Transmit Gain Optimization for Space Time Block Coding Wireless Systems with Co-channel Interference

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ABSTRACT

The combination of Orthogonal Frequency Division Multiplexing and space-time block coding is a promising technique for wireless broadband transmission. In a scenario where other devices generate interference, we propose a scheme where the transmit gains of each OFDM subchannel are adaptively chosen. As a design criteria we consider both the minimization of the interference and maximization of the signal to interference plus noise ratio at the detection point. As a particular case we consider also the situation of varying only the amplitude or the phase of the gains. Indeed, it turns out that when interference is present, an important role is played by the phase of the transmit gains, and for the case of two transmit antennas we derive the optimum phase of the transmit gains, under the assumption of equal amplitudes. As performance measure we used the achievable bit rate of the various solutions for a broadband indoor system denoted Windflex (European Project). Performance was compared also with the system capacity obtained by a novel close form expression.

I. INTRODUCTION

Space diversity has been recently considered with a growing interest for its ability to significantly improve the performances of wireless communications in non dispersive fading channels. In particular, space-time block coding (STBC) is attractive as a simple and effective technique that benefits from spatial diversity. First introduced by Alamouti for a communication system with up to two receive and transmit antennas [6], STBC was further generalized for a larger number of antennas [7].

At the same time, the need of high bit rates favors broadband communications, where the transmission channel is dispersive. The benefits of both spatial and frequency diversity can be easy achieved by the combination of STBC and orthogonal frequency division multiplexing (OFDM) [9], which divides the broadband channel into a number of orthogonal signals, which are modulated on equally spaced subcarriers. The combined use of STBC and OFDM has been recently considered for the deployment of wireless indoor networks in the European Wind-Flex project [8]. In these networks the devices are organized into synchronous piconets which can potentially interfere with each other and thus limit considerably the network throughput.

In a STBC OFDM system, according to the particular condition of both the channel and the interfering signals, adaptation of the antenna gains could be done for each of the OFDM subcarriers. However, a fully optimized system turns out to be exceedingly complex, hence we focus our investigation only on the transmit gain adaptation. Although in general, the transmit gains assume complex values, to limit complexity we also consider cases where gains have the same phase or the same amplitude. Moreover, in order to limit complexity, the receiver adopts maximum ratio combining whose optimization depends only on the channel and not on the interference signal. Within this framework, we consider two cost functions for the choice of the transmit gains, namely minimization of the power of the interference at the receiver (MI), and maximization of the signal to noise plus interference ratio.

In order to have an upper bound on the system performance we derive a novel expression of the capacity of a system with two receive antennas with adaptive transmit gains. In fact, previous results are limited to the system where each transmitted signal is the linear combination of all space-coded data [11].

Simulation results for the Wind-Flex scenario show that indeed there is a significant tradeoff between performance and computational complexity of the various solutions.

II. SYSTEM DESCRIPTION

An OFDM wireless system is considered, where data of each subcarrier is coded by a space-time block code and transmitted by N_t transmit antennas. The receiver is equipped with N_r receive antennas and it receives both the useful signal and interference generated by N interferers. We assume that the interferers use OFDM and are synchronous with the useful transmitter. Hence, by assuming that the cyclic prefix [9] is sufficiently long, the transmission and the interference channels are flat on each OFDM subcarrier. Note that if the interference signals are not synchronous, the cyclic prefix may not absorb the delays of all devices and both intersymbol interference and intercarrier interference will be present.

We indicate with $H_{k,\ell}^{(m)}$ the frequency response of the transmit channel from antenna k to antenna ℓ of the m-th OFDM subcarrier. With $G_{k,\ell}^{(m)}$ we indicate frequency response of the interference channel from the k-th interference antenna to the ℓ-th receive antenna of the m-th OFDM subcarrier. Perfect knowledge of the useful and interference frequency responses is assumed.

Before transmission, the coded data is scaled by the complex gain $\alpha_t^{(m)}$, for each transmit antenna $t = 1, 2, \ldots, N_t$ and each OFDM subcarrier $m = 0, 1, \ldots, M-1$. In order to set a constraint on the transmit total power, it must be

$$\sum_{t=1}^{N_t} |\alpha_t^{(m)}|^2 = 1\,. \tag{1}$$

Since the choice of the transmit gain is independent of the subcarrier, in the following we will omit the index (m).

The data signal is coded by space-time block coding, according to the schemes of [6, 7], and at the receiver maximum ratio combining (MRC) of the received signals is applied, according to the channel coefficients and the transmit gains. In particular, by indicating with $r_t^{(q)}$ the received signal at time t on the antenna q, the k-th transmitted signal of the s-th block is obtained by linear processing as

$$\tilde{u}_k^{(s)} = \sum_{t=1}^{N_t} \sum_{q=1}^{N_r} H^*_{\epsilon_t(k),q} \alpha^*_{\epsilon_t(k)} \delta_t(k) r^{(q)}_{s+t}\,, \tag{2}$$

where for each k, $\epsilon_q(k)$ is a permutation function of the indexes $\{1, 2, \ldots, N_r\}$ and $\{\delta_q(k)\}$ depend on the code. For example, for orthogonal design codes $\delta_q(k) \in \{-1, +1\}$, [7]. In the following, without loss of generality we will assume $s = 0$ and we will drop the indexes (s) and (k).

After the MRC, from (2) the power of the useful signal is

$$\sigma_u^2 = \left(\sum_{t=1}^{N_t} |\alpha_t|^2 \sum_{r=1}^{N_r} |H_{t,r}|^2 \right)^2 . \tag{3}$$

while by indicating with $i_t^{(r)}$ the interference signal received at time t on the r-th receive antenna, the power of the residual interference is

$$\sigma_i^2 = \mathrm{E}\left[\left| \sum_{t=1}^{N_t} \sum_{r=1}^{N_r} i_t^{(r)} \alpha^*_{\epsilon_t} H^*_{\epsilon_t,r} \delta_t \right|^2 \right] . \tag{4}$$

We indicate with σ_w the noise variance on each antenna of each subchannel, before combining.

Hence, the signal to noise plus interference ratio (SNIR) is given by

$$\Gamma = \frac{\sigma_u^2}{\sigma_w^2 \sigma_u + \mathrm{E}\left[\left| \sum_{t=1}^{N_t} \sum_{r=1}^{N_r} i_t^{(r)} \alpha^*_{\epsilon_t} H^*_{\epsilon_t,r} \delta_t \right|^2 \right]} . \tag{5}$$

III. TRANSMIT GAIN SELECTION

According to the information available at the transmitter and the overall complexity of the device, different criteria for the choice of the transmit gains may be considered.

As a first option we investigate the minimization of the interference (MI), regardless of the noise. However, this choice may decrease the power of the useful signal at the detection point and hence in general we consider as cost function the maximization of the SNIR (MSNIR).

As a reduced complexity solution we consider also the choice of transmit gains with equal amplitude (EA) or equal phase (power adaptation, EP). For both cases we adopt the MSNIR criterion.

A. Minimum interference (MI)

If the interference is the limiting factor for the communication, a reasonable target for the choice of the transmit gains is the minimization of the residual interference. In order to minimize (4) under constraint (1), we apply the Lagrange multiplier method. Let's indicate with f_m the inverse function of ϵ_t, i.e.

$$\epsilon_{f_m} = m\,. \tag{6}$$

By defining the matrix $\boldsymbol{B}$ with entries

$$[B]_{\ell,m} = \sum_{r=1}^{N_r} \sum_{q=1}^{N_r} \mathrm{E}\left[i_{f_\ell}^{(r)*} i_{f_m}^{(q)} \right] H^*_{m,r} \delta_{f_m} H_{\ell,q} \delta_{f_\ell}\,, \tag{7}$$

and the vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots \alpha_{N_t}]$ collecting the N_t transmit gains, the interference power (5) can be written in the quadratic form

$$\mathrm{E}\left[\left| \sum_{t=1}^{N_t} \sum_{r=1}^{N_r} i_t^{(r)} \alpha^*_{\epsilon_t} H^*_{\epsilon_t,r} \delta_t \right|^2 \right] = \boldsymbol{\alpha}^* \boldsymbol{B} \boldsymbol{\alpha}\,. \tag{8}$$

Then the minimization problem is solved by the following linear system of equations

$$\boldsymbol{B}\boldsymbol{\alpha} + \lambda \boldsymbol{\alpha} = 0\,, \tag{9}$$

under the constraint (1). From (9) we conclude that the minimization of the interference is archived when $\boldsymbol{\alpha}$ is the eigenvector of $\boldsymbol{B}$ corresponding to the minimum eigenvalue of $\boldsymbol{B}$.

Note that if the minimum eigenvalue of $\boldsymbol{B}$ is zero, then the interference can be completely canceled.

B. Maximum signal to noise plus interference ratio (`MSNIR`*)*

The minimization of the interference can lead to poor performance when the interference has a similar propagation characteristic of the useful channel, since the resulting received useful signal may also be particularly attenuated. Hence we consider here the more general target of maximizing the `SNIR` Γ under the constraint (1).

By applying the Lagrange multiplier method to (5) under the constraint (1) a non-linear system of equations is obtained. In order to find a solution we observe that by multiplying all transmit gains by a constant real positive value c^2, Γ is multiplied by c. Hence, in order to find the solution under the constraint (1) first a set of transmit gains $\{\tilde{\alpha}_t\}$ which maximize Γ is found and then (1) is satisfied by setting

$$\alpha_t = \frac{\tilde{\alpha}_t}{\sum_{t=1}^{N_t} |\tilde{\alpha}_t|^2} \, . \tag{10}$$

In order to maximize (5) we minimize its denominator

$$\sigma_w^2 \left(\sum_{t=1}^{N_t} |\tilde{\alpha}_t|^2 \sum_{r=1}^{N_r} |H_{t,r}|^2 \right) + \mathrm{E}\left[\left| \sum_{t=1}^{N_t} \sum_{r=1}^{N_r} i_t^{(r)} \alpha_{\epsilon_t}^* H_{\epsilon_t,r}^* \delta_t \right|^2 \right]$$

under the constraint that the numerator is a constant, i.e.

$$\sum_{t=1}^{N_t} |\tilde{\alpha}_t|^2 \sum_{r=1}^{N_r} |H_{t,r}|^2 = 1 \, . \tag{11}$$

Now, by defining the vector $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_{N_t}]$ with entries

$$\beta_n = \tilde{\alpha}_n \sqrt{\sum_{r=1}^{N_r} |H_{n,r}|^2} \tag{12}$$

and the matrix $\boldsymbol{A}$ with entries

$$[A]_{\ell,m} = \frac{[B]_{\ell,m}}{\sqrt{\sum_{r=1}^{N_r} |H_{\ell,r}|^2}} \, , \tag{13}$$

the Lagrange multiplier method yields the following system of equations

$$\boldsymbol{A}\boldsymbol{\beta} + \lambda \boldsymbol{\beta} = 0 \, , \tag{14a}$$

$$\sum_{t=1}^{N_t} |\beta_t|^2 = 1 \, . \tag{14b}$$

Hence, first we need to find the eigenvector $\boldsymbol{\beta}$ corresponding to its minimum eigenvalue of $\boldsymbol{A}$, then the coefficients $\{\tilde{\alpha}_n\}$ can be computed by (12). Lastly, in order to satisfy the constraint (1), we normalize $\{\tilde{\alpha}_n\}$ by (10).

Note that if the minimum eigenvalue is null, then there is no interference at the decision point and the `MSNIR` criterion is equivalent to the maximization of σ_u as given by (5). In this case, Γ is maximized by allocating all the power to the transmit antenna t with the maximum value of

$$\sum_{r=1}^{N_r} |H_{q,r}|^2 \, , \quad q = 1, 2, \ldots, N_t \, . \tag{15}$$

We examine now two particular cases for the transmit gains.

C. Equal phase (`EP`*)*

When only the gain amplitude adaptation is considered, this is equivalent to assume that $\{\alpha_t\}$ are real numbers. In this case, we maximize (5) under the constraint (1) and we consider only the real solution for the transmit gains. Hence, the transmit gains that solves the problem is the solution of the linear system of equations

$$\mathrm{Re}[\boldsymbol{A}]\boldsymbol{\beta} + \lambda \boldsymbol{\beta} = 0 \, , \tag{16}$$

where $\boldsymbol{A}$ and $\boldsymbol{\beta}$ are defined by (13) and (12), respectively. The linear system (16) must be solved under the constraint (1). In this case, the solution $\boldsymbol{\beta}$ is the eigenvector corresponding to the minimum eigenvalue of $\mathrm{Re}[\boldsymbol{A}]$.

D. Equal amplitude (`EA`*)*

We consider here the adaptation of only the phase of the transmit gains, i.e.

$$\alpha_t = \frac{e^{j\theta_t}}{\sqrt{N_t}} \, , \quad t = 1, 2, \ldots, N_t. \tag{17}$$

From (3) we note that by forming an equal gain amplitude, the power of the received user signal is independent of the transmit gains and the `MI` and the `MSNIR` criteria yield the same solution. Additionally, from (8) we have that it is not restrictive to set $\theta_1 = 0$.

Now, by imposing the constraint (17) to (8), we obtain a problem which in general does not have a close form solution, to the author's knowledge. However, a close form solution for the case $N_t = 2$ is straightforward. From (8), the interference power is minimized by minimizing the cost function

$$([\boldsymbol{B}]_{1,1} + [\boldsymbol{B}]_{2,2}) + 2|[\boldsymbol{B}]_{1,2}| \cos(\theta_1 + \angle[\boldsymbol{B}]_{1,2}) \, . \tag{18}$$

Hence the solution is

$$\theta_1 = \cos^{-1}\left(\frac{2|[\boldsymbol{B}]_{1,2}|}{[\boldsymbol{B}]_{1,1} + [\boldsymbol{B}]_{2,2}} \right) - \angle[\boldsymbol{B}]_{1,2}. \tag{19}$$

IV. **CAPACITY CONSIDERATIONS**

As an upper bound on the performance of a STBC with adaptive transmit gains, we give the capacity that can be achieved by a multi antenna system with adaptive transmit gains and when interference is present.

Let's define the matrix $\boldsymbol{H}$ having as entries $\{H_{k,n}\}$ for $k = 1, 2, \ldots, N_r$, $n = 1, 2, \ldots, N_t$, and let's denote with $\boldsymbol{R}_i$ the $N_r \times N_r$ autocorrelation matrix of the interference. Let's also indicate with $\boldsymbol{T}$ the $N_t \times N_t$ diagonal matrix having as entries $\{\alpha_n\}$.

From [1], the capacity of the considered multi antenna system is given by

$$C = \log_2 \frac{\det[\Gamma \boldsymbol{R}_i + \boldsymbol{I}_{N_r} + \Gamma \boldsymbol{H}\boldsymbol{T}\boldsymbol{T}^H \boldsymbol{H}^H]}{\det[\Gamma \boldsymbol{R}_i + \boldsymbol{I}_{N_r}]}, \quad (20)$$

Since the denominator of C in (20) does not depend on $\boldsymbol{T}$, the maximization of C with respect to $\boldsymbol{T}$ yields the following problem

$$\max_{\boldsymbol{T}} \log_2\{\det[\boldsymbol{I}_{N_r} + \Gamma(\boldsymbol{R}_i + \boldsymbol{H}\boldsymbol{T}\boldsymbol{T}^H\boldsymbol{H}^H)]\} \quad (21a)$$

$$\text{trace}\, \boldsymbol{T}\boldsymbol{T}^H = 1\,. \quad (21b)$$

In [11] Farrokhi *et al.* computed the matrix $\boldsymbol{T}$ that solve the above problem in the case $\boldsymbol{T}$ is not constrained to be diagonal. In this general case, (21) can be rewritten as

$$\max_{\boldsymbol{T}} \log_2\{\det(\boldsymbol{I}_{N_r} + \Gamma \tilde{\boldsymbol{H}}\boldsymbol{T}\boldsymbol{T}^H \tilde{\boldsymbol{H}}^H)\} \quad (22)$$

and the solution is attained by diagonalizing $\tilde{\boldsymbol{H}}\boldsymbol{T}\boldsymbol{T}^H\tilde{\boldsymbol{H}}^H$. Hence, by indicating with $\tilde{\boldsymbol{H}} = \boldsymbol{V}\boldsymbol{W}\boldsymbol{U}$ the SVD of $\tilde{\boldsymbol{H}}$, the optimum transmit matrix that maximizes the capacity is $\boldsymbol{T} = \boldsymbol{U}^H\boldsymbol{\Xi}$ where $\boldsymbol{\Xi}$ is a diagonal matrix with entries computed according to the water-filling principle [11].

Unfortunately, when we force $\boldsymbol{T}$ to be diagonal, the matrix $\tilde{\boldsymbol{H}}\boldsymbol{T}\boldsymbol{T}^H\tilde{\boldsymbol{H}}^H$ cannot be diagonalized and for the a system with any number of transmit antennas there is no a close solution to the problem, to the authors' knowledge. However, for the interesting case of $N_t = 2$ and a general number of receive antennas, we derive the transmit gains that maximizes the capacity.

By using the property $\det[\boldsymbol{I} + \boldsymbol{A}\boldsymbol{B}] = \det[\boldsymbol{I} + \boldsymbol{B}\boldsymbol{A}]$, the equation (22) can be rewritten as

$$\max_{\boldsymbol{T}} \log_2\{\det[\boldsymbol{I}_2 + \boldsymbol{Q}\boldsymbol{T}\boldsymbol{T}^H]\}\,, \quad (23)$$

where $\boldsymbol{Q} = \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}}$ is a 2×2 matrix with entries $[\boldsymbol{Q}]_{n,m}$, $m, n = 1, 2$. By applying the Lagrange multiplier method to (23) under the constraint (1), we obtain the system of equations

$$[\boldsymbol{Q}]_{1,1}\alpha_1^* + \det[\boldsymbol{Q}]|\alpha_2|^2\alpha_1^* + \lambda\alpha_1^* = 0 \quad (24a)$$

$$[\boldsymbol{Q}]_{2,2}\alpha_2^* + \det[\boldsymbol{Q}]|\alpha_1|^2\alpha_2^* + \lambda\alpha_2^* = 0\,, \quad (24b)$$

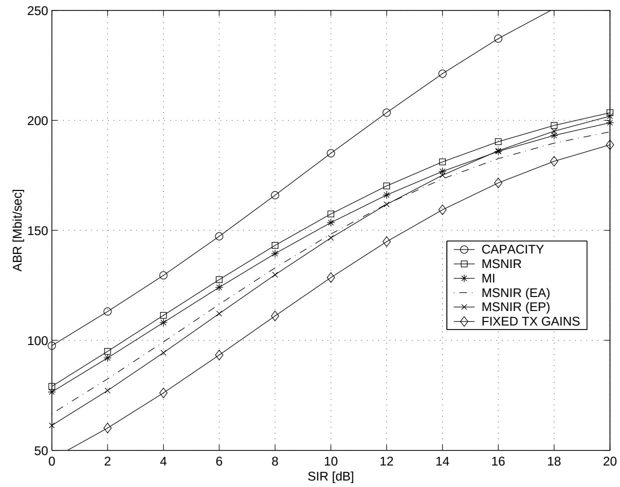

Fig. 1. Achievable bit rate as a function of the signal to interference ratio (SIR), for different transmit selection schemes. The average SNR at the channel output is 10dB.

where λ is the Lagrange multiplier. When

$$\frac{|[\boldsymbol{Q}]_{1,1} - [\boldsymbol{Q}]_{2,2}|}{\det[\boldsymbol{Q}]} \leq 1 \quad (25)$$

the transmit gains that maximize the capacity are given by

$$|\alpha_1|^2 = \frac{1}{2} + \frac{[\boldsymbol{Q}]_{1,1} - [\boldsymbol{Q}]_{2,2}}{2\det[\boldsymbol{Q}]} \quad (26a)$$

$$|\alpha_2|^2 = \frac{1}{2} + \frac{[\boldsymbol{Q}]_{2,2} - [\boldsymbol{Q}]_{1,1}}{2\det[\boldsymbol{Q}]}\,. \quad (26b)$$

If (25) is not satisfied, by indicating with $k = \text{argmax}_p\{[\boldsymbol{Q}]_{p,p}\}$ we set $\alpha_k = 1$, while the other gain is zero.

Note that, since only $\boldsymbol{T}\boldsymbol{T}^H$ is present in the capacity expression (20), the phases of the transmit gains do not affect the capacity.

V. **PERFORMANCE COMPARISON**

For the performance comparison we consider the channel model obtained by the measurements of the indoor radio channel at 17 GHz for the Wind-Flex European project [8]. An OFDM system with 64 subcarriers and a cyclic prefix of length 8 was simulated on a line of sight channel, with a transmission bandwidth of 50 MHz, a mean rms delay spread of 27 ns and an average SNR at the channel output of 10 dB. As a performance measure we use the bit rate that can be achieved by the system, assuming perfect channel loading and coding, namely

$$ABR = \frac{1}{T}\sum_{m=0}^{M-1} \log_2(1 + \Gamma_m)\,, \quad (27)$$

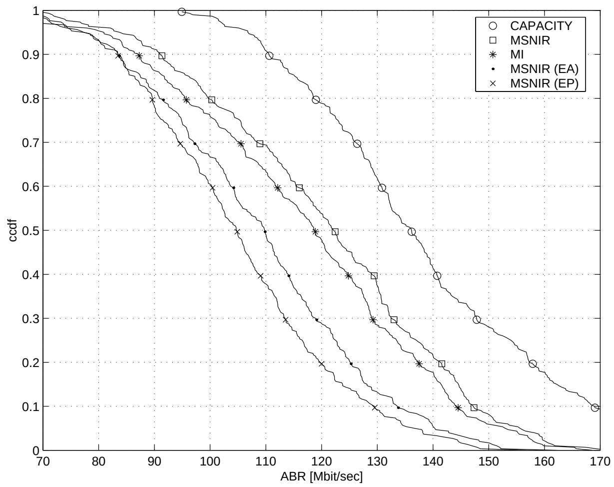

Fig. 2. Complementary cdf of the achievable bit rate for different transmit gains selection schemes. The average SNR is 10dB, while the average SIR is 5dB.

where Γ_m is the SNIR after the combining at the receiver on the m-th OFDM subcarrier. We considered a system with $N_t = N_r = 2$ and $N = 2$.

In the figures we indicate with SIR the signal to interference ratio at the transmitter, i.e. the ratio between the power transmitted by the useful device and the overall power transmitted by the interfering devices, while the transmission channel is assumed to have unitary gain on average.

Fig. 1 shows the ABR as a function of the SIR. For reference, we also plot the performance of the system with fixed transmit gains, $\alpha_1 = \alpha_2 = 1/\sqrt{2}$, indicated with the label `Fixed Tx gains`. From the figure we observe that for a SIR of 10 dB both the EA and the EP solutions outperform by about 3 dB the `Fixed Tx gains` technique, while being only 1 dB poorer than the optimum MSNIR solution.

Fig. 2 shows the complementary cumulative distribution function (ccdf) of the ABR for some schemes, in a scenario with a SIR of 5dB.

VI. CONCLUSIONS

Transmit gain optimization has been derived for STBC systems with a multiple transmit and receive antennas, when co-channel interference is present. The results hold for a receiver device using maximum ratio combining and with perfect knowledge of the channel and interference at the transmitter. Various criteria for the design of the transmit gains were investigated. A close form expression of the capacity of this system has been derived for the case of two transmit antennas. Simulations performed on a Wind-Flex scenario shows that a simple system as the equal amplitude gain method yields a significant improvement of the performance, when compared to a scheme with no adaptation of the transmitter.

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