# Distributed control for optimal reactive power compensation in smart microgrids

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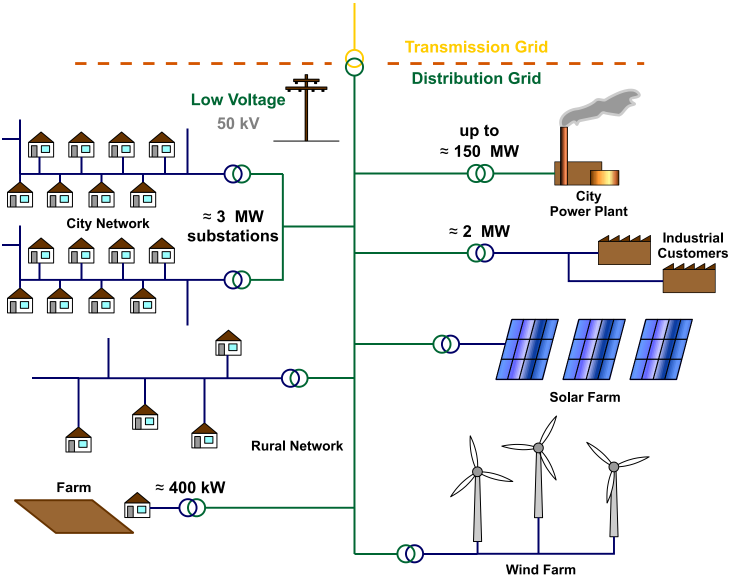
# REACTIVE POWER COMPENSATION

# Power distribution networks



# Microgrids

### Smart microgrid

We define a smart microgrid as a portion of the electrical power distribution network that hosts microgeneration devices (solar panels, ...) and is managed autonomously from the rest of the network.

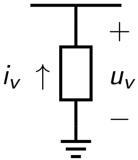
In particular, some microgrid controllers command the microgenerators in order to optimize the microgrid operation.

We focus on the problem of optimal reactive power compensation for the minimization of distribution losses.

# Reactive power

## Reactive power flows

Whenever a device in the grid injects (is supplied with) a current that is out of phase with the voltage, we have injection (delivery) of reactive power.




Adopting the phasorial notation for voltages and currents, we define the complex power

$$s_v = p_v + j q_v := u_v \bar{i}_v$$

## Reactive power "facts"

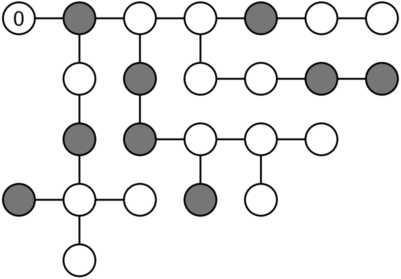
- Loads in the microgrid require reactive power
- reactive power can be obtained from the transmission grid or produced by the microgenerators in the grid
- producing reactive power has no fuel cost
- larger flows of reactive power correspond to quadratically larger power losses on the cables.

## Optimal reactive power compensation problem

Injecting reactive power in the grid as close as possible to the loads that need it, in order to minimize power distribution losses.

# MICROGRID MODEL

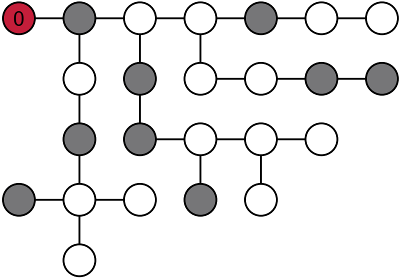
# Graph model

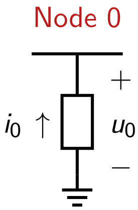



Nodes of the graph represent loads (in white) that cannot be controlled, and microgenerators (in gray) which can be commanded, can sense the grid, and can communicate.

Nodes are connected by a tree $\mathcal{T}$, representing the electrical connection (power lines) among them.

# Graph model

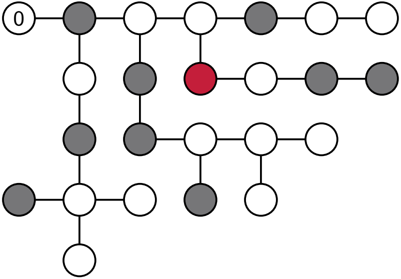


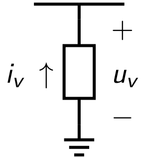




Node 0 represents the point of connection of the microgrid to the transmission grid.
Its voltage $u_0$ corresponds to the nominal voltage of the microgrid:

$$u_0 = U_0.$$

# Graph model



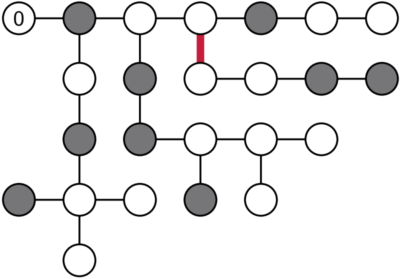



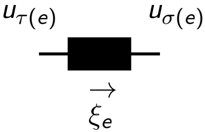

Node voltage $u_v$ and node current $i_v$ satisfy

$$u_v \bar{i}_v = s_v = S_v \left| \frac{u_v}{U_0} \right|^{\eta_v}$$

for microgenerators and loads (exponential / ZIP model).

# Graph model




Voltage drop $u_{\tau(e)} - u_{\sigma(e)}$ and the current $\xi_e$ flowing on the edge $e$ satisfy

$$u_{\tau(e)} - u_{\sigma(e)} = z_e \xi_e$$

where $z_e$ is the impedance of the power line $e$.

# Microgrid nonlinear equations

The voltages $u_v$ and the currents $i_v$ of the microgrid are therefore implicitely defined by the system of nonlinear equations

$$\begin{cases} Lu = i \\ u_v \bar{i}_v = S_v \left| \frac{u_v}{U_0} \right|^{\eta_v} & v \neq 0 \\ u_0 = U_0, \end{cases}$$

where $L$ is the weighted Laplacian of the graph

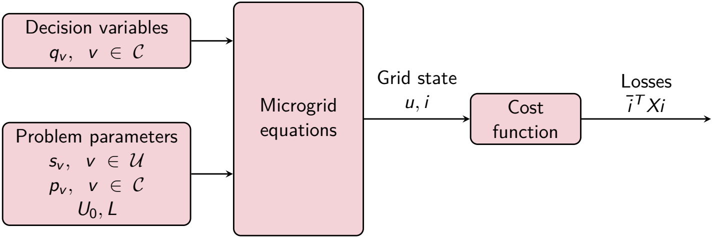
$$L = A^T Z^{-1} A$$

and $A$ is the incidence matrix of the graph.

# OPTIMIZATION PROBLEM

# Optimization problem

The optimization problem consists in deciding the reactive power injection at the microgenerators that minimizes power distribution losses.




In order to design an algorithm, we need to have an explicit expression for the grid state as a function of the decision variables.

# Explicit grid solution

## Approximate solution

We constructed the Taylor expansion of the system state for large nominal voltage $U_0$.

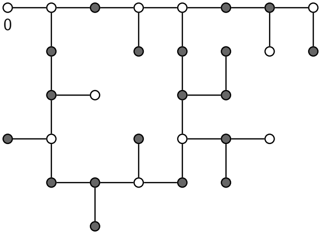
$$i_v(U_0) = \frac{\bar{S}_v}{\bar{U}_0} + \frac{\delta_v(U_0)}{\bar{U}_0}$$

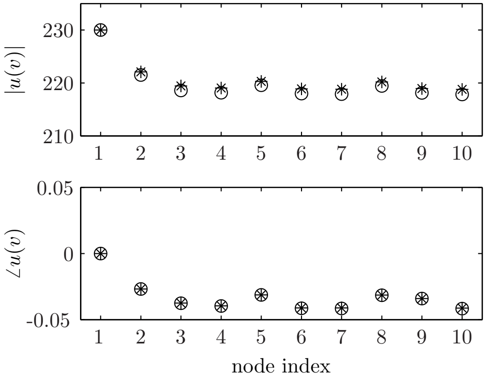
where $\delta_v(U_0)$ is infinitesimal when $U_0$ tends to infinity.

$$u_v(U_0) = U_0 + \frac{[X\bar{S}]_v}{\bar{U}_0} + \frac{\lambda_v(U_0)}{\bar{U}_0}$$

where $\lambda_v(U_0)$ is infinitesimal when $U_0$ tends to infinity.

# Approximation error



# Approximate problem

## Quadratic cost function

The approximate solution of the grid equations allows us to approximate the cost function (losses) as a convex quadratic function of the decision variables

$$J \approx \frac{1}{|U_0|^2} q^T X q \quad \text{subject to} \quad 1^T q = 0$$

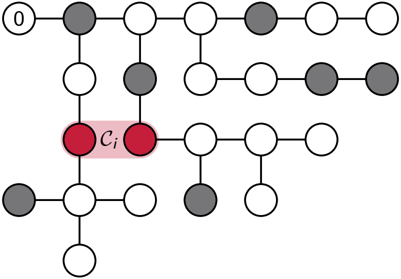
## Gradient estimation

Via local voltage measurements, agents can estimate the gradient.

$$\nabla J_v \approx -\frac{1}{|\mathcal{C}'|} \sum_{w \in \mathcal{C}'} |u_v||u_w| \sin(\angle u_v - \angle u_w - \theta)$$

# DISTRIBUTED ALGORITHM

# Iterative algorithm




Consider the family of subsets of $\mathcal{C}$

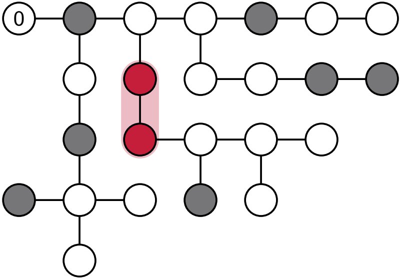
$$\{\mathcal{C}_1, \ldots, \mathcal{C}_\ell\}$$

such that $\bigcup_{i=1}^{\ell} \mathcal{C}_i = \mathcal{C}$.

Every time a set $\mathcal{C}_i$ is chosen

1) agents in $\mathcal{C}_i$ sense the network and obtain an estimate of the gradient;
2) they determine the update step that minimizes the given cost function, coordinating their action;
3) they actuate the system by updating their state $q_v$.

# Iterative algorithm

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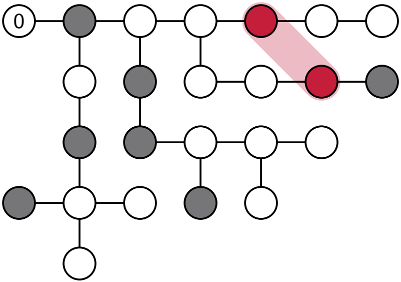


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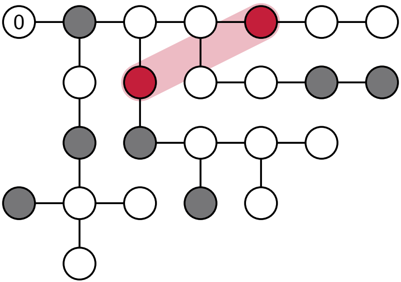


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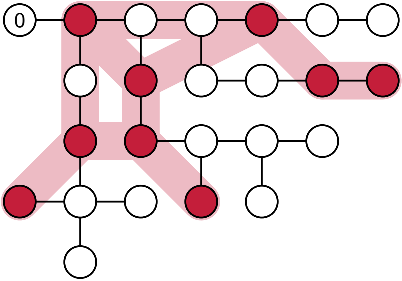
# Algorithm convergence

## Convergence result

Let $\mathcal{H}$ be an hypergraph whose edges $e \subseteq \mathcal{C}$ corresponds to the subproblems. Then the algorithm converges if and only if $\mathcal{H}$ is connected.



# Rate of convergence

Consider the expected cost

$$v(t) = \mathbb{E}[J(q(t)) - J(q^*)].$$

(Exponential) convergence rate

$$R = \sup_{q(0)\in\mathcal{S}} \limsup v(t)^{1/t}.$$

We characterized the convergence rate $R$ (via an upper bound) as a function of

- grid topology and parameters
- cluster size
- clustering strategy.

# Optimal clustering stategy

## Neighbor-to-neighbor communication

The optimal strategy consists in choosing clusters which resembles the physical interconnection of the electric network (edge-disjoint).

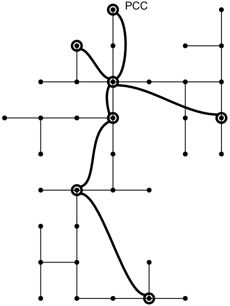
$$R = 1 - \frac{\left(\sum_{i=1}^{\ell} \rho_i |\mathcal{C}_i|\right) - 1}{m - 1}$$

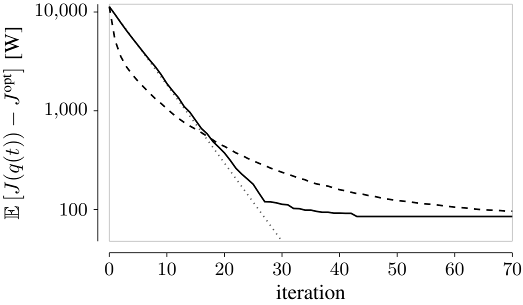
This result is interesting in the fact that they constrast with the phenomena generally observed in gossip consensus algorithms, in which long-distance communications are beneficial for the rate of convergence.

This is of course motivating, and suggests further investigation towards plug and play protocols, parallel implementation, communication over power lines, etc.

# Simulations

The algorithm behavior has been simulated on the IEEE 37 standard testbed.



# CONCLUSION

Two main contributions have been presented.

- **Microgrid power flows modeling**
  The proposed approximate solution of the nonlinear power flow equations is a powerful tool: it allows to cast the problem in a well-known framework and, more important, it shows how to obtain system-wide information (the gradient) via local measurements (voltages).
- **Randomized gossip-like algorithm**
  The proposed algorithm is a possible decentralized solution for this optimization problem.
  Its convergence is guaranteed, and its rate of convergence has been analyzed, yielding design rules to maximize performance.

Bolognani, S., and Zampieri, S. (2011).
A gossip-like distributed optimization algorithm for reactive power flow control.
Extended version available online on http://automatica.dei.unipd.it
*IFAC World Congress 2011*, Milano, Italy.

Bolognani, S., and Zampieri, S. (2011).
Distributed control for optimal reactive power compensation in smart microgrids.
Extended version available online on http://automatica.dei.unipd.it
*50th IEEE CDC*, Orlando (FL), USA.

Bolognani, S., and Zampieri, S. (2011).
A distributed control strategy for reactive power compensation in smart microgrids.
Available online on http://arxiv.org

# Thanks!

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