Some results on optimal estimation and control for lossy NCS

Luca Schenato
Networked Control Systems

Drive-by-wire systems

Wireless Sensor Networks

Swarm robotics

Traffic Control: Internet and transportation

Smart structures: adaptive space telescope

Smart materials: sheets of MEMS sensors and actuators

NCSs: physically distributed dynamical systems interconnected by a communication network
NCSs: what’s new for control?

Classical architecture: Centralized structure
NCSs: what’s new for control?

NCSs: Large scale distributed structure

Connectivity
Limited capacity

Interference
COMMUNICATION
NETWORK
Congestion

Packet loss
Random delay
Quantization

COMMUNICATION NETWORK

Plant

Packet loss
Random delay
Quantization

Connectivity
Limited capacity
Interdisciplinary research needed

**COMMUNICATIONS ENGINEERING**
- Comm. protocols for RT apps
- Packet loss and random delay
- Wireless Sensor Networks
- Bit rate and Inf. Theory

**SOFTWARE ENGINEERING**
- Embedded software design
- Middleware for NCS
- RT Operating Systems
- Layering abstraction for interoperability

**NETWORKED CONTROL SYSTEMS**
- Graph theory
- Distributed computation
- Complexity theory
- Consensus algorithms

**COMPUTER SCIENCE**
Interdisciplinary research needed

COMMUNICATIONS ENGINEERING
- Comm. protocols for RT apps
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NETWORKED CONTROL SYSTEMS
- Graph theory
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COMPUTER SCIENCE
- Average TimeSync (ATS): a distributed consensus protocol for sensor networks clock synchronization

Martedi prossimo
NCS example: Pursuit Evasion Games w Sensor Networks

Sensor nodes w/ motion sensors

Information flow from SN

Evaders

Pursuers
Motivating example: wireless sensor networks

Forest Temperature Monitoring
(data-extraction application)

Wildfire detection & tracking
(real-time application)

- Can we design optimal estimators that compensate for random delay and packet loss?
- What is the performance if we have packet arrival statistics?
- How can we compare different communication/routing protocols in terms of estimation performance?
Optimal LQG

\[ u^c_t = u^c_t \]

\[ x_{t+1} = Ax_t + Bu^a_t + w_t \]

\[ y_t = Cx_t + v_t \]

\[
\min_{u^c_1, \ldots, u^c_T} J = \sum_{t=1}^{T} E[x_t^T W x_t + u_t^T U u_t], \quad T \to \infty
\]

Sensors and actuators are co-located, i.e. no delay nor loss
1. Separation principle holds: Optimal controller = Optimal estimator design + Optimal state feedback design
2. Closed Loop system always stable (under standard cont/obs. hypotheses)
3. Gains $K,L$ are constant solution of Algebraic Riccati Equations
Optimal LQG control over DCN

**Actuators**
\[ u_t^a = \begin{cases} u_t^c - \tau \\ 0 \end{cases} \]

**Plant**
\[ x_{t+1} = Ax_t + Bu_t^a + w_t \]

**Sensors**
\[ y_t = Cx_t + v_t \]

**Controller?**

**DIGITAL COMMUNICATION NETWORK**

**Controller**

Random delay or drop

\[ u_t^c - \tau_c \rightarrow \]

\[ \text{ACK?} \]

\[ Y_t - \tau_s \rightarrow \]
Some consideration on the separation principle

\[ u^c_t = u^c_t \]

\[ x_{t+1} = Ax_t + Bu^a_t + w_t \]

\[ y_t = Cx_t + v_t \]

**Actuators**

\[ u^c_t = u^c_t \]

**Plant**

\[ x_{t+1} = Ax_t + Bu^a_t + w_t \]

**Sensors**

\[ y_t = Cx_t + v_t \]

**State feedback**

\[ u^c_t = L\hat{x}_t \]

**Kalman filter**

\[ \hat{x}_t = A\hat{x}_{t-1} + Bu^c_{t-1} + K(y_t - C\hat{x}_t) \]

Random delay
Packet loss

\[ \hat{x}_t = E[x_t | y_t, y_{t-1}, \ldots, y_0, u^a_{t-1}, \ldots, u^a_1] \]

\[ = f(y_t, y_{t-1}, \ldots, y_0) \]

if \((u^a_{t-1}, \ldots u^a_1)\) known \(\Rightarrow e_t = x_t - \hat{x}_t = f(y_t, y_1, \ldots, y_1, y_0)\)
Modeling of Digital Communication Network (DCN)

Assumptions:
(1) Quantization noise $<<$ sensor noise
(2) Packet-rate limited (bit-rate)
(3) No transmission noise (data corrupted=dropped packet)
(4) Packets are time-stamped

Random delay & Packet loss ($\tau=1$) at receiver
Estimation modeling

PLANT

\[ x_{t+1} = Ax_t + w_t \]
\[ y_t = Cx_t + v_t \]

Digital Communication Network

ESTIMATOR

Buffer
\[ y_1 \ast y_3 \ast y_4 \ast \ldots \]

\[ \hat{x}_t \]

- No packet arrives
  \[ t = 3 \]
  \[ y_3 \rightarrow [y_1 \ y_3] \]

- Packet out of order
  \[ t = 4 \]
  \[ y_2 \rightarrow [y_1 \ y_2 \ y_3] \]

- Multiple packets arrive
  \[ t = 5 \]
  \[ y_4, y_6 \rightarrow [y_1 \ y_2 \ y_3 \ y_4 \ y_6] \]

\[ \ddots \]
Minimum variance estimation

\[ \hat{x}_t = \mathbb{E}[x_t | \{y_k\}] \text{ available at estimator at time } t \]

**PLANT**

\[
\begin{align*}
  x_{t+1} &= Ax_t + w_t \\
  y_t &= Cx_t + v_t
\end{align*}
\]

**Digital Communication Network**

**ESTIMATOR**

\[
\gamma_k^t = \begin{cases} 
  1 & \text{if } y_k \text{ arrived before or at time } t, \ t \geq k \\
  0 & \text{otherwise}
\end{cases}
\]

\[
\tilde{y}_k = \gamma_k^t (Cx_k + v_k) = C_k^t x_k + u^t
\]

**Kalman time-varying linear system**

\[ \hat{x}_t = \mathbb{E}[x_t | \tilde{y}_1, \ldots, \tilde{y}_t, \gamma_1^t, \ldots, \gamma_t^t] \]
Minimum variance estimation

\[ t = 3 \]

\[ \hat{x}_0 \]

\[ x^+ = A\hat{x} \]
\[ P^+ = APA^T + Q \]

\[ x^+ = A\hat{x} + K_k^t(\hat{y}_k^t - CA\hat{x}) \]
\[ P^+ = APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \]

\( \gamma = 0 \)

\( \gamma = 1 \)

Lyapunov Equation (unstable)

Riccati Equation (stable)
Minimum variance estimation

\[ t = 3 \]

\[
\begin{bmatrix}
  y_2 \\
  y_3 \\
  \hat{x}_1 
\end{bmatrix}
\]

\[ \gamma = 0 \]

\[
\begin{align*}
  \dot{x}^+ &= A\dot{x} \\
  P^+ &= APA^T + Q
\end{align*}
\]

Lyapunov Equation (unstable)

\[ \gamma = 1 \]

\[
\begin{align*}
  \dot{x}^+ &= A\dot{x} + K_k^t(\tilde{y}_k^t - CA\dot{x}) \\
  P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T
\end{align*}
\]

Riccati Equation (stable)
Minimum variance estimation

\[ t = 3 \]
\[ \begin{array}{c}
\text{y2} \\
\text{y3} \\
\hline
\text{\( \hat{x}_2 \)}
\end{array} \]

\[ \gamma = 0 \]
\[ \begin{align*}
\dot{x}^+ &= A\hat{x} \\
P^+ &= APA^T + Q
\end{align*} \]
Lyapunov Equation (unstable)

\[ \gamma = 1 \]
\[ \begin{align*}
\dot{x}^+ &= A\hat{x} + K_k^t(y_k^t - CA\hat{x}) \\
P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T
\end{align*} \]
Riccati Equation (stable)
Minimum variance estimation

\[ t = 3 \]

\[ \begin{array}{c}
  y_2 \\
  y_3 \\
  \tilde{x}_3 \\
\end{array} \]

\[ \gamma = 0 \]

\[ \begin{align*}
  \dot{x}^+ &= A\dot{x} \\
  P^+ &= APA^T + Q
\end{align*} \]

Lyapunov Equation
(unstable)

\[ \gamma = 1 \]

\[ \begin{align*}
  \dot{x}^+ &= A\dot{x} + K_t (\tilde{y}_k - CA\tilde{x}) \\
  P^+ &= APA^T + Q - APCT(CPC^T + R)^{-1}CPAT
\end{align*} \]

Riccati Equation
(stable)
Minimum variance estimation

Lyapunov Equation (unstable)

$$\begin{align*}
\hat{x}^+ &= A\hat{x} \\
P^+ &= APA^T + Q
\end{align*}$$

Riccati Equation (stable)

$$\begin{align*}
\hat{x}^+ &= A\hat{x} + K_k^t (\tilde{y}_k^t - CA\hat{x}) \\
P^+ &= APA^T + Q - APC^T (CPC^T + R)^{-1} CPA^T
\end{align*}$$
Minimum variance estimation

\[
\begin{align*}
\hat{x}_1 &= t = 4 \\
y_2 &\quad y_3
\end{align*}
\]

Lyapunov Equation (unstable)

\[
\begin{align*}
\dot{x}^+ &= A\hat{x} \\
P^+ &= APA^T + Q
\end{align*}
\]

Riccati Equation (stable)

\[
\begin{align*}
\dot{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k - CA\hat{x}) \\
P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T
\end{align*}
\]

\(\gamma = 0\)

\(\gamma = 1\)
Minimum variance estimation

\[ t = 4 \]

Lyapunov Equation (unstable)
\[
\begin{align*}
\dot{x}^+ &= A\dot{x} \\
P^+ &= APA^T + Q
\end{align*}
\]

Riccati Equation (stable)
\[
\begin{align*}
\dot{x}^+ &= A\dot{x} + K_k(\tilde{y}_k^T - CA\tilde{x}) \\
P^+ &= APA^T + Q - APCT(CPCT + R)^{-1}CPAT
\end{align*}
\]
Minimum variance estimation

\[ t = 4 \]

\[ y_2 \quad y_3 \]

\[ \hat{x}_3 \]

\[ \gamma = 0 \]

Lyapunov Equation (unstable)

\[ \hat{x}^+ = A\hat{x} \]
\[ P^+ = APA^T + Q \]

\[ \gamma = 1 \]

Riccati Equation (stable)

\[ \hat{x}^+ = A\hat{x} + K^T_k (\tilde{y}_k - C\hat{x}) \]
\[ P^+ = APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \]
Minimum variance estimation

\[ t = 4 \]

\[
\begin{pmatrix}
y_2 \\
y_3 \\
\hat{x}_4
\end{pmatrix}
\]

\[ \gamma = 0 \]

\[
\hat{x}^+ = A\hat{x} \\
P^+ = APA^T + Q
\]

Lyapunov Equation (unstable)

\[ \gamma = 1 \]

\[
\hat{x}^+ = A\hat{x} + K_k(\tilde{y}_k - CA\hat{x}) \\
P^+ = APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T
\]

Riccati Equation (stable)
Minimum variance estimation

\[ t = 5 \]

\[ y_1 \rightarrow \begin{array}{c} y_1 \ y_2 \ y_3 \\ \hat{x}_4 \end{array} \]

\[ \gamma = 0 \]

\[ \begin{align*}
\hat{x}^+ &= A\hat{x} \\
P^+ &= APA^T + Q
\end{align*} \]

Lyapunov Equation (unstable)

\[ \gamma = 1 \]

\[ \begin{align*}
\hat{x}^+ &= A\hat{x} + K_k^t(y_k^t - CA\hat{x}) \\
P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T
\end{align*} \]

Riccati Equation (stable)
Properties of Optimal Estimator

- Optimal for any arrival process
- Stochastic time-varying gain $K_t = K(\gamma_1, \ldots, \gamma_t)$
- Possibly infinite memory buffer
- Inversion of up to $t$ matrices at any time $t$

\[
\begin{align*}
\hat{x}_t &= A\hat{x} + \gamma_t y_t P C^T (C P C^T + R)^{-1} (\bar{y}_t - C A \hat{x}) \\
P^{-1} &= A P A^T + Q - \gamma_{t-N} A P C^T (C P C^T + R)^{-1} C P A^T
\end{align*}
\]
Minimum variance estimation

\[ t = 4 \]

\[ \hat{y}_2 \hat{y}_3 \]

\[ \hat{x}_2 \]

\[ \gamma = 0 \]

\[ \hat{x}^+ = A\hat{x} \]
\[ P^+ = APA^T + Q \]

\[ \gamma = 1 \]

\[ \hat{x}^+ = A\hat{x} + K_k(\hat{y}_k - CA\hat{x}) \]
\[ P^+ = APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \]

Lyapunov Equation (unstable)

Riccati Equation (stable)
Minimum variance estimation

Lyapunov Equation (unstable)

\[
\begin{align*}
\dot{x}^+ &= A\hat{x} \\
P^+ &= APA^T + Q
\end{align*}
\]

Riccati Equation (stable)

\[
\begin{align*}
\dot{x}^+ &= A\hat{x} + K_k^t(y_k^t - CA\hat{x}) \\
P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T
\end{align*}
\]

\[\gamma = 0\]

\[\gamma = 1\]
What about stability and performance?

Additional assumption on arrival sequence necessary: i.i.d. arrival with stationary distribution

\( \tau_k \): delay of packet \( y_k \), \( \tau_k = \infty \) if \( y_k \) never arrives

\[
\begin{align*}
\lambda_h \triangleq & \mathbb{P}[\tau_k \leq h], \\
\lambda_{loss} \triangleq & \mathbb{P}[\tau_k = \infty]
\end{align*}
\]
Optimal estimation with constant gains and buffer finite memory

\[ \{K_h\}_{h=0}^{N-1}, \quad N \text{ static gains} \]

\[ \tilde{x}^+ = A\tilde{x} + \gamma^t_{t-h} K_h (\tilde{y}^t_{t-h} - CA\tilde{x}), \quad h = N - 1, \ldots, 0 \]

- Does not require any matrix inversion
- Simple to implement
- Upper bound for optimal estimator: \( P_t \leq \tilde{P}_{t|t} \implies \mathbb{E}_\gamma[P_{t|t}] \leq \mathbb{E}_\gamma[\tilde{P}_{t|t}] = \overline{P}_{t|t} \)
- \( N \) is design parameter

GOAL: compute \( \overline{P}_{t|t} \)
Suboptimal minimum variance estimation

\[ t = 3 \]

\[
\begin{bmatrix}
K_3 & K_2 & K_1
\end{bmatrix}
\]

\[ y_2 \ y_3 \]

\[ \hat{x}_1 \]

\[ \gamma = 0 \]

\[ \hat{x}^+ = A\hat{x} \]

Open loop

\[ \gamma = 1 \]

\[ \hat{x}^+ = A\hat{x} + K_l(\tilde{y}_k - CA\hat{x}) \]

Closed loop
Suboptimal minimum variance estimation

\[ t = 4 \]

Lyapunov Equation (unstable)

\[ \gamma = 0 \]

\[ \hat{x}^+ = A\hat{x} \]

Riccati Equation (stable)

\[ \gamma = 1 \]

\[ \hat{x}^+ = A\hat{x} + K_l(\tilde{y}_k^t - CA\hat{x}) \]
Steady state estimation error

**Fixed gains:**

\[ \mathcal{L}_\lambda(K, P) = \lambda A(I-KC)P(I-KC)^T A^T + (1-\lambda) APA^T + Q + \lambda AKR K^T A^T \]

\[
\bar{P} = \mathcal{L}_{\lambda_{N-1}}(K_{N-1}, \bar{P}) \\
\bar{P}^+ = \mathcal{L}_{\lambda_k}(K_k, \bar{P}), \quad k = N-2, \ldots, 0 \\
\lim_{t \to \infty} \bar{P}_{t|t} = \bar{P}
\]

**Optimal fixed gains:**

\[ \Phi_\lambda(P) = APA^T + Q - \lambda APCT^T (CPC^T + R)^{-1} CP A^T \]

Modified Algebraic Riccati Equation (MARE)

\[ \Phi_{\lambda_{N-1}}(P_{N-1}) = \Phi_{\lambda_{N-1}}(P_{N-1}) \]

\[ \Phi_{\lambda_k}(P_{k+1}), \quad k = N-2, \ldots, 0 \]

\[ K_k = P_k C^T (CP_k C^T + R)^{-1} \]

(offline computation)
Numerical example (1)

Discrete time linearized inverted pendulum:

\[ A = \begin{bmatrix} 1.01 & 0.05 \\ 0.05 & 1.01 \end{bmatrix}, \quad C = [1 \ 0], \quad R = 1, \quad Q = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 1 \end{bmatrix} \]
Time-varying arrival probability distribution

\[ \lambda^1 \quad 0 \leq t \leq 50 \]

\[ \lambda^2 \quad t > 50 \]
Back to the control problem

Actuators
\[ u_t^a = u_t^c \]

Plant
\[ x_{t+1} = Ax_t + Bu_t^a + w_t \]

Sensors
\[ y_t = Cx_t + v_t \]

State feedback
\[ u_t^c = L\hat{x}_t \]

Static Kalman filter
\[ \hat{x}_t = A\hat{x}_{t-1} + B u_{t-1}^c + K(y_t - C\hat{x}_t) \]
Back to the control problem

\[ u_t^c = u_t^c \]

**Actuators**

**Plant**

\[ x_{t+1} = Ax_t + Bu_t^a + w_t \]

**Sensors**

\[ y_t = Cx_t + v_t \]

\[ z^{-1} \]

**State feedback**

\[ u_t^c = L\hat{x}_t \]

**Time-varying Kalman filter w/ memory**

\[ \hat{x}_t = A\hat{x}_{t-1} + Bu_{t-1}^c + K_t(y_t - C\hat{x}_t) \]

\[ \hat{x}_t = E[x_t|y_t, y_{t-1}, \ldots, y_0, u_{t-1}^a, \ldots, u_1^a] \]

if \( u_{t-1}^c \neq u_{t-1}^a \) \( \implies e_t = x_t - \hat{x}_t = f(y_t, \ldots, y_0, u_t^c, \ldots, u_0^c, u_1^a, \ldots, u_1^a) \)

\[ P_{t|t-1} = AP_{t-1|t-1}A^T + Q + B(u_{t-1}^a - u_{t-1}^c)(u_{t-1}^a - u_{t-1}^c)^TB^T \]

Estimation error coupled with control action \( \Rightarrow \) no separation principle
LQG over TCP-like (ACK-based) protocols

Actuators

\[ u^a_t = \begin{cases} u^c_t & \nu_t = 1 \\ 0 & \nu_t = 0 \end{cases} \]

Plant

\[ x_{t+1} = Ax_t + Bu^a_t + w_t \]

Sensors

\[ y_t = Cx_t + v_t \]

Packet loss

\[ \nu_t \]

State feedback

\[ u^c_t = L \nu \tilde{x}_t \]

Time-varying Kalman filter w/ memory

\[ \tilde{x}_t = A \tilde{x}_{t-1} + Bu^a_{t-1} + K_t (y_t - C \tilde{x}_t) \]

Separation principle hold (I know exactly \( u^a_{t-1} \))

\( \nu_t \) Bernoulli rand. var and independent of observation arrival process

Static state feedback, \( L \) solution of dual MARE
LQG over UDP-like (no-ACK) protocols

- LQG problem still well defined: \( \min_{u^c_1, \ldots, u^c_t} E[\sum_{h=1}^{t} x_t^T W x_t + (u^a_t)^T U u^a_t] \)
- No separation principle hold (\( u^{a}_{t-1} \) NOT known exactly)
- ... but still have some statistical information about \( u^{a}_{t-1} \)
LQG over UDP-like (no-ACK) protocols

- Bernoulli arrival process $P[\nu_t = 1] = \bar{\nu}, P[\gamma_t = 1] = \bar{\gamma}$
- $\bar{\nu}u^c_t = E[u^a_{t-1}]$
- Sub-optimal controller forced to be state estimator+state feedback
- Optimal choice of $K,L$ is unique solution of 4 coupled Riccati-like equations

"Compensability and Optimal Compensation of systems with white parameters", De Koning, TAC'92
LQG as optimization problem

\[ \text{Min}_{K,L} \quad \text{Trace} \left( \begin{bmatrix} W & 0 \\ 0 & \bar{\nu} L^T U L \end{bmatrix} P \right) \]

\[ s.t. \quad P = \mathbb{E} \left[ \begin{bmatrix} A & -\nu_k B L \\ \gamma_k K C & A - \bar{\nu} B L - \gamma_k K C \end{bmatrix} P \begin{bmatrix} A & -\nu_k B L \\ \gamma_k K C & A - \bar{\nu} B L - \gamma_k K C \end{bmatrix}^T \right] + \begin{bmatrix} Q & 0 \\ 0 & \bar{\gamma} K R K^T \end{bmatrix} \]

- Non convex problem even for \( \nu_k = 1 \), i.e. classic LQG
- Classic and TCP-based LQG become convex when exploiting optimality conditions like uncorrelation between estimate and error estimate \( \mathbb{E} [x(x - \hat{x})^T] = 0 \)
- For UDP-like problem non convex but unique solution using Homotopy and Degree Theory (DeKoning, Athans, Bernstein) (maybe using Sum-of-Squares?)
- Stability on \( \gamma \) and \( \nu \) is coupled
Side note: Kalman filter is not always optimal!

- Kalman filter always gives smallest estimate error regardless of controller $L$
- If controller $L \neq L_{LQ}$, then performance improves if my estimate is “bad”!
Numerical example: TCP vs UDP
To hold or to zero control input?

Most common strategy:

\[ g(u_{t-1}^a) = 0 \]  
zero-input strategy \((\text{mathematically appealing})\)

\[ g(u_{t-1}^a) = u_{t-1}^a \]  
hold-input strategy \((\text{most natural})\)
To hold or to zero control input: no noise (jump linear systems)

**Zero-input Strategy**

\[ u_k^p = \nu_k u_k^c \]
\[ x_{k+1} = Ax_k + Bu_k^p \]
\[ u_k^c = L_z x_k \]

\[ J_z^* = \min_{L_z} E[\sum_{t=1}^{\infty} x_t^T W x_t + (u_t^a)^T U u_t^a] \]

**Hold-input Strategy**

\[ u_k^p = \nu_k u_k^c + (1 - \nu_k)u_{k-1}^p \]
\[ x_{k+1} = Ax_k + Bu_k^p \]
\[ u_k^d = L_h x_k \]

\[ J_h^* = \min_{L_h} E[\sum_{t=1}^{\infty} x_t^T W x_t + (u_t^a)^T U u_t^a] \]

Using cost-to-go function (dynamic programming)

\[ J_z^* = E[x_0^T S_z x_0] \]
\[ J_h^* = E[x_0^T S_h x_0] \]

\[ S_z = \Phi_z(S_z) \rightarrow \text{Riccati-like equation} \rightarrow S_h = \Phi_h(S_h) \]
\[ L_z^* = f_z(S_z) \]
\[ L_h^* = f_h(S_h) \]
Example: unstable scalar system

- $A=1.2, U=0$ (fastest convergence)
- $A=1.2, U=10$ (small input)
Conjecture:

- Separation principle hold
- Optimal function \( g(u_{t-1}^a) = \rho u_{t-1} \)
- Design parameter \( L, l, \rho \) obtained via LQ-like optimal state feedback
"Optimal LQG control across a packet-dropping link", Gupta, Spanos, Murray, Submitted to Sys.Cont.Lett. 05
"Estimation under controlled and uncontrolled communications in networked control systems", Xu, Hespanha, CDC 05

Smart sensors & smart actuators

Actuators

Plant

Sensors

\[ u^a_t = ? \]

\[ x_{t+1} = Ax_t + Bu^a_t + w_t \]

\[ y_t = Cx_t + v_t \]

\[ \hat{x}_t \]

\[ \hat{x}_t = E[x_t|y_6, y_5, \ldots, y_1] = E[x_7|x_6] \]
Numerical example: remote vs co-located controller
Takeaway points

- Input packet loss more dangerous than measurement packet loss
- TCP-like protocols help controller design as compared to UDP-like (but harder for communication designer)
- If you can, place controller near actuator
- If you can, send estimate rather than raw measurement
- Zero-input control seems to give smaller closed loop state error ($||x_t||$) than hold-input (but higher input)
- Trade-off in terms of performance, buffer length, computational resources (matrix inversion) when random delay
- Can help comparing different communication protocols from a real-time application performance
Future work

- Multiple sensors:
  - data fusion, i.e. $y_1, \ldots, y_m$ arrive at different times
  - distributed estimation & consensus $E[x|y_1, \ldots, y_N] = E[x|\hat{x}_{s_1}, \hat{x}_{s_N}]$
- Multiple actuators
  - trade-off between distributed control & centralized coordination