Combined Speed and Current Model Predictive Control with Inherent Field-Weakening Features for PMSM Drives

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May 5th, 2008

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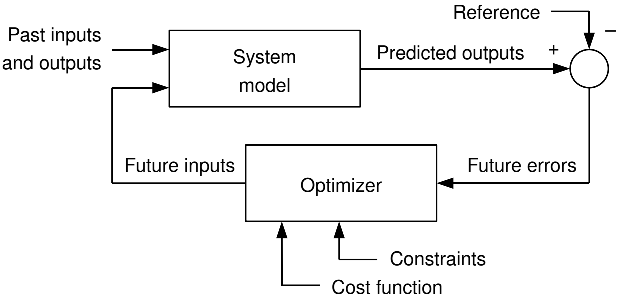

Receding horizon strategy

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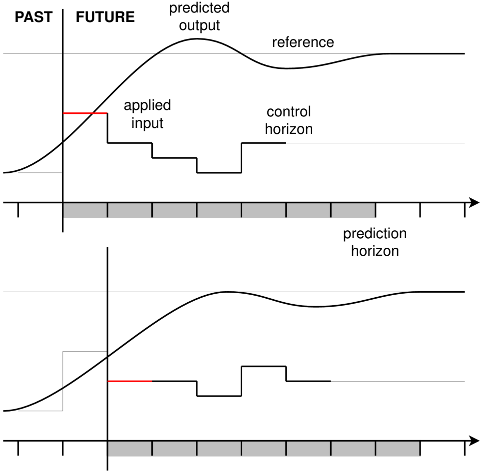

Mathematical framework

One of the mathematical framework that can be used for MPC is state space (linear) modeling with linear constraints.

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)\\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)\end{aligned}$$

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This setup simplifies the resulting control law.

Fields of application

Model Predictive Control is particularly effective for plants that:

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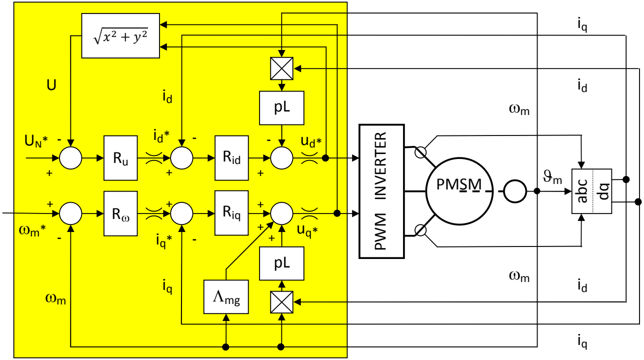
Electrical drives fit in this category!

Once faster hardware and algorithm are available, MPC can be considered when designing an electrical motor drive, for example for a PMSM drive.

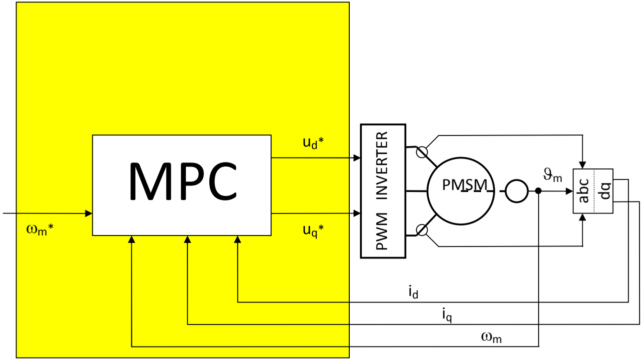
Controller structure



PID cascade controller

Classical solution: an inner loop for the current control and an outer loop for speed control. Bounds are enforced in every controller and flux weakening requires an external algorithm.

Controller structure



One MPC single controller

One only MPC controller is driven by the speed reference and commands the voltages that feed the motor. Bounds are enforced inside the controller.

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which are continuous-time, non linear equations, even if iron saturation is assumed negligible (constant parameters), because of the motional coupling terms.

Linearized discrete–time model

The first step in MPC design consists in obtaining a linear, discrete–time model for the system. This mainly means getting rid of the motional coupling terms $\omega_{me} L i_d$ and $\omega_{me} L i_q$.

$$\mathbf{x} = \begin{bmatrix} i_d & i_q & \widehat{\omega_{me} i_q} & \omega_{me} \end{bmatrix}^T$$

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}} = \begin{bmatrix} -\frac{R}{L} & 0 & 1 & 0 \\ -\omega_B & -\frac{R}{L} & 0 & -\frac{\Lambda_{mg}}{L} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{pk_t}{J} & 0 & -\frac{B}{J} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

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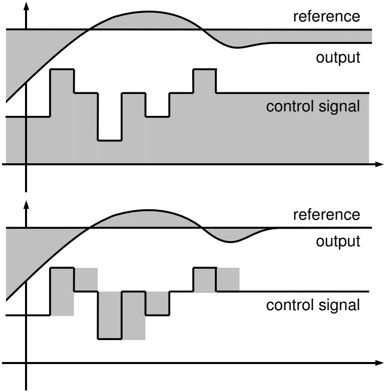

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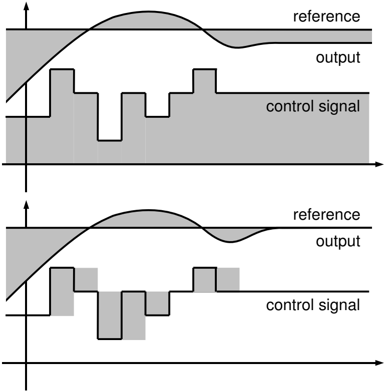
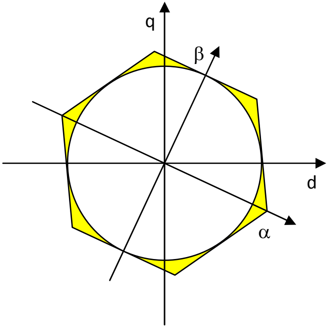


After this augmentation, the system state has dimension 7.

Input and state constraints



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Due to inverter limitation, the voltage vector has to stay inside an hexagon which in fixed in the $\alpha - \beta$ reference frame. In the $d - q$ frame this can be approximated by a circular region.

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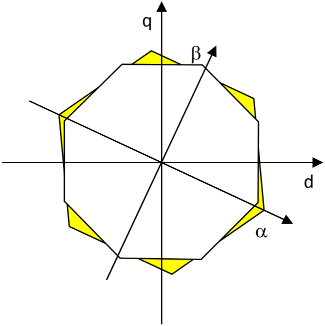

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To have linear contraints in the $d - q$ frame, an octagonal region is adopted.

Cost function

Once the model of the system is fully defined, the cost function has to be tuned.

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- γ_{i_d}
  penalizes the direct current i_d, so that it is kept to zero unless it is really needed (i.e. for flux weakening)

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The matrices **Q** and **R** contain weighting coefficients for the state variables and the input signals.

- γ_{i_d}
- γ_{i_q}
  slightly penalizes the quadrature current i_q (mainly for stability purposes)

Cost function

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- γ_{i_d}
- γ_{i_q}
- $\gamma_{\omega_{me}}$

  weighs the speed error with respect to the gived speed reference

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- γ_{i_d}
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  weights the input signal variations (it has to be non–zero to have unique solution of the optimization problem)

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These parameters are mainly tuned in a trial–and–error process based on simulations.

Affine state feedback control law

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Moreover, the resulting control that is commanded (the first sample of the optimal input sequence) results to be an affine feedback of the state defined in different regions of the state space.

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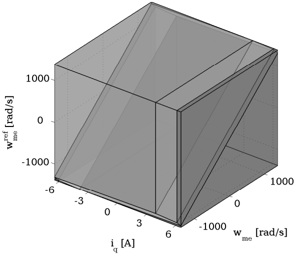

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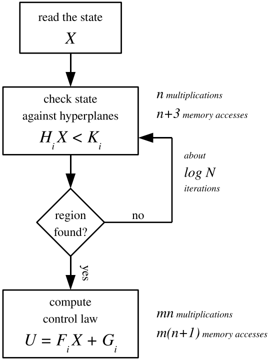
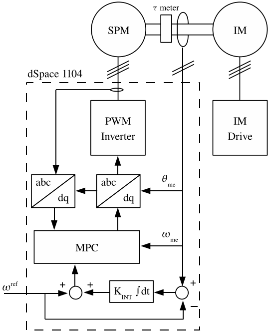
It is not necessary to inspect all the regions! A dichotomic search can be done comparing the current state against a series of hyperplanes.
A binary search tree is built off-line. The online part of the algorithm is minimal!



Test bench



The MPC controller we just designed has been implemented on a dSpace 1104 fast prototyping board.

The *Multi Parametric Toolbox (MPT)* for *Matlab* provides both a solver for the optimization problem and some routines to generate the region partition and the binary search tree for simulation and actual implementation.

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Speed reference step 1200 to 1800 rpm – no FW is needed.

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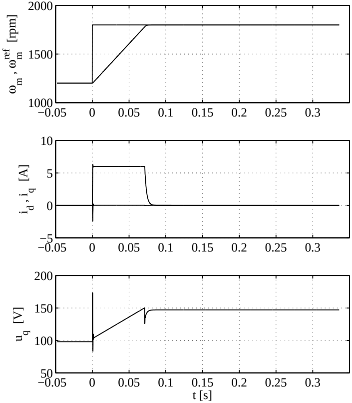

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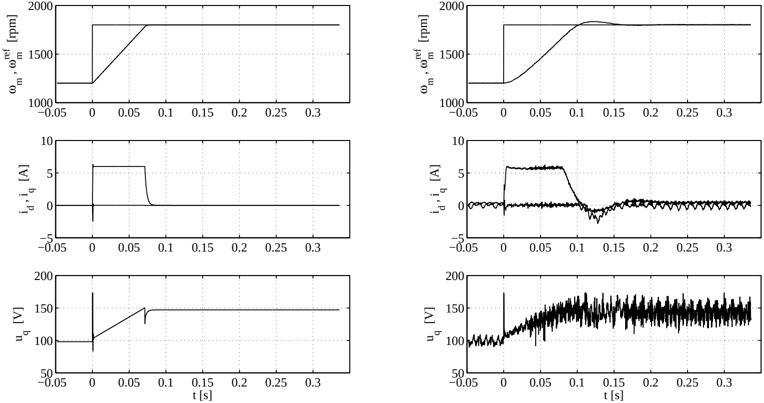

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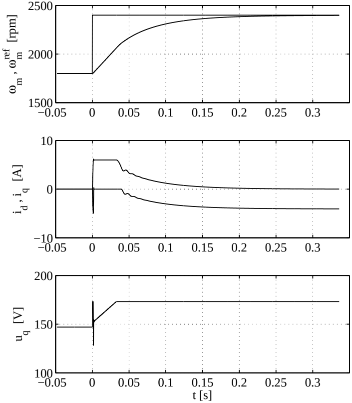

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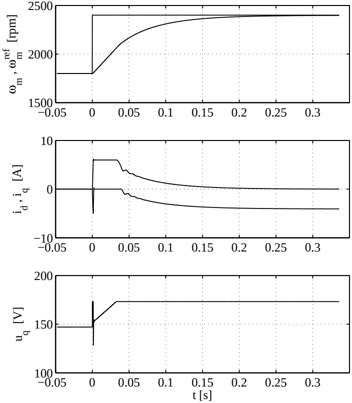

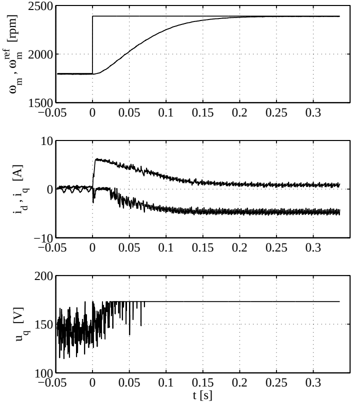

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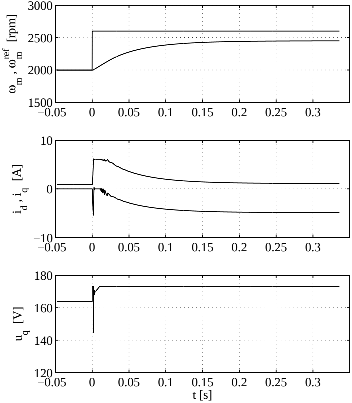

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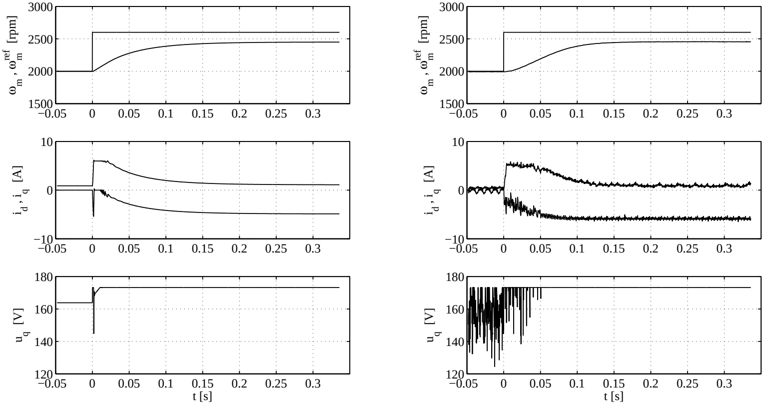

MPC for electrical drives

The proposed MPC controller for both speed and current of a PMSM drive, is able to command the motor both below and above base speed, by an effective and inherent flux weakening.

Details of the design, of the control algorithm generation, and of the actual implementation have been provided in the paper.

Simulation and experimental results prove the promising performance of the proposed controller.

Pros and Cons

Pros:

- one single controller effectively drives the motor in all its operating regions
- bounds are automatically enforced, with no need of saturations and anti–windup strategies
- more complex cost functions may be adopted
- reference and load dynamics can be added in the model

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Cons:

- the computational effort is higher than most of the conventional controllers
- drive and controller parameter changes require off-line algorithm tuning
- model mismatch has to be taken into account

Possible future works

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- testing on different drives, SPM motors with wider flux weakening region, IPM motors
- direct implementation with FPGA
- stability and robustness analysis with respect to parameters mismatch

Thank you!