A distributed control strategy for reactive power compensation in smart microgrids

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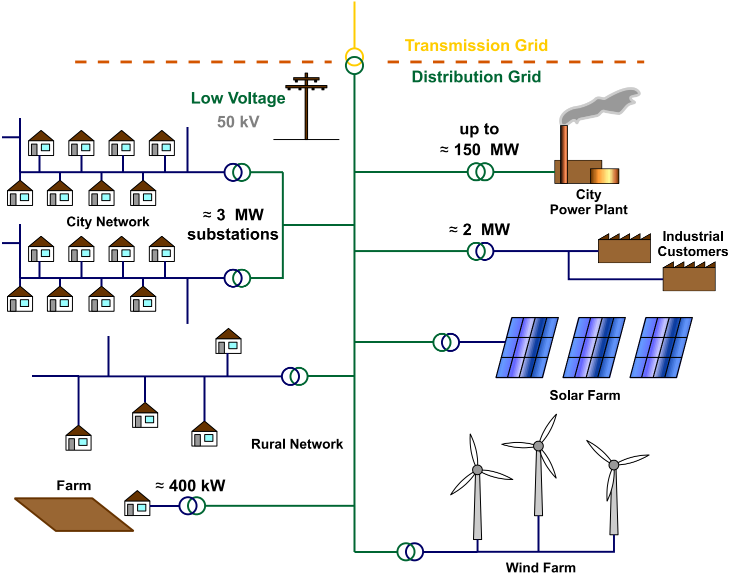
REACTIVE POWER COMPENSATION

Power distribution networks



Microgrids

Smart microgrid

We define a smart microgrid as a portion of the electrical power distribution network that hosts microgeneration devices (solar panels, ...) and is managed autonomously from the rest of the network.

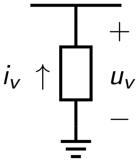
In particular, some microgrid controllers command the microgenerators in order to optimize the microgrid operation.

We focus on the problem of optimal reactive power compensation for the minimization of distribution losses.

Reactive power

Reactive power flows

Whenever a device in the grid injects (is supplied with) a current that is out of phase with the voltage, we have injection (delivery) of reactive power.



Adopting the phasorial notation for voltages and currents, we define the complex power

$$s_v = p_v + j q_v := u_v \bar{i}_v$$

Reactive power "facts"

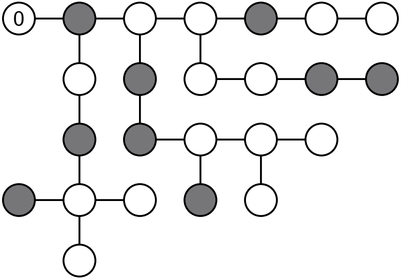
- Loads in the microgrid require reactive power
- reactive power can be obtained from the transmission grid or produced by the microgenerators in the grid
- producing reactive power has no fuel cost
- larger flows of reactive power correspond to quadratically larger power losses on the cables.

Optimal reactive power compensation problem

Injecting reactive power in the grid as close as possible to the loads that need it, in order to minimize power distribution losses.

MICROGRID MODEL

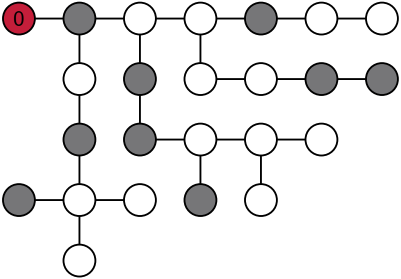
Graph model

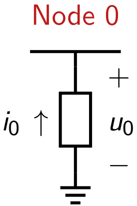


Nodes of the graph represent loads (in white) that cannot be controlled, and microgenerators (in gray) which can be commanded, can sense the grid, and can communicate.

Nodes are connected by a tree $\mathcal{T}$, representing the electrical connection (power lines) among them.

Graph model

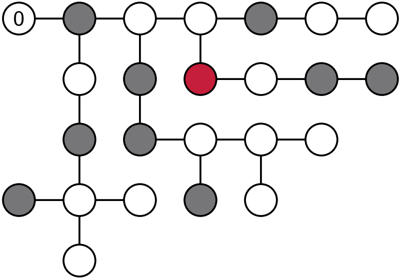




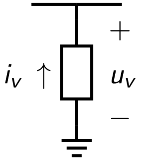
Node 0 represents the point of connection of the microgrid to the transmission grid.
Its voltage u_0 corresponds to the nominal voltage of the microgrid:

$$u_0 = U_0.$$

Graph model



Nodes $v \neq 0$

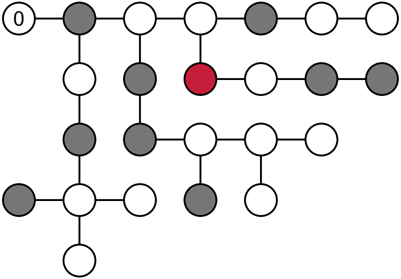


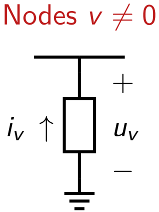
Node voltage u_v and node current i_v satisfy

$$s_v = u_v \bar{i}_v = S_v \left| \frac{u_v}{U_0} \right|^{\eta_v}$$

for microgenerators and loads (exponential / ZIP model).

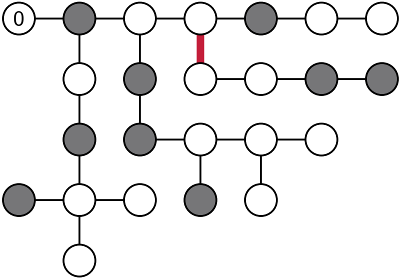
Graph model



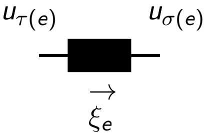


- $\eta_v = 0$: constant power devices
  (including microgenerators, which receive complex power references)
- $\eta_v = 1$: constant current devices
- $\eta_v = 2$: constant impedance devices

Graph model



Edges e



Voltage drop $u_{\tau(e)} - u_{\sigma(e)}$ and the current ξ_e flowing on the edge e satisfy

$$u_{\tau(e)} - u_{\sigma(e)} = z_e \xi_e$$

where z_e is the impedance of the power line e.

Microgrid nonlinear equations

The voltages u_v and the currents i_v of the microgrid are therefore implicitely defined by the system of nonlinear equations

$$\begin{cases} Lu = i \\ u_v \bar{i}_v = S_v \left| \frac{u_v}{U_0} \right|^{\eta_v} & \qquad v \neq 0 \\ u_0 = U_0, \end{cases}$$

where L is the weighted Laplacian of the graph

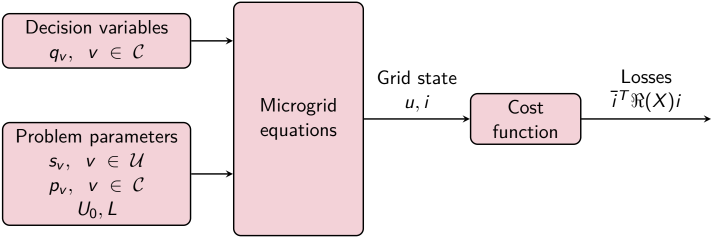
$$L = A^T Z^{-1} A$$

and A is the incidence matrix of the graph.

OPTIMIZATION PROBLEM

Optimization problem

The optimization problem consists in deciding the reactive power injection at the microgenerators that minimizes power distribution losses.



In order to design an algorithm we need to have an explicit expression for the grid state as a function of the decision variables.

Explicit grid solution

Approximate solution

We constructed the Taylor expansion of the system state for large nominal voltage U_0.

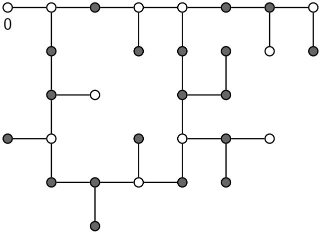
$$i_v(U_0) = \frac{\bar{S}_v}{\bar{U}_0} + \frac{\delta_v(U_0)}{\bar{U}_0}$$

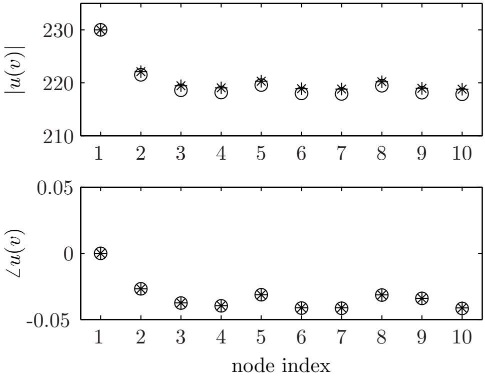
where $\delta_v(U_0)$ is infinitesimal when U_0 tends to infinity.

$$u_v(U_0) = U_0 + \frac{[X\bar{S}]_v}{\bar{U}_0} + \frac{\lambda_v(U_0)}{\bar{U}_0}$$

where $\lambda_v(U_0)$ is infinitesimal when U_0 tends to infinity.

Approximation error





This model extends the DC power flow model, by relaxing the assumption of zero losses (i.e. inductive lines).

Approximate problem

The approximate solution of the grid equations allows us to rewrite the cost function (losses) as a quadratic function of the decision variables.

$$J = \frac{1}{|U_0|^2} p^T \, \Re(X) \, p + \frac{1}{|U_0|^2} q^T \, \Re(X) \, q + \frac{1}{|U_0|^2} \tilde{J}(q, U_0)$$

where $\tilde{J}(U_0)$ is infinitesimal for large U_0, and q satisfies $1^T q = 0$.

Quadratic cost function

We approximated the original problem as a convex quadratic optimization problem subject to a linear equality constraint.

DISTRIBUTED ALGORITHM

Motivation for a distributed algorithm

Employing the closed form solution for the solution of the quadratic (linearly constrained) optimization problem is practically impossible:

- complete knowledge of the system structure and of the system state is required
- coordination and communication among all agents is required
- compensators
  - are in large number
  - can connect and disconnect
  - have limited communication capabilities.

Distributed architecture

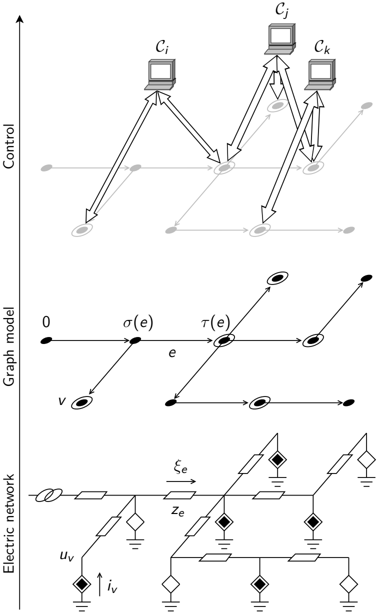
Consider the family of subsets of $\mathcal{C}$

$$\{\mathcal{C}_1, \ldots, \mathcal{C}_\ell\}$$

such that $\bigcup_{i=1}^{\ell} \mathcal{C}_i = \mathcal{C}$.

Let each cluster be managed by an intelligent unit (possibly, one of the compensators), which

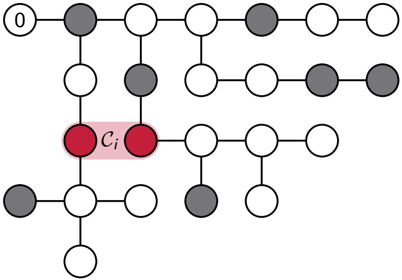
- knows the relative position of the compensators
- collects data from the compensators
- processes the collected data
- commands the compensators.



Iterative algorithm

At every time step

1) a cluster $\mathcal{C}_i$ is chosen;
2) agents in $\mathcal{C}_i$ coordinate their action and determine the optimal update step that minimizes the global cost function;
3) they actuate the system by updating their state $q_v, v \in \mathcal{C}_i$, while the other compensators keep their state constant.



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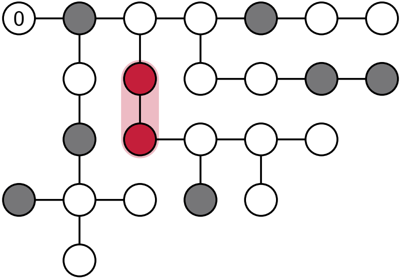

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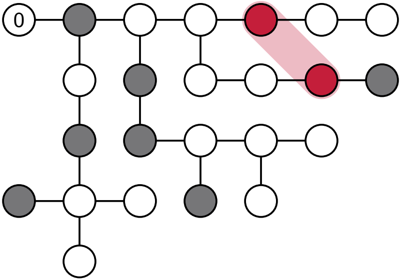

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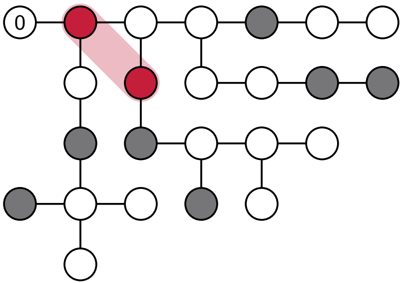

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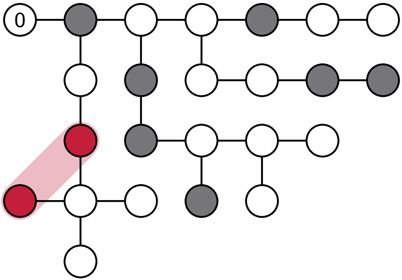

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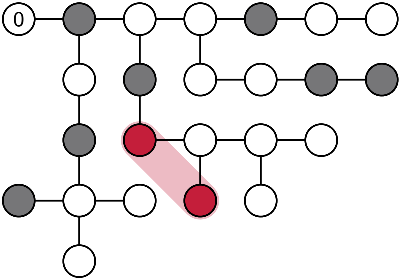

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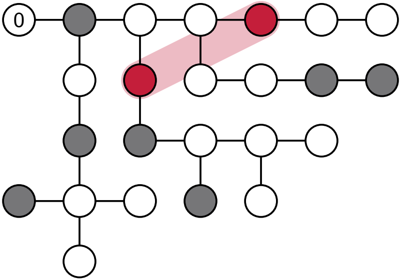


Computation of the optimal step for $\mathcal{C}_i$

The optimal update that has to be performed by cluster $\mathcal{C}_i$ is given by the (constrained) Newton step:

$$\begin{cases} q_h^{\text{opt, } i} = q_h & \text{for each } h \notin \mathcal{C}_i, \\ q_h^{\text{opt, } i} = q_h - \sum\limits_{k \in \mathcal{C}_i} \Gamma_{hk}^{(i)} \, \nabla J_k & \text{for each } h \in \mathcal{C}_i, \end{cases}$$

where

- $\Gamma^{(i)}$ is function of the Hessian $\Re(X)$,
- ∇J is the gradient.

In general, these are global quantities.

However, according to the approximate model for the power system state, both $\Gamma_{hk}^{(i)}$ and ∇J_k can be obtained from local data.

Computation of the optimal step for $\mathcal{C}_i$

Hessian estimation

$$\Gamma^{(i)} = -2 \left(\Omega_i \, R_{\text{eff}}^{(i)} \, \Omega_i \right)^{\sharp},$$

where

- $R_{\text{eff}}^{(i)}$ is the matrix of the cluster mutual effective resistances
- Ω_i is a projection operator.

Gradient estimation

$$\nabla J_k \approx -\cos\theta \frac{1}{|\mathcal{C}_i|} \sum_{w \in \mathcal{C}_i} |u_k||u_w| \sin(\angle u_k - \angle u_w - \theta),$$

where

- u_w are the voltage measurements inside $\mathcal{C}_i$
- θ is a known grid parameter.

Resulting algorithm

We therefore obtained the following distributed control algorithm.

Offline initialization

Each cluster computes $\Gamma^{(i)}$ according to the electric distance among compensators in the cluster.

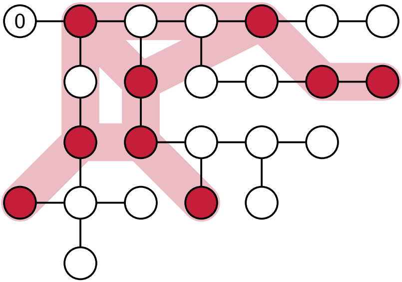
Online iterative algorithm

1. a cluster $\mathcal{C}_i$ is chosen;
2. agents not in $\mathcal{C}_i$ hold their state constant;
3. agents in $\mathcal{C}_i$
   3.1 measure their voltage and estimate ∇J;
   3.2 compute the optimal update step $-\Gamma^{(i)} \nabla J$;
   3.3 update their state;

Algorithm convergence

Convergence result

Let $\mathcal{H}$ be an hypergraph whose hyperedges corresponds to the clusters $\{\mathcal{C}_i\}$.
Then the algorithm converges if and only if $\mathcal{H}$ is connected.



Rate of convergence

Consider the expected cost

$$v(t) = \mathbb{E}[J(q(t)) - J(q^*)].$$

(Exponential) convergence rate

$$R = \sup_{q(0)} \limsup v(t)^{1/t}.$$

We characterized the convergence rate R as a function of

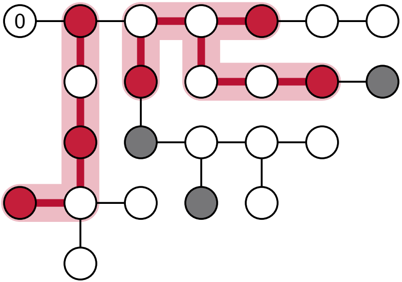
- grid topology and parameters
- clustering strategy.

Neighbor-to-neighbor communication

Edge-disjoint clusters

Let $\mathcal{P}_{hk}$ be the electric path between node h and k.
Let $\mathcal{P}_{\mathcal{C}_i} = \bigcup_{h,k \in \mathcal{C}_i} \mathcal{P}_{hk}$.
Two clusters i, j are edge-disjoint if

$$\mathcal{P}_{\mathcal{C}_i} \cap \mathcal{P}_{\mathcal{C}_j} = \emptyset.$$



Optimal clustering strategy

The optimal strategy consists in choosing clusters which resembles the physical interconnection of the electric network.

Best achievable performance

If all clusters are edge-disjoint, then

$$R = 1 - \frac{\left(\sum_{i=1}^{\ell} \rho_i |\mathcal{C}_i|\right) - 1}{m - 1} = R^{\mathsf{OPT}},$$

where

- ρ_i is the probability of choosing cluster $\mathcal{C}_i$,
- $|\mathcal{C}_i|$ is the size of cluster $\mathcal{C}_i$,
- m is the number of compensators.

Optimal clustering stategy

This result is interesting in the fact that it constrasts with the phenomena generally observed in gossip consensus algorithms, in which long-distance communications are beneficial for the rate of convergence.

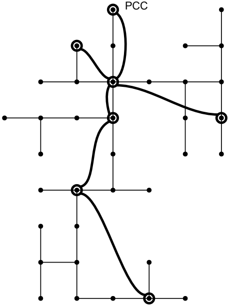
$$J = \frac{1}{|U_0|^2} q^T \, \Re(X) \, q \quad \text{subject to} \quad 1^T q = 0.$$

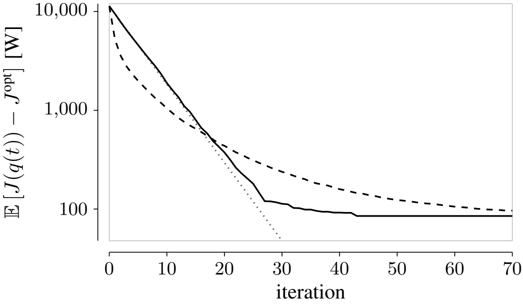
This is of course motivating, and suggests further investigation towards

- plug and play protocols,
- parallel implementation,
- communication over power lines.

Simulations

The algorithm behavior has been simulated on the IEEE 37 standard testbed.





dotted line – best possible performance
solid line – edge-disjoint clusters (see left)
dashed line – complete communication graph

CONCLUSIONS AND REMARKS

Conclusions

Two main contributions have been presented.

Microgrid power flows modeling

The proposed approximate solution of the nonlinear power flow equations is a powerful tool:

- it extends the DC model to generic line impedances
- it allows to cast the problem into a well-known framework
- it shows how to obtain system-wide information (the gradient, the hessian) from local measurements (voltages, electric distance).

Remark

Other applications of distributed optimization share this same feature (radio power control, congestion avoidance protocols in data networks).

In these applications, the iterative tuning of the decision variables associated to each agent (radio power, transmission rate) depends on congestion indices that are function of the entire state of the system. However, these indices can be detected locally by each agent by measuring some feedback signals: error rates, delays, signal-to-noise ratios, etc.

Conclusions

Randomized gossip-like algorithm

The proposed algorithm is one possible decentralized solution for this optimization problem.
Its convergence is guaranteed, and its rate of convergence has been analyzed, yielding design rules to maximize performance.

Other algorithms can be considered (subgradient methods, ADMM, ...). However, this specific problem features

- a non-separable cost function,
- a global complicating constraint,
- the requirement of primal feasibility at every time.

Future work

Among the very next steps on this specific topic, are:

- modelization of time varying demand (dynamic optimization)
- adoption of a meaningful performance metric for the dynamic problem
- introduction of box constraints of the agents
- study of the algorithm robustness against measurement noise
- design of a pricing mechanism for reactive power for fair rewarding of the agents.

Bolognani, S., and Zampieri, S. (2011).
A gossip-like distributed optimization algorithm for reactive power flow control.
Extended version available online on http://automatica.dei.unipd.it
IFAC World Congress 2011, Milano, Italy.

Bolognani, S., and Zampieri, S. (2011).
Distributed control for optimal reactive power compensation in smart microgrids.
Extended version available online on http://automatica.dei.unipd.it
50th IEEE CDC, Orlando (FL), USA.

Bolognani, S., and Zampieri, S. (2011).
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Available online on http://arxiv.org

Thanks!

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