \sum_{j=0}^{L-1}\sum_{N_u=0}^{+\infty}\big(1 - (\bar{P}_1^{\le j})^{N_u}\big)\frac{(\mu\mathcal{A})^{N_u}e^{-\mu\mathcal{A}}}{N_u!} \\ &= L - \sum_{j=0}^{L-1} e^{-\mu\mathcal{A}(1-\bar{P}_1^{\le j})} \\ &= \sum_{j=0}^{L-1}\left[1 - e^{-\mu\mathcal{A}(1-\bar{P}_1^{\le j})}\right]\,. \end{aligned} \tag{9}$$

4.2 Reliability Performance

The average probability that a user requires more than j rounds to recover all data packets is $\bar{P_1}^{>j} = 1 - \bar{P_1}^{\le j}$ (see (4)). If L is the maximum number of transmission rounds allowed, then $\bar{P_1}^{>L}$ is the probability that a given user fails to recover the K original data packets.

The average number of users in this condition, $\mathrm{E}[N_u^{\mathrm{ko}}]$, can be found recalling that all users are assumed independent of each other in terms of packet error events. Conditioning on the number of users N_u, we have:

$$\mathrm{E}[N_u^{\mathrm{ko}}|N_u] = N_u\bar{P_1}^{>L}\,, \quad \mathrm{E}[N_u^{\mathrm{ko}}] = \mu\mathcal{A}\bar{P_1}^{>L}\,. \tag{10}$$

Therefore, the average number of transmissions given that there are N_u users is:

$$\mathrm{E}[\mathrm{tx}|N_u] = K + \xi\mathrm{E}[j|N_u]\,, \tag{11}$$

which can be averaged over N_u to yield

$$\begin{aligned} E[\mathrm{tx}] &= \sum_{N_u=0}^{+\infty}(K + \xi\mathrm{E}[j|N_u])\frac{(\mu\mathcal{A})^{N_u}e^{-\mu\mathcal{A}}}{N_u!} \\ &= K + \xi\mathrm{E}[j]\,. \end{aligned} \tag{12}$$

4.3 Delay Performance

The delay will partly depend on the MAC protocol in use. Here, however, we consider the following simplified model. Assume

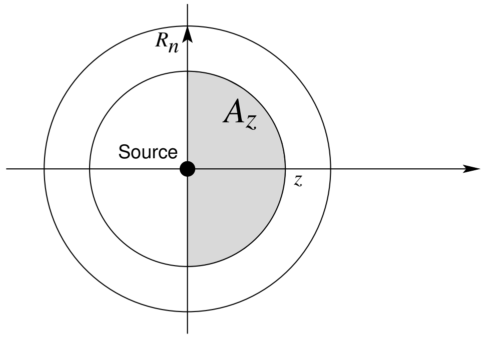

Figure 2: Illustration of the area $\mathcal{A}_z$ for the definition of advancement.

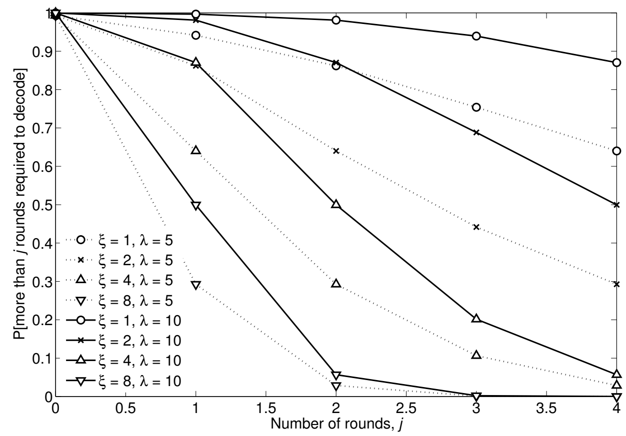

Figure 3: Probability that more than j rounds are required so that all nodes within the desired coverage range R_n are successful for varying ξ and λ.

again that we want to reach all receivers within a transmission range R_n. Let $B(R_n)$ be the bandwidth allocated for the broadcast transmission in this case. Let $T_D(R_n) = S/B(R_n)$ and $T_A(R_n)$ be the transmission time of a data packet and a feedback packet, respectively, where we recall that S is the transmit packet size. We assume in the following that $T_A(R_n) = T_D(R_n)/10$. The average delay for the transmission of the K original packets to all receivers within the coverage area $\mathcal{A}$ is computed as

$$D = \big(K + \mathrm{E}[j]\xi\big)T_D(R_n) + \big(\mathrm{E}[j]-1\big)\big(2\tau_{R_n} + T_A(R_n)\big) + \tau_{R_n}\,, \tag{13}$$

where τ_{R_n} is the propagation time required by the acoustic waves to cover a distance R_n. Note that D increases with R_n because of two separate factors, namely the increase of τ_{R_n}, and the increase of the packet transmission time. In fact, the latter factor depends on the distance through the inverse of the transmit bandwidth $B(d)$, which is in turn a decreasing function of d.

4.4 Per-hop Advancement

In the following we say that a broadcast transmission has achieved an advancement z if all nodes within a distance less than or equal to z successfully decode the message within a maximum number of rounds L. This is equivalent to requiring that the circular area $\mathcal{A}_z = \pi z^2/2$ depicted in Fig. 2 contains only successful nodes. We observe that this problem is circularly symmetric as seen by the source.

The first step to compute the average advancement is to fix a certain direction and an advancement z towards that direction, and to compute the probability that a certain user, randomly located in $\mathcal{A}_z$, is successful. To do this we integrate $P_1^{\leq L}\big(p(\ell)\big)$ over $\mathcal{A}_z$, where $p(\ell)$ is the packet error probability as a function of the distance ℓ between the transmitter (the source) and the receiving node. Due to the circular simmetry of the problem, the integration can be performed as follows:

$$P_{\mathcal{A}_z}(z) = \int_0^z P_1^{\leq L}\big(p(\ell)\big)\frac{2\ell}{z^2}\mathrm{d}\ell\,. \tag{14}$$

Note that the expression of $P_{\mathcal{A}_z}(z)$ is equivalent to (4). In fact, they represent very similar problems, since the calculation of $P_{\mathcal{A}_z}(z)$ requires to average $P_1^{\leq j}$ over the interval $[0, z]$ instead of $[0, R_n]$, with the understanding that we require the user to be successful at any moment within the L transmission rounds, thus posing $j = L$ in (14). We can now compute the probability that all nodes are successful within $\mathcal{A}_z$, which represents the tail distribution of the advancement. Note that since all nodes within $\mathcal{A}_z$ are successful, the advancement is at least z, because further nodes outside $\mathcal{A}_z$, could be successful as well. Hence, the complementary distribution function of the advancement can be found as

$$\begin{aligned} F(z) &= P[\mathrm{adv} > z] = \sum_{N_u=0}^{+\infty} \frac{(\mu\mathcal{A}_z)^{N_u}}{N_u!} e^{-\mu\mathcal{A}_z}\big(P_{\mathcal{A}_z}(z)\big)^{N_u} \\ &= e^{-\mu\mathcal{A}_z(1-P_{\mathcal{A}_z}(z))}\,, \end{aligned} \tag{15}$$

which represents the probability that no node is unsuccessful in $\mathcal{A}_z$. The average per-hop advancement is finally obtained as

$$\mathrm{E}[z] = \int_0^{R_n} F(z)\mathrm{d}z\,. \tag{16}$$

5. RESULTS

In this section we present some results about the performance of our fountain codes-based broadcasting scheme. The behavior of the data dissemination protocol is as described at the beginning of section 4. We assume that the number of packets that make up the whole message is $K = 32$, each having a length of $S = 1000$ bits. We recall that R_n is the *nominal* coverage range and represents the distance which can be reached with a PER of 0.25 (which entails the usage of a specific transmission power and bandwidth, calculated as in section 4.1). We fix the maximum number of transmission rounds to $L = 5$ and pursue an analysis of broadcasting performance by varying R_n, and the number of redundancy packets per round from $\xi = 1$ (very limited correcting power) to $\xi = 8$ (high success probability but likely waste of redundancy). The user density per unit area μ is varied so as to yield $\lambda = 5$ or $\lambda = 10$ users within a $5\,\mathrm{km}$ range. Transmissions take place according to the propagation model described in section 4, where we set the spreading parameter k to a practical value of 1.5 in order to approximate shallow water conditions. We assumed the sound speed constant and equal to $1.5\,\mathrm{km/s}$.

The first result illustrates the performance of the fountain code with $R_n = 5\,\mathrm{km}$. In Fig. 3 we show the probability that more that j rounds are required for all nodes to complete the reception of the message successfully, i.e., the average value of (6) taken over N_u. As expected, all curves decrease with increasing number of rounds, as more incremental redundancy is sent to the receivers and the probability of being unsuccessful becomes progressively smaller. The curves depend on the amount of incremental redundancy, ξ,

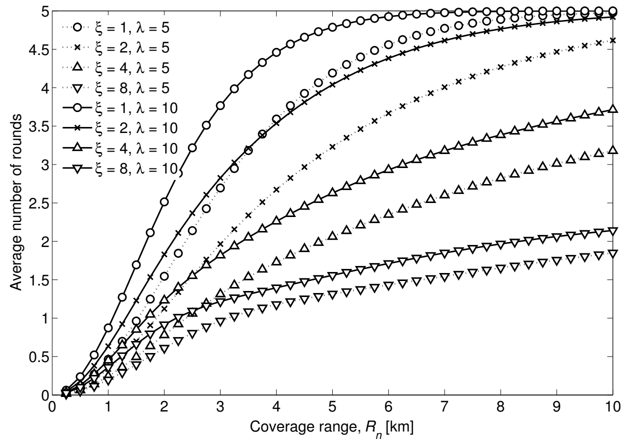

Figure 4: Average number of rounds, $\mathrm{E}[j]$, as a function of the desired coverage range R_n for varying ξ and λ.

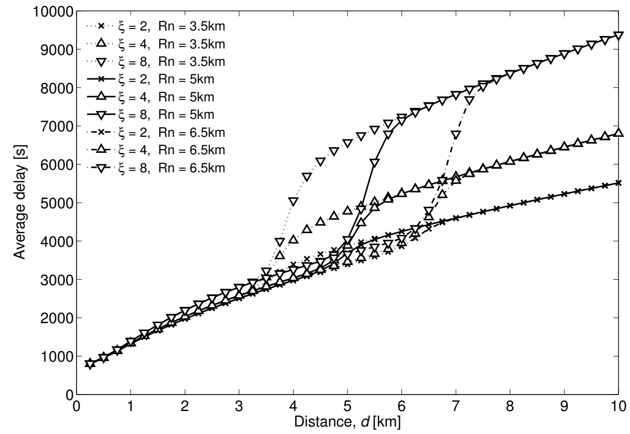

Figure 5: Average transmission delay, D, as a function of the distance d for varying ξ and R_n, with $\lambda = 10$.

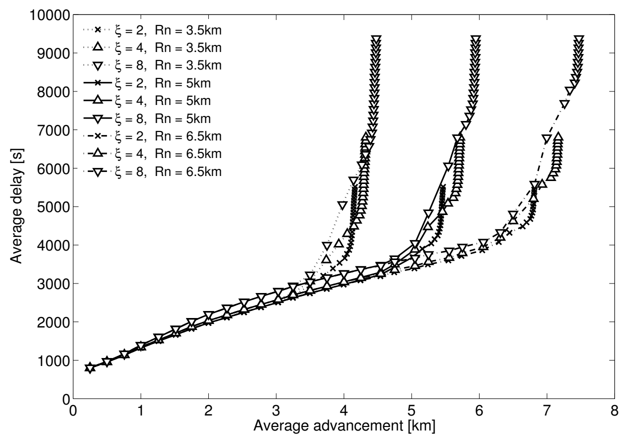

Figure 6: Tradeoff between transmission delay D and expected advancement $\mathrm{E}[z]$. Curves are obtained by keeping the transmission power (i.e., R_n) fixed and varying d. An increase of d corresponds to spanning the curves from the bottom-left to the top-right. λ is set to 10 users for all curves.

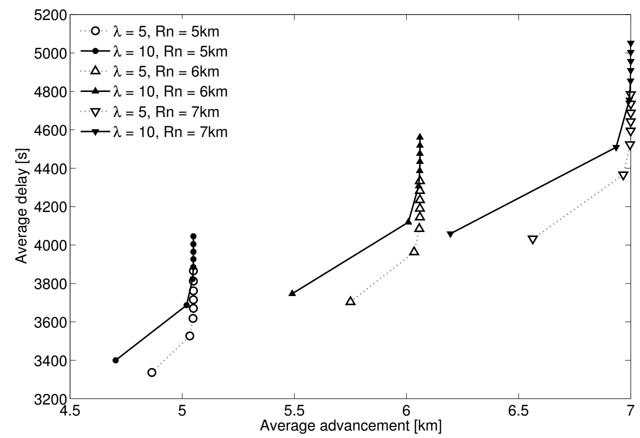

Figure 7: Tradeoff between transmission delay D and expected advancement $\mathrm{E}[z]$. Curves are obtained by keeping the transmission power (i.e., R_n) fixed and varying ξ. An increase of ξ corresponds to spanning the curves from left to right.

and on the average number of receivers to serve: the probability that all users are successful at the end of a round increases for smaller λ and larger ξ. Correspondingly, the average number of rounds required on average so that all users are successful decreases for increasing ξ. This is shown in Fig. 4, where we plot $\mathrm{E}[j]$ versus R_n. As expected, the number of rounds increases with R_n and λ. In fact, for a fixed λ, a larger R_n corresponds to a greater number of users to serve.

Fig. 5 shows the average delay for a single hop broadcast (see (13)). This figure is obtained fixing R_n and plotting the delay as a function of the distance d, i.e., the transmission power is set to cover all users within R_n while we exploit the correction capability of the protocol to actually cover all users within distance d. All curves have a phase transition around R_n. For, e.g., $R_n = 5\,\mathrm{km}$ we see that the delay is still tolerable for $d \approx 5\,\mathrm{km}$ and that the use of the fountain code guarantees a larger coverage. This, however, occurs at the price of more transmission rounds and thus longer delays. A further increase of d (beyond $\approx 6\,\mathrm{km}$) makes it unlikely to successfully cover the farthest users. In this case the protocol uses the maximum number of rounds L, and transmits the maximum number of packets $K + L\xi$. The delay, after this point, still increases linearly with distance due to the increase of the round-trip time with d (see (13)).

In Fig. 6 we report a tradeoff between the transmission delay and the expected advancement $\mathrm{E}[z]$. Note that, as expected, $\mathrm{E}[z]$ saturates for increasing d. Also, a larger ξ can help in achieving larger advancements. However, we do not observe a substantial performance improvement from, e.g., $\xi = 4$ to $\xi = 8$. This is to say that additional redundancy is beneficial but should be used wisely. This fact is further emphasized in Fig. 7 where we show the tradeoff in terms of delay versus $\mathrm{E}[z]$ by varying the number of redundant packets per round, ξ. From this figure it is clear that increasing ξ from one to two, in general, leads to good improvements for $\mathrm{E}[z]$ with no substantial increase of the delay, D. This is particularly true at high densities, where fountain codes scale well. Further increasing ξ from two to four (or larger), however, only yields marginal improvements for $\mathrm{E}[z]$, while having a detrimental effect on D.

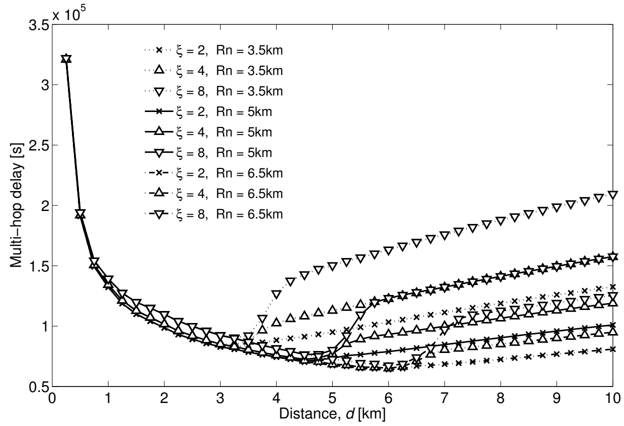

Figure 8: Multi-hop delay, to cover a distance of $Z = 100\,\mathrm{km}$, as a function of the single hop distance d. Curves are plotted for various values of ξ and R_n, with $\lambda = 10$.

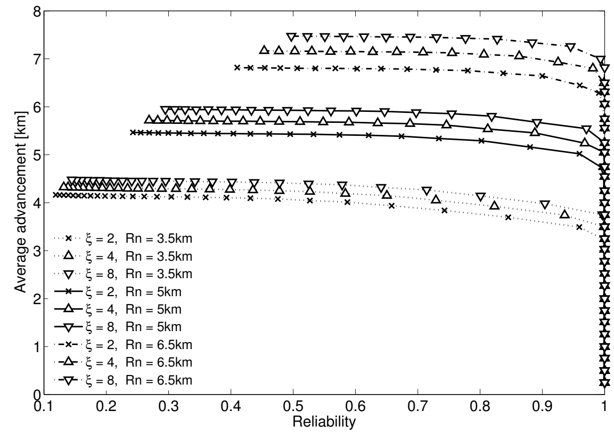

Figure 9: $\mathrm{E}[z]$ as a function of the reliability by varying ξ and R_n. An increasing distance d traverses the curves from right to left.

In Fig. 8, we look at the multi-hop performance in terms of (average) time taken to cover a distance of $Z = 100\,\mathrm{km}$. This metric is approximated here as $ZD/\mathrm{E}[z]$, where D is the single hop delay of (13). Different curves are plotted for various values of ξ and R_n. For given R_n (thus for given transmission power) and ξ there is an optimal distance d, which is close to R_n. Similarly to what we observed for Fig. 7, we see that the use of ξ beyond two has the effect of substantially worsening the multi-hop performance. Once again, the adoption of a too aggressive coding scheme is not recommended as, beyond a certain point, it only leads to additional delays, with small improvements in terms of covered distance.

Finally, in Fig. 9 we show the tradeoff between expected advancement $\mathrm{E}[z]$ and reliability. For each curve in the graph we fix R_n. Thus, we consider a given d and compute $\mathrm{E}[z]$ for all nodes within d as well as the fraction of these nodes which reliably receive the message. As expected, the reliability is very close to one whenever d is small. However, as we keep increasing d, $\mathrm{E}[z]$ flattens out with a consequent (and substantial) decrease of the reliability. This operating region should be avoided as the broadcast protocol is no longer effective here.

We conclude by observing that fountain coding and incremental redundancy are very effective in terms of error correction capability and needed transmission rounds, see Figs. 3 and 4. The transmission delays in underwater channels are however so high that they reduce this gain, as can be seen from the above delay performance, see Fig. 7. For this reason, a correct design should avoid using too large a number of redundant transmissions.

6. CONCLUSIONS

In this paper, we analyzed the performance of hybrid ARQ based on fountain codes as applied to broadcasting in underwater networks. The specific features of the channel, such as the variation of the available bandwidth with distance, have been explicitly taken into account. Our model allows to understand the interplay between coding and network parameters, and their effect on the performance of underwater broadcasting.

Our future work deals with policies to find, in a distributed manner, the right amount of redundancy as a function of node positions and density. Other future directions include the comparison between fountain codes and fixed-rate packet codes (such as Reed-Solomon), and a comparison with other broadcasting approaches for radio networks as applied to underwater networks.

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