

A linear dynamic model for microgrid voltages in presence of distributed generation

Saverio Bolognani, Guido Cavraro, Federico Cerruti, and Alessandro Costabeber
 Department of Information Engineering
 University of Padova, Italy

Abstract—We consider the scenario of a low voltage microgrid populated by a number of distributed microgenerators. We focus on the problem of obtaining a dynamic model that describes the input-output relation between complex power commands sent to the microgenerator inverters and the voltage measurements across the network. Such a model is intended as a necessary tool in the design of distributed and centralized control algorithms for the provision of ancillary services in the power distribution grid. Because this model is to be used for the design of such algorithms, we look for an analytical derivation instead of a simulative tool. The proposed model is linear and explicitly contains the network parameters and topology. Simulation shows how the proposed model approximates well the behavior of the original nonlinear system.

I. INTRODUCTION

One of the major features that is going to characterize future *smart grids* is the appearance of a large number of microgenerators connected to the LV power distribution network.

This scenario poses a number of nontrivial challenges, together with exciting opportunities. The management of future *microgrids* (portions of the power distribution network) requires that many control and optimization algorithms are executed at the same time: generation-demand matching protocols, energy market mechanisms, algorithms for optimal energy use and quality of service, and many others. The complexity of such scenario and the different time scales of these control tasks, suggest that a *layered* architecture should be adopted. In a layered architecture (similarly to the ISO/OSI model in data communication networks), different algorithms coexist at different levels and at different time scales. Lower level algorithms, in charge of controlling the specific physical devices, obtain references and commands from higher levels of the architecture. Higher level algorithms command many instances of the lower level ones, based on a simplified model of their behavior and on the aggregated data provided by the underlying layers.

One possible application of this structure to the control of a smart microgrid has been depicted in Figure 1 (see also [1] and references therein). Algorithms residing in the top layer (*tertiary control*) dispatch active power generation on the

The research leading to these results has received funding from the European Union Seventh Framework Programme [FP7/2007-2013] under grant agreements no. 257462 HYCON2 Network of excellence and no. 223866 FeedNetBack.

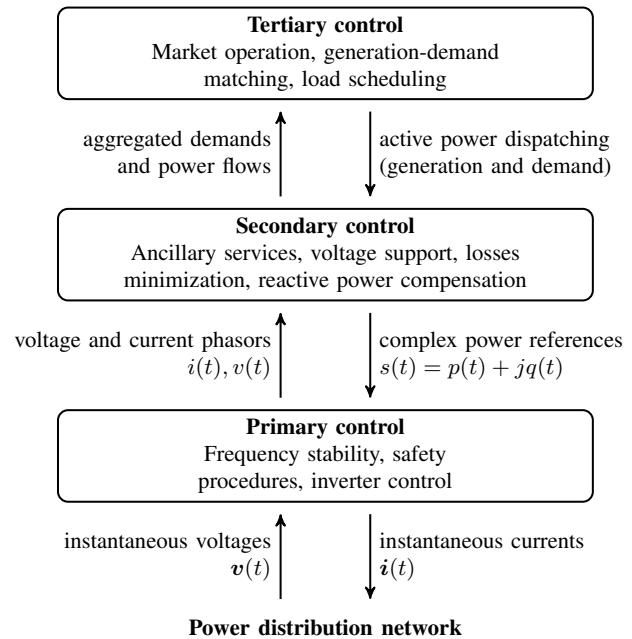


Figure 1. A possible layered architecture for the simultaneous execution of different algorithms in a smart microgrid.

basis of economic reasons, measured and predicted aggregate demand, and availability of the microgenerators.

Secondary control algorithms, on the other hand, take care of sharing the commanded power references among microgenerators, while satisfying a number of operational constraints (voltage limits, stability, congestion avoidance) and optimizing the network operation (power losses minimization, improved power quality). These algorithms require some knowledge of the grid parameters and on the system state.

Primary control algorithms are executed on a local level (at the single inverter), with very little or no communication between the devices, and on a faster time scale; on the basis of the reference signals that they receive, they practically command the power converters that equip each inverter, ensuring frequency stability, avoiding detrimental interactions, and ensuring safety of operation.

We focus here on the middle layer in this architecture. In particular, ongoing studies are exploring the possibility of controlling the microgeneration devices for the provision of ancillary services inside the power distribution network

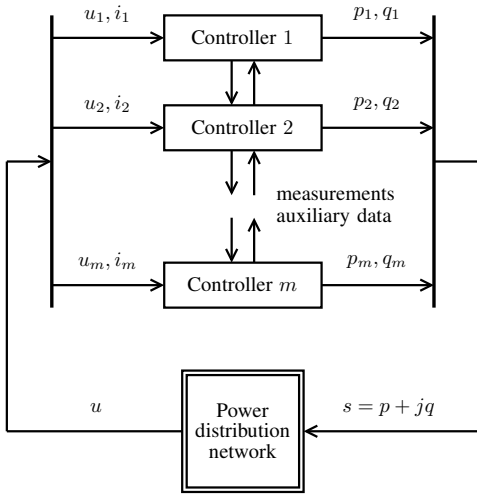


Figure 2. The structure of a generic algorithm for the sharing of active (p) and reactive (q) power references among the distributed microgenerators in a microgrid, based on measured voltage phasors (u).

(reactive power compensation, voltage support, etc.), as explained for example in [2]. This direction of investigation yielded a series of algorithms of different fashions and with different requirements in terms of coordination of the inverters, ranging from centralized architectures based on telemetry and on the presence of a central controller (as in [3], [4], [5]), to distributed ones where each inverter operates with little or no communication with the others (as in [6], [7], [8]).

A generic algorithm of this type can be represented as in Figure 2, where it has been highlighted how the controller of each microgenerator decide on its own power reference according to local measurements and according to the data collected from other controllers.

In accordance to what is the state of the art in the control of power converters [9], [10], [1] and in microgrid state measurement and estimation [11], [12], we assume here that reference signals consist in *active and reactive power signals*, while measurements consist in *phasor measurements*.

For an appropriate design of such algorithms, it is necessary to have a model capable of describing the dynamic behavior of the measured signals (voltage phasors) as a function of the complex power references sent to the microgenerators.

A static model (typically the solution of steady state power flow equations) is unlikely to be enough for such task. Consider for example the frequent case of iterative algorithms, that require a repeated execution of

- 1) measurement of voltages on the grid;
- 2) communication and coordination among inverters and/or with a central controller;
- 3) implementation of the algorithm via data processing;
- 4) actuation of the systems by commanding active and reactive power injected by each microgenerator.

For these iterative algorithms, it is critical to estimate the propagation time of the control action across the microgrid. In other words, we need to understand how long after

changing the amount of injected active and reactive power at the inverters, the grid voltages reach their state state value. Only by answering this question it is possible to decide on the time interval between consecutive iterations of the algorithm, and on the appropriate feedback gain that guarantee stability in presence of such delays. This question has an almost immediate answer in the case of linear loads (constant impedance loads). However, modern power distribution networks are populated by a large amount of non-linear loads, in particular constant power loads (including the microgenerators themselves), whose interconnection produce richer and more complex dynamics.

In this work, we aim at obtaining a model of the input-output relation between injected power and voltages that can be used for control design purposes; we therefore focus on the subclass of linear models, for which plenty of analysis and control design methods exist.

The tools that we will use for the derivation of such model, are the tools of *complex systems*, in which the complexity of the entire dynamical system is not given by the complex behavior of the single components, but by the interconnection of a large number of them. Indeed, the presented results show that a complex behavior *emerges* even if a very simple nonlinear model is chosen for each node, once the nodes are interconnected. On the other hand, by choosing a simple model for the individual devices, it is possible to study the behavior of the whole system in an analytical and rigorous way, understanding how the resulting dynamical behavior depends on the system parameters (power line impedances, network topology, power demands).

This approach is quite new in the field of power systems, but seems to be a powerful tool when the power grid becomes populated by a large number of interacting devices, which is the case in the future power distribution network.

II. MATHEMATICAL PRELIMINARIES AND NOTATION

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \sigma, \tau)$ be a *directed graph*, where \mathcal{V} is the set of *nodes*, \mathcal{E} is the set of *edges*, and $\sigma, \tau : \mathcal{E} \rightarrow \mathcal{V}$ are two functions such that edge $e \in \mathcal{E}$ goes from the source node $\sigma(e)$ to the terminal node $\tau(e)$.

Let us define a *path* \mathcal{P}_{hk} from node h to node k as a sequence of consecutive edges (regardless of their orientation and without repetitions) connecting node h and k . \mathcal{G} is a *tree* if for every pair of nodes $i, j \in \mathcal{V}$ there is exactly one path connecting them.

If v is a node of the graph, we define by $\mathbf{1}_v$ the vector of size equal to the number of nodes of \mathcal{G} , and whose value is 1 in the position corresponding to v , and 0 everywhere else. We denote by $\mathbf{1}$ the vector of all ones, of the same size.

We denote by $\bar{\cdot}$ the operation of element-wise complex conjugation, and by T the operation of matrix transposition.

The *incidence matrix* A of a graph is defined via its elements

$$A_{ev} = \begin{cases} -1 & \text{if } v = \sigma(e) \\ 1 & \text{if } v = \tau(e) \\ 0 & \text{otherwise.} \end{cases}$$

If the graph \mathcal{G} is connected, then $\mathbf{1}$ is the only vector in $\ker A$.

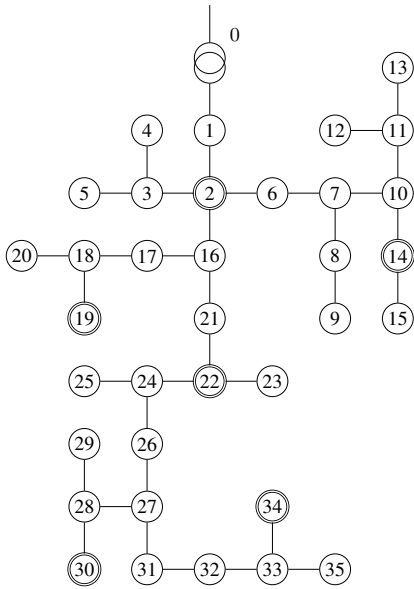


Figure 3. Graph model of a microgrid (based on the IEEE37 test feeder). The nodes represent microgenerators (circled), loads, and the PCC (node 0).

III. DISTRIBUTION NETWORK MODEL

For the purpose of this paper, we model a microgrid as a directed tree \mathcal{G} , in which edges represent the power lines, and nodes (indexed from 0 to n) represent both loads and generators that are connected to the grid (see Figure 3). These include the residential and industrial consumers, distributed microgenerators, and also the point of connection to the transmission grid (called point of common coupling, or PCC, and indexed as 0).

We consider single-phase or balanced three-phase scenarios. We also limit our study to the steady state behavior of the system, when all voltages and currents are sinusoidal signals at the same frequency ω_0 . Each voltage and current can therefore be represented via a complex number (*phasor*) whose absolute value corresponds to its root-mean-square value, and whose phase corresponds to its phase in $t = 0$. Therefore the complex number y represents the signal $\mathbf{y}(t) = |y|\sqrt{2}\sin(\omega_0 t + \angle y)$.

In this notation, the steady state of a microgrid is described by the following system variables:

- u_v , $v = 0, \dots, n$, i.e. the grid voltage at node v ;
- i_v , $v = 0, \dots, n$, i.e. the current injected by node v into the grid.

Each node v of the microgrid is characterized by a law relating its injected current i_v with its voltage u_v .

We model node 0, corresponding to the PCC, as a constant voltage generator, i.e.

$$u_0 = U_0. \quad (1)$$

For the remaining nodes (loads and microgenerators), we adopt the following dynamic model:

$$\tau_v \frac{di_v}{dt} = -i_v + \frac{\bar{s}_v}{\bar{u}_v}, \quad v = 1, \dots, n, \quad (2)$$

where τ_v is the characteristic time constant, and s_v is a constant reference complex power (and, we recall, $\bar{\cdot}$ means complex conjugate). Notice indeed that the steady state of (2) is

$$i_v = \frac{\bar{s}_v}{\bar{u}_v},$$

which is a static constant power model for each node v . In other words, we assumed that loads and microgenerators behave as constant power loads with a first-order dynamic.

This model corresponds to the widely adopted model introduced in [13], [14], [15] on the basis of experimental data. According to the literature, it describes quite well the behavior of the vast majority of microgeneration devices [16], [10], and is also a good approximation for many industrial and residential loads.

The time constant τ_v can differ a lot from node to node, ranging from extremely fast nodes (microgenerators and inverters in general) to slower ones (thermal loads, tap changers).

We however neglected the possibility that the steady state power reference depends on the voltage of the node (as in constant-impedance and constant-current loads). Notice however that for the purpose of this study (the transient of the measured signals across the microgrid) the steady state of the individual node has little effect. As said in the Introduction, the complex behavior of the global system emerges because of the interconnection of a large number of nodes, each one with a very simple, nonlinear, dynamic behavior.

Notice that in (2) we are describing the transient of a system in a phasorial notation, which should rigorously be used for steady state behavior only. This is however acceptable if the typical transient duration that we are considering are longer than the fundamental period of the signals. More on this issue can be found in [17].

To catch the effect of the interconnection of many nodes via the electrical network, we need to express u_v on the right side of (2) as a function of the currents injected by all the nodes.

To do that, we consider the incidence matrix A of \mathcal{G} , and the diagonal matrix Z whose elements are the impedances z_e of the edges of the graph. We can then define the complex-valued Laplacian of the graph as

$$\mathbf{L} = \mathbf{A}^T \mathbf{Z}^{-1} \mathbf{A}.$$

By writing Kirchoff current law for each node of the graph, we obtain the following relation between voltages u_v and currents i_v :

$$\mathbf{L} \begin{bmatrix} u_0 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} i_0 \\ \vdots \\ i_n \end{bmatrix}, \quad (3)$$

where the left-hand side describe edge currents in terms of voltage difference across the edges and of edge impedances, and the right-hand side is the set of injected currents.

It is possible to show that, given \mathbf{L} , there exists a unique, symmetric, matrix \mathbf{X} such that

$$\begin{cases} \mathbf{X} \mathbf{L} = \mathbf{I} - \mathbf{1} \mathbf{1}_0^T \\ \mathbf{X} \mathbf{1}_0 = \mathbf{0}. \end{cases} \quad (4)$$

It can also be shown that the elements of the matrix \mathbf{X} can be directly obtained from the network topology and parameters. If \mathcal{P}_{0h} and \mathcal{P}_{0k} are the paths connecting the PCC to nodes h and k respectively, then the element \mathbf{X}_{hk} is the impedance of the intersection between the two paths, i.e.

$$\mathbf{X}_{hk} = \sum_{e \in \mathcal{P}_{0h} \cap \mathcal{P}_{0k}} z_e. \quad (5)$$

Note that \mathcal{P}_{00} is empty, and therefore the first row and column of \mathbf{X} are zero, i.e.

$$\mathbf{X} = \begin{bmatrix} 0 & 0 \\ 0 & X \end{bmatrix}. \quad (6)$$

By left-multiplying (3) by \mathbf{X} , we can express voltages as an affine function of the currents:

$$\mathbf{X}\mathbf{L} \begin{bmatrix} u_0 \\ \vdots \\ u_n \end{bmatrix} = \mathbf{X} \begin{bmatrix} i_0 \\ \vdots \\ i_n \end{bmatrix}$$

and therefore, via (4)

$$\begin{bmatrix} u_0 \\ \vdots \\ u_n \end{bmatrix} = \mathbf{X} \begin{bmatrix} i_0 \\ \vdots \\ i_n \end{bmatrix} + u_0 \mathbf{1}. \quad (7)$$

Based on (1), we can eliminate the voltage and current of the PCC (which are trivially defined as $u_0 = U_0$ and $i_0 = -\sum_{v \in \mathcal{V} \setminus \{0\}} i_v$) in (7), and via (6) we obtain

$$u = \mathbf{X}i + U_0 \mathbf{1}, \quad (8)$$

where u and i are the n -dimensional complex-valued vectors

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix}.$$

Finally, we can plug the expression (8) for voltages in the first order dynamical equations (2), obtaining

$$\tau_v \frac{di_v}{dt} = -i_v + \frac{\bar{s}_v}{\epsilon \mathbf{1}_v^T \bar{X} i + \bar{U}_0}, \quad v = 1, \dots, n. \quad (9)$$

This model is a n -dimensional nonlinear dynamic system, in which the state of the system corresponds to the currents injected by the nodes, and the coupling between the individual nodes is due to the matrix X in the denominator. Together with (8), it provides a nonlinear input-output relation between power references s_v and node voltages u_v .

IV. APPROXIMATED MODEL

While model (9) could be easily used for simulation, its nonlinear nature makes it unfit for a mathematical analysis and for its use in the design of a controller. For this reason, we will now derive an approximated linear version of (9).

Let $\epsilon = 1/\bar{U}_0$. Equations (9) can be rewritten as

$$\tau_v \frac{di_v}{dt} = f_v(i, \epsilon) := -i_v + \frac{\epsilon \bar{s}_v}{\epsilon \mathbf{1}_v^T \bar{X} i + 1}, \quad v = 1, \dots, n. \quad (10)$$

We now rewrite (10) via a second-order Taylor expansion of $f_v(i, \epsilon)$ around $\epsilon = 0$, i.e.

$$f_v(i, \epsilon) \approx f_v(i, 0) + \left. \frac{\partial f_v(i, \epsilon)}{\partial \epsilon} \right|_{\epsilon=0} \epsilon + \frac{1}{2} \left. \frac{\partial^2 f_v(i, \epsilon)}{\partial \epsilon^2} \right|_{\epsilon=0} \epsilon^2 \quad (11)$$

where

$$\begin{aligned} f_v(i, \epsilon)|_{\epsilon=0} &= -i_v \\ \left. \frac{\partial f_v(i, \epsilon)}{\partial \epsilon} \right|_{\epsilon=0} &= \left. \frac{\bar{s}_v (\epsilon \mathbf{1}_v^T \bar{X} i + 1) - \bar{s}_v \epsilon \mathbf{1}_v^T \bar{X} i}{(\epsilon \mathbf{1}_v^T \bar{X} i + 1)^2} \right|_{\epsilon=0} = \bar{s}_v \\ \left. \frac{\partial^2 f_v(i, \epsilon)}{\partial \epsilon^2} \right|_{\epsilon=0} &= \left. \frac{-2\bar{s}_v (\epsilon \mathbf{1}_v^T \bar{X} i + 1) \mathbf{1}_v^T \bar{X} i}{(\epsilon \mathbf{1}_v^T \bar{X} i + 1)^4} \right|_{\epsilon=0} \\ &= -2\bar{s}_v \mathbf{1}_v^T \bar{X} i. \end{aligned}$$

Then, by (11), each node dynamics is approximated with

$$\tau_v \frac{di_v}{dt} \approx -i(v) + \epsilon \bar{s}_v - \epsilon^2 \bar{s}_v \mathbf{1}_v^T \bar{X} i. \quad (12)$$

While the first and the second term depend only by the current and power reference of each node, the third term models the coupling between all the grid nodes via the matrix X . We then obtain the approximated system

$$T \frac{di}{dt} = -i + \epsilon \bar{s} - \epsilon^2 \bar{S} \bar{X} i$$

where

$$T = \begin{bmatrix} \tau_1 & 0 & \dots \\ 0 & \tau_2 & 0 \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & \tau_n \end{bmatrix}, \quad S = \begin{bmatrix} s_1 & 0 & \dots \\ 0 & s_2 & 0 \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & s_n \end{bmatrix}, \quad \text{and } s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}.$$

To obtain a linear system, we augment the state i and the input s by including their complex conjugate \bar{i} and \bar{s} , thus obtaining a $2n$ -dimensional system in the form

$$\begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i \\ \bar{i} \end{bmatrix} = F \begin{bmatrix} i \\ \bar{i} \end{bmatrix} + G \begin{bmatrix} s \\ \bar{s} \end{bmatrix} \quad (13)$$

where

$$F = \begin{bmatrix} -I & -\epsilon^2 \bar{S} \bar{X} \\ -\epsilon^2 S X & -I \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} 0 & \epsilon I \\ \epsilon I & 0 \end{bmatrix}.$$

This system is a linear, time-varying system, as the input s is also present in the state update matrix F . However, if we assume that power references change slowly compared to the transient of the system, then we can assume that \bar{S} are constant and (13) becomes a linear, time invariant system.

Together with (8), we have therefore obtained an approximate, linear, description of the dynamic input-output relation between the complex power references s_v (inputs) and the node voltages u_v (outputs).

This result allows us to employ the tools of linear systems (eigenvalue analysis, step response analysis, transfer functions and Bode plots) to study the dynamic behavior of such system and to design the appropriate controller.

Moreover, the explicit presence of the line impedances (via the matrix X) and power demands in the model, allows to understand how these parameters affect its dynamic behavior.

As an example of what can be done in this sense, we report here a preliminary result regarding the eigenvalues of the matrix F . The proof of this result is omitted due to space limitations.

Result. The eigenvalues of F corresponds, up to an approximation error that vanishes for small ϵ ,

$$\Lambda = \left\{ -\frac{1}{\tau_v} \pm \frac{|\epsilon||s_v||X_{vv}|}{\tau_v} \right\}.$$

Remember now that according to (5), X_{vv} is the impedance of the path from the PCC to node v . Therefore this results shows how the eigenvalues of the global system depends on the time constant of each node, together with its power demand and its distance from the PCC. As the settling time (and, ultimately, the stability) depend on the largest of these eigenvalues, this results gives important information about the loads which are critical in this sense. It is clear how the fact of having explicit parametric dependence in the proposed model (13) allows some type of analysis that would be impossible if only a simulation tool was available.

V. SIMULATIONS

In this Section we validate the approximation that yielded to the linear model (13), by comparing its behavior with the nonlinear dynamics described by (9).

As a low voltage testbed is currently missing in the literature, we considered a 4.8 kV testbed inspired from the standard test feeder IEEE37 [18]. We however assumed that load are balanced, and therefore all currents and signals can be described in a single-phase phasorial notation. As shown in Figure 3, we assumed that some of the nodes are microgenerators connected to the microgrid via some power inverters. Following the modeling proposed in Section III, we assumed that every node (but the PCC) behaves as a constant-power device with a first order dynamic. A quite short time constant of 1.6 ms has been chosen to describe the fast dynamic behavior of the microgenerator inverters, while the time constants of the loads have been distributed between 200 ms and 10 s.

A step change of 60 kVAR in the reference for the injected reactive power has then been commanded to the microgenerator connected to node 30. The first graph in Figure 4 shows how the voltage at the same node (30) exhibit a rich dynamic behavior, due to the coupling of many nonlinear systems. Indeed, even if the inverter dynamic response is very fast, the slower behavior of the loads affect the voltage response: after an initial quick rise, the voltage approaches its steady state value quite slowly.

The dotted line in the same graph show the value of the voltage u_{30}^{ss} , which is the voltage corresponding to the solution of the static power flow equations. The second and third panel in Figure 4 show the absolute value and the angle of the error between u_{30} and u_{30}^{ss} . The dashed line corresponds to the linear approximated model (13), which appears to be extremely close to the output of the nonlinear model.

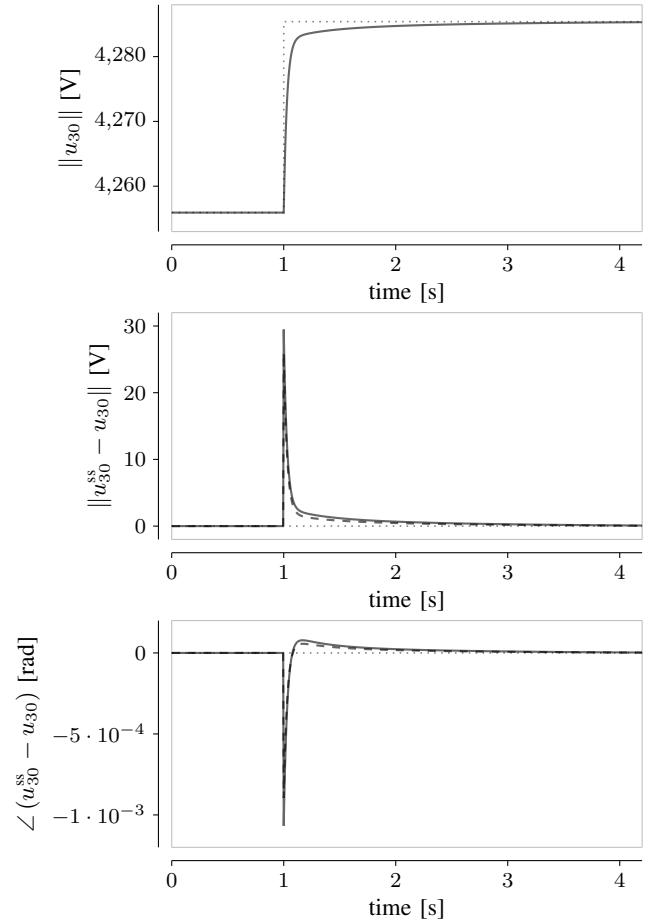


Figure 4. Voltage at node 30, after a step change in the reactive power injected by node 30. The dashed in the second and third panel is the output of the approximate linear model.

In Figure 5, the same quantities have been plotted for another node (22), which kept its power reference constant while node 30 was actuating the system. This figure shows how the effect of actuation in one point of the microgrid propagates to other nodes, causing similar transients in the voltage measurements of other nodes. It also shows how the approximated linear model correctly describes this behavior.

Figure 6 shows how the same step change in the reactive power injected by node 30 affects the amount of reactive power flowing into the microgrid from the PCC. The proposed model is capable of modeling this behavior too, and provides extremely useful information for those algorithms (like [3]) in which a central controller measures the power flow at the PCC and, on the basis of that measurement, commands the microgenerators in the microgrid. For the controller to be stable and effective, it is critical to have this kind of tools to estimate which is the time delay that have to be waited before sensing the PCC power flow again.

VI. CONCLUSIONS

In this paper we focused on a specific problem: obtaining a dynamic model for the input-output relation between the

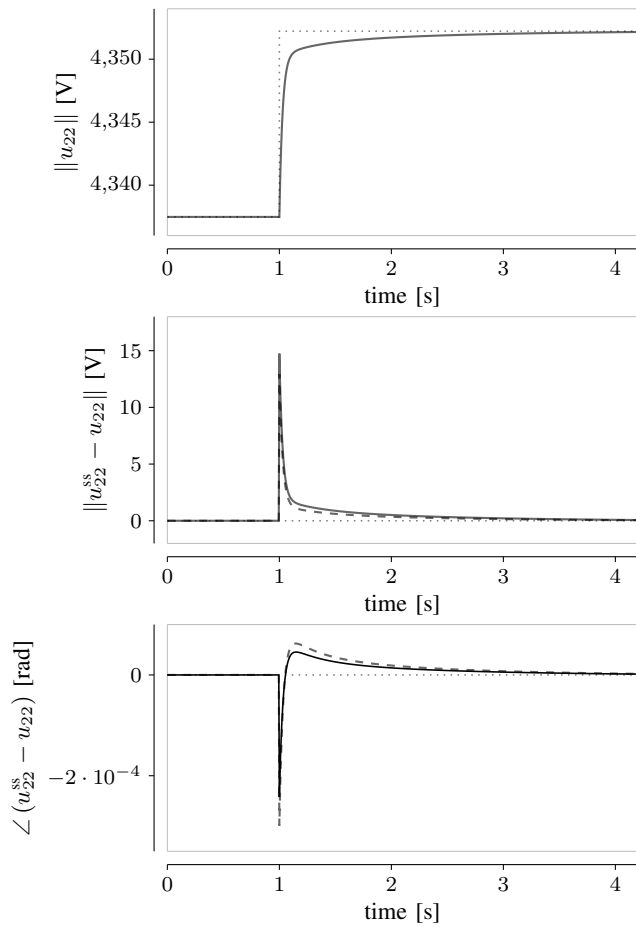


Figure 5. Voltage at node 22, after a step change in the reactive power injected by node 30. The dashed in the second and third panel is the output of the approximate linear model.

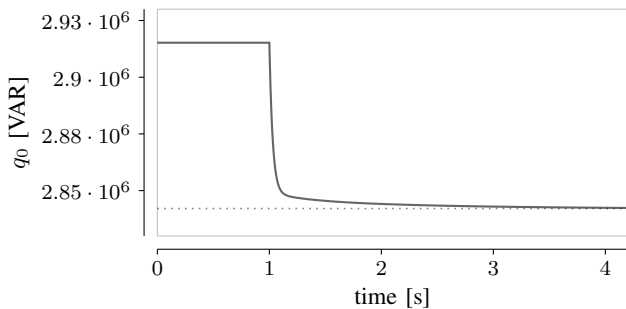


Figure 6. Reactive power flowing through the PCC, after a step change in the reactive power injected by node 30.

complex power references commanded to the microgenerators dispersed in a microgrid, and their phasorial voltage measurements. The obtained approximated model exhibits many remarkable features:

- it is a linear system, meaning that classical analysis and design tools can be employed;
- the network topology and parameters are clearly recognizable in the model (which is not true, in general, in

simulative approaches);

- the model catches the most relevant phenomena emerging because of the interconnection of a large number of simple nonlinear loads;
- the model gives important insight and guidelines for the design of the algorithms intended to provide ancillary services in the power distribution networks, as it provides estimates on the time of propagation and settling time after iterative actuations of the system.

The next step that will necessarily be explored is to validate this model with more detailed simulators (or experimental testbeds), in which loads with different steady state characteristics and different dynamic behaviors coexist.

REFERENCES

- [1] K. De Brabandere, K. Vanthournout, J. Driesen, G. Deconinck, and R. Belmans, "Control of microgrids," in *IEEE Power Engineering Society General Meeting*, 2007.
- [2] M. Prodanovic, K. De Brabandere, J. Van den Keybus, T. Green, and J. Driesen, "Harmonic and reactive power compensation as ancillary services in inverter-based distributed generation," *IET Generation, Transmission & Distribution*, vol. 1, no. 3, pp. 432–438, 2007.
- [3] E. Tedeschi, P. Tenti, and P. Mattavelli, "Synergistic control and cooperative operation of distributed harmonic and reactive compensators," in *Proceedings of the IEEE Power Electronics Specialists Conference (PESC 2008)*, 2008.
- [4] P. Tenti, A. Costabeber, and P. Mattavelli, "Improving power quality and distribution efficiency in micro-grids by cooperative control of switching power interfaces," in *IPEC 2010*, 2010.
- [5] A. Cagnano, E. De Tuglie, M. Liserre, and R. Mastromauro, "On-line optimal reactive power control strategy of pv-inverters," *IEEE Transactions on Industrial Electronics*, to appear.
- [6] P. Tenti, D. Trombetti, A. Costabeber, and P. Mattavelli, "Distribution loss minimization by token ring control of power electronic interfaces in residential micro-grids," in *ISIE 2010*, 2010.
- [7] S. Bolognani and S. Zampieri, "A gossip-like distributed optimization algorithm for reactive power flow control," in *Proc. IFAC World Congress 2011*, Milano, Italy, Aug. 2011.
- [8] K. Turitsyn, P. Šulc, S. Backhaus, and M. Chertkov, "Options for control of reactive power by distributed photovoltaic generators," *Proceedings of the IEEE*, vol. 99, no. 6, pp. 1063–1073, Jun. 2011.
- [9] M. C. Chandorkar, D. M. Divan, and R. Adapa, "Control of parallel connected inverters in standalone AC suppliesystems," *IEEE Transactions on Industrial Applications*, vol. 29, no. 1, pp. 136–143, Jan. 1993.
- [10] T. C. Green and M. Prodanović, "Control of inverter-based micro-grids," *Electric Power Systems Research*, vol. 77, no. 9, pp. 1204–1213, Jul. 2007.
- [11] A. G. Phadke, "Synchronized phasor measurements in power systems," *IEEE Computer Applications in Power*, vol. 6, no. 2, pp. 10–15, Apr. 1993.
- [12] A. G. Phadke and J. S. Thorp, *Synchronized phasor measurements and their applications*. Springer, 2008.
- [13] D. J. Hill, "Nonlinear dynamic load models with recovery for voltage stability studies," *IEEE Transactions on Power Systems*, vol. 8, no. 1, pp. 166–176, Feb. 1993.
- [14] D. Karlsson and D. J. Hill, "Modelling and identification of nonlinear dynamic loads in power systems," *IEEE Transactions on Power Systems*, vol. 9, no. 1, pp. 157–164, Feb. 1994.
- [15] W. Xu and Y. Mansour, "Voltage stability analysis using generic dynamic load models," *IEEE Transactions on Power Systems*, vol. 9, no. 1, pp. 479–493, Feb. 1994.
- [16] J. A. Lopes, C. L. Moreira, and A. G. Madureira, "Defining control strategies for microgrids islanded operation," *IEEE Transactions Power Systems*, vol. 21, no. 2, pp. 916–924, May 2006.
- [17] A. G. Phadke and B. Kasztenny, "Synchronized phasor and frequency measurement under transient conditions," *IEEE Transactions on Power Delivery*, vol. 24, no. 1, pp. 89–95, Jan. 2009.
- [18] W. H. Kersting, "Radial distribution test feeders," in *IEEE Power Engineering Society Winter Meeting*, vol. 2, Jan. 2001, pp. 908–912.