Review of[5]
Graph Theory and Interconnection Networks
by Lih-Hsing Hsu and Cheng-Kuan Lin
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1 Introduction

Interconnection networks appear in almost all systems where some components communicate: the telephone network and the communication system of a parallel/distributed computing platform are some notable examples. When designing the interconnection network of a system, many different issues should be taken into account, ranging from the physical properties of connections to communication protocols. One of the first problems to deal with is: which topology do we use for connecting nodes? Since an interconnection network can be seen as a graph where vertexes represent nodes and edges the links among them, graph theory has provided a valuable mathematical tool for the design and analysis of the topology of an interconnection network.

There are several ways to interconnect n nodes, and each one has its pros and cons. For example, a complete graph guarantees connectivity even if $n-2$ nodes or links break down, but it requires an high number ($n(n-1)/2$) of links. On the other hand, a linear array requires only $n-1$ links, but just one fault disconnects the system. A number of graphs with interesting properties, regarding for example connectivity and fault tolerance, have been presented in the literature. The reviewed book *Graph Theory and Interconnection Networks* presents the structures and properties of some of these graphs and provides the background in graph theory for understanding them.

As the title suggests, the book is divided into two parts. The first one covers some classical notions and results in graph theory, such as graph coloring, matching and connectivity. The intent of this part is not to provide a profound coverage of graph theory, but just to put down necessary tools for studying the interconnection networks described in the following chapters. The second part of the book introduces some graph properties of interest in the design of interconnection networks, in particular connectivity, fault tolerance, Hamiltonian cycles, and diagnosability; these properties are also discussed for some graphs, like hypercubes, star and pancake graphs.

2 Summary

The book consists of 21 chapters which are partitioned into two parts. The first part (Chapters 1–10, pages 1–170) introduces some general notions and results in graph theory, while the second one (Chapters 11–21, pages 171–685) focuses on some graph properties regarding connectivity, fault tolerance, Hamiltonian cycles and diagnosability, and discusses them for some graphs.

Part I. *Chapter 1* is the mandatory chapter introducing the essential terminology, like graph, path and cycle. Some basic results, in particular on vertex degrees, are also given.

Chapters 2 builds on notions given in the previous chapter, and provides some examples of isomorphic graphs and of graphs containing a mesh or an hypercube even when k edges are removed (k-edge fault tolerant graphs).

Chapter 3 deals with the diameters of some graphs: shuffle-cubes, de Bruijn, star and pancake graphs. Few words are also devoted to routing algorithms for the aforementioned interconnections.

Chapter 4 is about trees and covers breadth and depth-first searches, tree traversals, number of (binary) trees.

Chapter 5 is devoted to Eulerian graphs. After describing the classical results, the chapter provides some applications, like the Chinese postman problem.

Chapter 6 is on graph matching and includes perfect matching as well as the more general notion of k-factor.

Chapter 7 deals with different measures of graph connectivity (e.g., k-connectivity, superconnectivity) and analyzes their relations; in particular, the relations between vertex cuts and the number of pairwise vertex-independent paths are shown (Menger's theorem).

Chapter 8 is about graph coloring, in particular vertex coloring: here we find bounds on the chromatic number and properties on color-critical graphs. The chapter closes with some remarks on edge coloring.

Chapter 9 covers Hamiltonian graphs. The chapter describes sufficient and necessary conditions of Hamiltonian graphs, the Hamiltonian-connectivity, and mutual independent Hamiltonian paths.

Chapter 10 closes the first part with some results on planar graphs.

Part II. *Chapter 11* deals with k-fault-tolerant Hamiltonian and k-fault-tolerant Hamiltonian-connected graphs. The first ones are graphs that contain an Hamiltonian circuit even with k edge or vertex faults; the second ones are graphs where each pair of vertexes is connected by an Hamiltonian path even with k faults. After describing some methods for constructing these kinds of graphs, some examples are investigated, including Peterson networks and pancake graphs.

Chapter 12 focuses on the special case of cubic 1-fault-tolerant Hamiltonian graphs. The chapter provides some construction schemes and discusses some graphs, like a variation of the Honeycomb mesh.

Chapter 13 covers k-fault-tolerant Hamiltonian-laceable graphs, that is, bipartite graphs where an Hamiltonian path exists between each pair of vertexes belonging to distinct sets even when k faults occur. This property is an extension of the aforementioned k-fault-tolerant Hamiltonian-connectivity which does not apply to bipartite graphs. As usual, after some general results, the chapter focuses on Hamiltonian-laceable graphs, like hypercubes and star graphs.

Chapter 14 covers k^*-connected graphs, which are graphs where each pair of vertexes is connected by k distinct paths which span all the vertexes (these paths compose a k^*-container of the vertex pair), and the extension to bipartite graphs, named k^*-laceable graphs, which requires the two vertexes to belong to distinct sets. These properties are discussed for some graphs, such as hypercubes and crossed cubes.

Chapter 15 focuses on cubic 3^*-connected and 3^*-laceable graphs, investigating their relations with 1-fault-tolerant Hamiltonian and Hamiltonian-laceable graphs.

Chapter 16 deals with the k^*-diameter of k^*-connected graphs. The length of a k^*-container is the length of its longest path, and the k^*-distance of a vertex pair is the minimum among the lengths of all the k^*-containers associated with the pair. Then, the k^*-diameter is defined as the maximum k^*-distance between any two vertexes. The chapter investigates the k^*-diameter of star graphs and hypercubes.

Chapter 17 deals with pancyclic and panconnected graphs and their bipartite versions: a graph of n vertexes is pancyclic if it contains a cycle of length ℓ, for each ℓ from 3 to n; a graph is panconnected if between each pair of two distinct vertexes there exists a path of length ℓ, for each ℓ ranging from their distance to n. As usual, these properties are discussed for some interconnections, as hypercubes and augmented cubes.

Chapter 18 is devoted to k-mutually independent Hamiltonian graphs, which are graphs where k mutually independent Hamiltonian cycles start in each vertex of the graph. The chapter provides bounds on k for many graphs, like hypercubes, pancake and star graphs.

Chapter 19 deals with k-mutually independent Hamiltonian-connected graphs, that is, graphs where each pair of vertexes is connected by k mutually independent Hamiltonian paths. As in the previous chapter, bounds on k for different graphs are provided.

Chapter 20 is completely devoted to wrapped butterfly graphs and studies how the aforementioned properties apply to these graphs. Indeed, since a wrapped butterfly of n vertexes is bipartite if and only if n is even, it is preferable to discuss its topological properties in a unique chapter.

The conclusive *Chapter 21* covers the diagnosability of a network, that is, how the network can identify all the faulty nodes. The chapter describes two models for self-diagnosis, namely the Preparata, Metze and Chien model and the Maeng and Malek model. Then, some graphs are studied under both models.

3 Opinion

As evidenced by the Summary, *Graph Theory and Interconnection Networks* is almost devoted to advanced properties of some graphs representing interconnection networks. The first part provides only the basic background for handling the second one, and hence some graph theoretical topics may not be found in the book, such as Ramsey theory and random graphs (see, e.g., [1]). Also, the book does not cover, with few exceptions, classical interconnections like linear arrays and meshes, parallel and routing algorithms, performance evaluation and deadlocks (see, e.g., [3, 2]).

In my opinion, the main target of the book is a veteran researcher working on the topologies of interconnection networks. This reader may use this book as a reference where looking for results concerning the topological properties of some interconnections, such as hypercubes, butterflies, star and pancake graphs. The book may be also used by a researcher wishing to begin working on interconnection topologies since the first part provides the background required in the whole book. However, this reader may find some difficulties due to the lack of exercises and non-technical sections describing some concepts and results.

A remarkable feature of this book is the generous use of figures for explaining concepts and proofs. Furthermore, I also appreciate its completeness, since almost all of the statements are proved and suitable references to the literature are provided. Nevertheless, there are some aspects of the book that in my opinion should be improved in a future edition. The first one is that looking for a result in the book is not simple since the index is incomplete and there is no symbol index, which may be useful when you forget the meaning of a symbol. The second aspect is that there should be more non-technical sections with the purpose of describing at high level concepts and results and of giving an overview of the content presented in a chapter: these sections may be useful for newbie researchers in interconnection networks.

References

[1] Reinhard Diestel. *Graph Theory (Graduate Texts in Mathematics)*. Springer, 2005.

[2] Jose Duato and Sudhakar Yalamanchili and Lionel M. Ni. *Interconnection Networks*. M. Kaufmann Publishers, 2002.

[3] Frank Thomson Leighton. *Introduction to parallel algorithms and architectures: arrays, trees, hypercubes*. M. Kaufmann Publishers, 1992.